HYDRAULICS

WITH

WORKING TABLES

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PREFACE TO THIRD EDITION

In this edition the book has been brought thoroughly up to date and subjected to careful and drastic revision. The chief object is, as before, to deal thoroughly with the facts, laws, and principles of Hydraulics, and to keep always in view their practical aspects.

The enormous waste caused by the use of erroneous co-efficients is known to all. Another important object is to remedy this. The use of old and inaccurate figures—as some recent papers show—is not uncommon. Fresh discussions on all the most important co-efficients are now given and specific recommendations are made. A new set of co-efficients for pipes is given.

Numerous examples of practical problems are included, and full sets of tables for working them out.

The large quantity of new and original matter which—as some reviewers were good enough to say—characterised the previous editions is reproduced in an improved form, and fresh matter has been added on weirs and weir-like conditions, on discharge measurement by means of pipe diaphragms, on standing waves and the practical use now being made of them in America, and on the laws governing silting and scour. The difficult question of the surface curve—upstream of weirs, etc.—is made clearer, and a simple method of proceedure, applicable to a vast number of cases and avoiding the use of the erroneous backwater function, is put forward.

Some remarks are made on the best practical forms for the chief formulæ, and also some remarks—much needed—on the practice of basing formulæ on certain selected experiments while rejecting others which are as good or better.

Obsolete matter and needlessly long mathematical investigations are avoided.

It is hoped that the book will meet all the requirements both of the student and of the engineer.

E. S. B.

Guildford,
1st Sept 1920.

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CHAPTER I

INTRODUCTION

SECTION I.—PRELIMINARY REMARKS AND DEFINITIONS

1. **Hydraulics.**—Hydraulics is the science in which the flow of water, occurring under the conditions ordinarily met with in Engineering practice, is dealt with. Based on the exact sciences of hydrostatics and dynamics, it is itself a practical, not an exact, science. Its principal laws are founded on theory, but owing to imperfections in theoretical knowledge, the algebraic formulæ employed to embody these laws are somewhat imperfect and contain elements which are empirical, that is, derived from observation and not from theory. The science of Hydraulics is concerned with the discussion of laws, principles, and formulæ, of such observed phenomena as are connected with them, and of their practical application. The quantities dealt with are generally velocities and discharges, but sometimes they are pressures or energies. It is frequently necessary in Hydraulics to refer to particular works or machines, but this is done to afford practical illustrations of the application of the laws and principles. Descriptions of works or machines form part of Hydraulic Engineering and not of Hydraulics, and the same remark applies to statistical information on subjects such as Rainfall. Some description of Hydraulic Fieldwork is included in this work for reasons given below (chap. ii. art. 25). The laws governing the power of a stream to move solids by rolling or carrying them are intimately connected with the laws of flow and are naturally included.

2. **Fluids, Streams, and Channels.**—A 'fluid' is a substance which offers no resistance to distortion or change of form. Fluids are divided into 'compressible fluids' or 'gases,' such as air, and 'incompressible fluids' or 'liquids,' such as water. Perfect fluids are not met with, all being more or less 'viscous,' that is, offering some resistance, though it may be very small, to change of form. A 'stream' is a mass of fluid having a general movement of

translation. It is generally bounded laterally by solid substances which form its 'channel.' If the channel completely encloses the stream, and is in contact with it all round, as in a pipe running full, it is called a 'closed channel'; but if the upper surface of the stream is 'free,' as in a river or in a pipe running partly full, it is an 'open channel.' An 'eddy' is a portion of fluid whose particles have movements which are irregular and generally more or less rotatory; it may be either stationary or moving with respect to other objects. The 'axis' of a stream or channel is a line centrally situated and parallel to the direction of flow. In an open channel its exact position need not be fixed, but in a pipe it is supposed to pass through the centre of gravity of each cross-section.

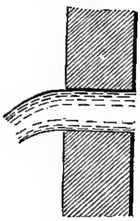


Fig. 1.

An 'orifice' or 'short tube' (Fig. 1) is a short closed channel expanding abruptly, or at least very rapidly, at both its upstream and downstream ends. A short open channel similarly circumstanced (Fig. 2) is called a 'weir,' provided the expansions are wholly or partly in a vertical direction. When they are wholly lateral it is called a 'contracted channel.' All these short channels will collectively be termed 'apertures,' and 'channel' will be used for channels of considerable length.

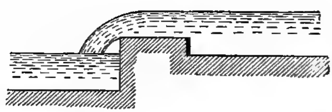


Fig. 2.

The stream issuing from an orifice or pipe is called a 'jet,' that falling from a weir a 'sheet.' Except in the case of a jet issuing under water a stream bounded by other fluid of the same kind is called a 'current.'

3. Velocity and Discharge.—The direction of the flow of a stream is in general parallel to the axis, but it is not always so at each individual point. If at any point the flow is not parallel to the axis, the velocity at that point may be resolved into two components, one of which is parallel to the axis and the other at right angles to it. The component parallel to the axis is termed the 'forward velocity.' A 'cross-section' of a stream is a section at right angles to the axis. The velocities at all points in the cross-section of a stream are not equal. A curve whose abscissas represent distances along a line in the plane of the cross-section, and whose ordinates represent forward velocities, is called a 'velocity curve.' The 'discharge' of a stream at any cross-section is the volume of water passing the cross-section in the unit of time. The 'mean velocity' at any cross-section is the mean of the different forward velocities. It is the discharge of the stream divided by the area of the cross-section. Thus

$$V=\frac{Q}{A} \text{ or } Q=AV \quad . \; . \; . \; (1).$$

This is the first elementary formula of Hydraulics. Except when velocities at individual points are under consideration, the term 'velocity' is generally used instead of 'mean velocity.'

As long as the conditions under which flow takes place at any given cross-section of a stream remain constant, the velocity and discharge are constant, that is, they are the same in succeeding equal intervals of time. In this case the flow is said to be 'steady.' As soon as the conditions change, the velocity and discharge usually change, and the flow is then said to be unsteady. Owing to the introduction or abstraction of water by subsidiary channels, leakage, or evaporation, the discharges at successive cross-sections of a stream may be unequal, but the flow may still be steady. Flow is unsteady only when the discharge varies with the time, and not when it merely varies with the place. In Hydraulics, flow is always assumed to be steady unless the contrary is expressly stated. For instance, in the statement that a rise of surface level gives an increase in velocity, it must be understood that this refers to the period after the surface has risen, and not to that while it is rising. In any length of stream in which the flow is steady, and in which no water is lost or gained, the discharges at all cross-sections are equal, or

$$Q=A_1V_1=A_2V_2=\text{etc.}, \quad . \; . \; . \; (2)$$

where A_1, A_2, etc., are the areas of the cross-sections, and V_1, V_2, etc., the mean velocities. In other words, the mean velocity at any cross-section is inversely as the sectional area.

Section II.—Phenomena observed in Flowing Water

4. Irregular Character of Motion.—In flowing water the free surface oscillates, especially in large and rapid streams. The oscillation is probably greater near the sides than at the centre. The motion of the water is also irregular. Except under peculiar conditions the fluid particles do not move in parallel lines, or 'stream-lines,' but their paths continually cross each other, and the velocity and direction of motion at any point vary every instant. The stream is, in fact, a mass of small eddies. The irregularities of motion increase with the roughness of the channel and with the velocity of the stream. They are especially great in open channels. Eddies produced at the bed are constantly rising to the surface. Floats dropped in at one point in quick succession move neither along the same paths nor with the same velocities. In experiments made by Francis, whitewash discharged into a stream four inches above the bed came to the surface in a length which was equal to ten to thirty times the depth, and was less, the rougher the channel. The eddies are strongest where they originate, namely, at the border of the stream. To compensate for the upward eddies there must, of course, be downward currents, but they are diffused and hardly noticeable. The resistance to flow caused by all these irregular movements is enormously greater than that which would exist in stream-line motion.

Although the velocity and water-level at any point fluctuate every moment as above described, the average values obtained in successive periods of time of longer duration are more or less constant. The velocities obtained at any point in successive seconds will, perhaps, vary by 20 per cent.; those obtained in successive minutes will vary much less; and those in successive periods of five minutes each probably scarcely at all. The same is true of the direction of the flow. For the water-level the averages of several observations obtained in periods of a minute each will probably agree very closely. A velocity curve obtained from a few observations is generally irregular, but one obtained from a large number is regular. If the flow is not steady, the average velocities and water-levels obtained in successive long periods of time may, of course, vary, but they will exhibit a regular change. When velocity and water-level are spoken of, the average values and not the momentary values are meant, and this remark applies to the foregoing definition of steady flow. The discharge at any cross-section, if considered in its momentary aspect, is probably never steady. The irregularity of the motion of water renders the theoretical investigation of flow extremely difficult, and no complete theory has yet been propounded.

5. Contraction and Expansion.—Except under an infinite force, a body cannot, without either coming to rest or describing a curve, change its direction of motion. Acting in obedience to this law, water cannot turn sharp round a corner. Wherever any sharp salient angle *A* or *B* (Fig. 3) occurs in a channel, or at the entrance of an aperture, the water travelling along the lines *GA*, *HB* cannot turn suddenly and follow the lines *AC*, *BD*. It follows the lines *AE*, *BF*, which are curves. At *A* and *B* the radii of the curves may be very small, but the curves doubtless touch the lines *GA*, *HB*. This phenomenon is known as 'contraction.' The stream contracts from *AB* to *EF*. If the channel or aperture extends far enough, the stream expands again and fills it at *MN*, the spaces *AME*, *BNF* containing eddies. These have, however, little or no forward movement, and are not part of the stream. There are also eddies at *K*, *L*. In a case of abrupt enlargement (Fig. 4) the stream expands gradually, and there are eddies in the corners. Similar phenomena occur at abrupt bends, bifurcations, and junctions. For a closed channel or an orifice, Fig. 3 represents any longitudinal section. For an open channel or a weir, it represents a plan or a horizontal section, and its lower part—from *PQR* downwards—a vertical section. And similarly with Fig. 4. Sometimes still or 'dead' water may replace part of an eddy. The term eddy will be used to include it.

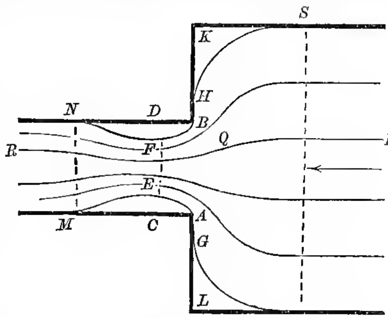


Fig. 3.

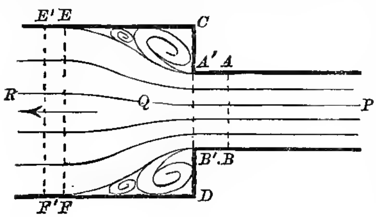


Fig. 4.

Section III.—Useful Figures

6. Weights and Measures.—The following table[1] gives the weight of distilled water for various temperatures. The weights of clear river and spring water are practically the same as the above. For all ordinary practical purposes the weight of fresh water may be taken to be 62·4 lbs. per cubic foot when clear, and 62·5 lbs. or 1000 ounces when containing sediment. Water is compressed by about one twenty-thousandth part of its bulk by a pressure of one atmosphere. Sea-water weighs about 64 lbs. per cubic foot. Water usually contains a small quantity of air in solution.

[1] Smith's *Hydraulics*, chap. i.

Temperature (Fahrenheit.)	Pounds per Cubic Foot.	Temperature (Fahrenheit).	Pounds per Cubic Foot.	Temperature (Fahrenheit).	Pounds per Cubic Foot.
32°	62·42	95°	62·06	160°	61·01
35°	62·42	100°	62·00	165°	60·90
39·3°	62·424	105°	61·93	170°	60·80
45°	62·42	110°	61·86	175°	60·69
50°	62·41	115°	61·79	180°	60·59
55°	62·39	120°	61·72	185°	60·48
60°	62·37	125°	61·64	190°	60·36
65°	62·34	130°	61·55	195°	60·25
70°	62·30	135°	61·47	200°	60·14
75°	62·26	140°	61·39	205°	60·02
80°	62·22	145°	61·30	210°	59·89
85°	62·17	150°	61·20	212°	59·84
90°	62·12	155°	61·11		

An Imperial gallon of water contains $\frac{1}{6\cdot232}$ cubic feet, and weighs almost exactly 10 lbs. A United States gallon is five-sixths of an Imperial gallon. A metre is 3·2809 feet, a cubic metre 35·317 cubic feet, a kilogram 2·2055 pounds avoirdupois, and a litre 61·027 cubic inches or ·2201 gallons. A cubic metre of water weighs 1000 kilograms. The metric system being that chiefly employed on the continent of Europe, these figures may be useful in the conversion of figures given in reports of foreign experiments or investigations. A French inch is ·02707 of a metre or ·0888 of an English foot.

The units employed in this work are the foot, the second, and the pound. Thus velocities and discharges are in feet or cubic feet per second, weights in pounds per cubic foot.

7. **Gravity and Air Pressure.**—The force of gravity, denoted by g, is generally assumed to be 32·2, that is, it is supposed to increase the velocity of a falling body by 32·2 feet per second, and $\sqrt{2g}$, a quantity very frequently occurring in hydraulics, is then 8·025. These figures are suitable for Great Britain and Canada, but the force of gravity varies with the locality, increasing with the latitude and decreasing with the height above sea-level. At the Equator at the sea-level g is 32·09, and at the Pole at the sea-level it is 32·26. The mean values of g and $\sqrt{2g}$ for ordinary elevations and for latitudes up to 70° are 32·16 and 8·02 respectively. These are suitable for the United States, India, and Australia, and are adopted in this work. They, however, differ by only ·12 per cent. and ·06 per cent. respectively from the values given above, and ordinarily this difference is of no account whatever. An increase of elevation of 5000 feet decreases g by only ·016 and $\sqrt{2g}$ by ·002.

The pressure of the atmosphere near the sea-level is about 14·7 lbs. per square inch, and is equivalent to about 30 inches of mercury or 34 feet of water. According to the 'English system' of computation by 'atmospheres,' one atmosphere is equivalent to 29·905 inches of mercury in London at a temperature of 32° Fahrenheit. The French system gives a pressure which is greater in the ratio of 1 to ·9997. For elevations above the sea-level the atmospheric pressure decreases. Up to a height of 6000 feet the reduction for every thousand feet is about ·5 lb. per square inch, or 1 inch of mercury, or 1·13 feet of water. Above 6000 feet the reduction is less rapid, amounting to 1·9 lbs. per square inch in rising from 6000 to 11,000 feet.

Section IV.—History and Remarks

8. Historical Summary.—A historical sketch of Hydraulics given in the *Encyclopædia Britannica* [1] comprises the names of Castelli, Torricelli, Pascal, Mariotte, Newton, Pitot, Bernouilli, D'Alembert, Dubuat, Bossut, Prony, Eytelwein, Mallet, Vici, Hachette, and Bidone. To these may be added Michelotti, D'Aubuisson, Castel, and Borda.

Coming to specific branches of Hydraulics and recent periods, flow in pipes has been made the subject of experiment and investigation by Weisbach, Coulomb, Venturi, Couplet, Darcy, Lampe, Hagen, Poiseuille, Reynolds, Smith, and Stearns, and flow through apertures by Poncelet, Lesbros, Weisbach, Rennie, Blackwell, Boileau, Ellis, Bornemann, Thompson, Francis,[2] Unwin, Fteley and Stearns,[3] Herschel, Steckel, Fanning, and Smith.[4] Many of the experiments on pipes and apertures have been discussed and summarised by Fanning [5] and Smith,[4] both of whom have compiled tables of co-efficients for pipes and apertures. Since then further important experiments have been made on weirs by Bazin,[6] on weirs and pipes by various American engineers,[7] and on orifices, weirs, and pipes by others who are mentioned in chapters iii. to v.

[1] *Encyclopædia Britannica.* 9th Edition. Article 'Hydromechanics.'

[2] *Lowell Hydraulic Experiments.*

[3] *Transactions of the American Society of Civil Engineers*, vol. xii.

[4] *Hydraulics.*

[5] *Treatise on Water Supply Engineering.*

[6] *Annales des Ponts et Chaussées.* 6th Series, Tomes 16 and 19, and 7th Series, Tomes 2, 7, 12, and 15. A *résumé* is given in *L'Écoulement en Déversoir.*

[7] *Transactions of the American Society of Civil Engineers*, vols. xix.,

Regarding flow in open channels, extensive observations and investigations have been made by Darcy and Bazin [1] on small channels, by Humphreys and Abbott [2] on the Mississippi, and by Cunningham [3] on large canals. Many observations have also been made by German engineers and some by Revy [4] on the great South American rivers. In this branch of Hydraulics the Swiss engineers Ganguillet and Kutter have analysed most of the chief experiments,[5] including some made by themselves, and arrived at a series of co-efficients for mean velocity. Their writings have been translated and commented on by Jackson,[6] who has framed tables of co-efficients [7] based on their researches. Finally Bazin has reviewed the whole subject [8] and arrived at some fresh co-efficients. Investigations have been made by Francis [9] on rod-floats, by Stearns [10] on current-meters, and by Kennedy [11] on the silt-transporting power of streams.

9. **Remarks.**—The different branches of Hydraulics are shown by the headings of chapters iii. to x. of this work. In the following chapter the whole subject is considered in a general manner. This enables us to dispose once for all of many points which would otherwise have had to be mentioned in more than one of the subsequent chapters. Moreover, the different branches are not always divided by such hard and fast lines as might appear; there are many points common to two branches, and the preliminary consideration of the various branches of the subject in connection with one another instead of separately will be advantageous.

xxii., xxvi., xxviii., xxxii., xxxiv., xxxv., xxxvi., xxxviii., xl., xli., xlii., xliv., xlvi., xlvii.

[1] *Recherches Hydrauliques.*

[2] *Report on the Physics and Hydraulics of the Mississippi River.*

[3] *Roorkee Hydraulic Experiments.*

[4] *Hydraulics of Great Rivers.*

[5] *A General Formula for the Uniform Flow of Water in Rivers and other Channels.* Translated by Hering and Trautwine.

[6] *The New Formula for Mean Velocity in Rivers and Canals.* Translated by Jackson. For other writers see chap. vi.

[7] *Canal and Culvert Tables.*

[8] *Étude d'une Nouvelle Formule pour Canaux Découverts.*

[9] *Lowell Hydraulic Experiments.*

[10] *Transactions of the American Society of Civil Engineers*, vol. xii.

[11] *Minutes of Proceedings, Institution of Civil Engineers*, vol. cxix.

CHAPTER II

GENERAL PRINCIPLES AND FORMULÆ

SECTION I.—FIRST PRINCIPLES

1. Bernouilli's Theorem.—Let Fig. 5 represent a body of still water, the openings at F and V being supposed to be closed. The

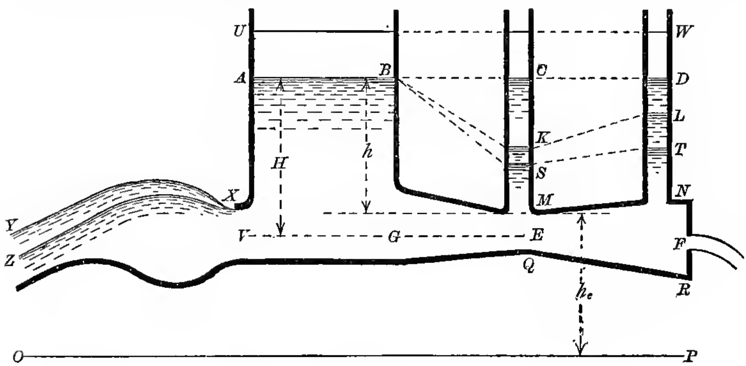


FIG. 5.

water in the tubes at C, D stands at the same level as AB. The 'head' or 'hydrostatic head' over any point is its depth below the plane AB. This plane is sometimes called the 'plane of charge.' The pressure is as the head. If P is the pressure per square foot at the depth H, and W the weight of one cubic foot of water, then $P = WH$ or $H = \frac{P}{W}$. The head H is said to be that 'due to' the pressure P.

Every particle of water in the reservoir possesses the same degree of potential energy. Comparing a particle at the depth H with one at the surface, the one possesses energy in virtue of its pressure, the other in virtue of its elevation.

Let an orifice be opened at F so that water flows along the pipe GEF, and let the reservoir be large, so that the water in it has no velocity and the surface AB is unaltered. The pressure in the water flowing in the pipe is reduced, and the water-levels in the tubes fall to K, L. The heights KM, LN are as the pressures at M and N, and they are called the 'hydraulic heads' or 'pressure heads.' The tubes are called 'pressure columns,' and the line BKL the line of 'hydraulic gradient.' Let p be the pressure at M, and h_p the pressure head. Then $h_p=\frac{p}{w}$. Let V be the velocity in the pipe at M and let $h_v=\frac{V^2}{2g}$. Then h_v is the 'velocity head.' It is the height through which a body falls under the influence of gravity in an unresisting medium in acquiring the velocity V, or the height to which it could be made to rise by parting with its velocity. Let it be supposed that there are no resistances to the motion of the water, so that no energy is consumed in overcoming them. Then by the law of the conservation of energy the total energy of any moving particle of water remains as before. Whatever is lost as pressure is gained as velocity. The head CK lost in pressure is the velocity head h_v. Thus

$$h=h_p+h_v \quad . \quad . \quad . \quad (3),$$

or the pressure head added to the velocity head is the hydrostatic head. This equation, due to Bernouilli, is the basis of all theoretical hydraulic formulæ. It obviously applies to any point in the pipe.

It has been seen that the pressure at M is as the height KM. Assume that the velocities at all points in the cross-section MQ are equal. Let H_p and H_v be the pressure head and velocity head at E, then $$H=H_p+H_v\,;\ h=h_p+h_v.$$
But since the velocities are equal, $H_v=h_v$; therefore $H_p-h_p=H-h$, or the change in pressure in passing from M to E is the same as it was when there was no flow. The pressure head at E is KE, and the pressure at any point in the cross-section is as its depth below K.

Let OP be a datum-line and let h_e be the 'head of elevation' of any point M above OP. Then $h+h_e$ is constant for all points in the system, and therefore

$$h_p+h_v+h_e=K \quad . \quad . \quad . \quad (4)$$

where K is constant. This is Bernouilli's theorem more fully stated. The total energy possessed by a particle of water is the sum of the energies due to its pressure, velocity, and elevation.

If instead of a pipe we consider an open channel XY, the results obtained will be the same as before. If pressure columns were used the water in them would not rise above the surface XY. At each point in the surface the pressure head is zero and the velocity head is equal to the hydrostatic head. If the velocities at all points in a cross-section are assumed to be equal, the law of change of pressure with depth is the same as before.

Since the area NR is greater than MQ, the velocity is less and the pressure greater. Thus from K to L there is a rise in the hydraulic gradient. Similarly, in the open stream there is a rise where the sectional area is increasing.

The pressure in a body of flowing water can never be negative, as the continuity of the liquid would be broken.

2. Loss of Head from Resistances.—Practically a certain amount of head h' is always expended in overcoming resistances, due to the friction of the water on its channel and to the internal movements of the water, so that the total head diminishes in going along the stream in the direction of the flow. In other words, the pressure head and velocity head do not together equal the hydrostatic head. The difference is the 'head lost.' The actual water-levels would in practice be S, T, and CS, DT would be the total losses of pressure head up to the points M and N. As head is lost, the work which the water is capable of doing in virtue of its elevation, pressure, and velocity is diminished. If h' is the head lost by resistance between two cross-sections, then

$$h' = h - \frac{V_2^2 - V_1^2}{2g} \quad . \quad . \quad . \quad (5),$$

or the head lost is equal to the fall in the surface or line of gradient less the increase in the velocity head. The same is true of the open channel. The surface would be XZ instead of XY.

3. Atmospheric and other Pressures.—Generally a body of water is subjected to the atmospheric pressure P_a. The head due to this pressure is $\frac{P_a}{W}$, and this has to be added in order to obtain the total head over any point. The case is the same as if the water-surface at each point were raised from AD to UW by a height $\frac{P_a}{W}$. But usually—as in the preceding demonstrations—the relative heads over two or more points are considered, the pressure of the atmosphere affects all parts equally and is left out of consideration. If, however, different portions of the water are subjected to pressures of different intensities caused, say, by partly exhausted air, by steam, or by a weighted piston, the water-surface of each portion of the system must be considered as being raised by a height $\frac{P}{W}$, where P is the intensity of the special pressure acting on it.

Section II.—Flow through Apertures.

4. Definitions.—An aperture is said to be 'in a thin wall' when its upstream edge is sharp (Figs. 6 and 7), and the 'wall' or structure containing the aperture is thin, or is bevelled or stepped, so that the stream after passing the edge springs clear and does not touch it again. An aperture like that shown in Fig. 1 or Fig. 2, page 2, may have its upstream edge sharp, but it does not come within the definition.[1] A rounded or 'bell-mouthed' orifice (Fig. 8) is one in which the sides are curved, so that the tangents at C, D are parallel, and the stream after passing *CD* does not contract. A weir of analogous shape may be formed by rounding the angle between the top and the upstream side or 'face,' and by prolonging the sidewalls *AB* (Fig. 8A) upstream.

Fig. 6.

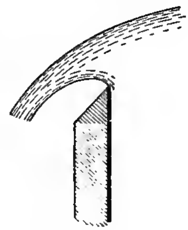


Fig. 7.

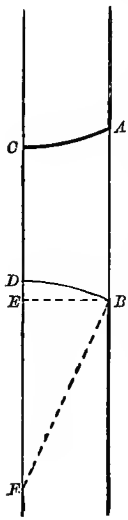


Fig. 8.

The upstream surface of the wall surrounding an aperture will be called the 'margin.' The margin is said to be 'clear' when it is free from projections, leakages, or anything which would interfere with the free flow of water along the wall towards the aperture. The clear margin, if not otherwise limited, is bounded by the sides of the reservoir or channel, or by any other aperture existing in the same wall. When an aperture has sharp edges an increase in the clear margin, up to a certain limit, increases the degree of contraction. When this limit has been reached the contraction is said to be 'complete.'

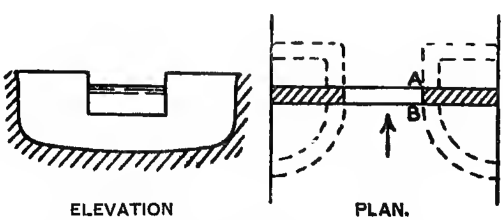


Fig. 8A.

[1] For this reason the expression 'sharp-edged,' used by some recent writers in preference to the old one of 'in a thin wall,' is not suitable.

5. **Flow through Orifices.**—Let H be the height of the free surface (Fig. 9) above the centre of gravity of the small orifice C, D, or E, and let V be the velocity of the issuing jet. Both the jet and the free surface AB are supposed to be subject to the atmospheric pressure P_a. The total head over the orifice is $H+\frac{P_a}{W}$, and the pressure in and upon the issuing jet is P_a. Then from equation 3 (page 10), supposing no head to be lost in overcoming resistances,

$$H+\frac{P_a}{W}=\frac{P_a}{W}+\frac{V^2}{2g},$$

or

$$V=\sqrt{2gH} \quad . \quad . \quad . \quad (6).$$

All formulæ for flow from apertures are modifications of this. The velocity $\sqrt{2gH}$ is called the 'theoretical velocity.' It is the same as would be acquired by a body falling from rest in a vacuum through a height H. If the jet issues vertically upwards it will, in the absence of all resistance except gravity, rise to the level of AB. The velocity depends only on H and not on the direction in which the jet issues. If AGR is a parabola with axis vertical and parameter $2g$, the theoretical velocities of jets issuing at F, M, N are as the ordinates FG, MK, NR. Practically owing to resistances caused by friction and internal movements of the water, the velocity of efflux is less than the theoretical velocity, and is given by the formula

$$V=c_v\sqrt{2gH} \quad . \quad . \quad (7),$$

where c_v is a 'co-efficient of velocity' whose mean value for the two kinds of orifices under consideration is about ·97.

Instead of assuming the water in the reservoir to have no appreciable motion, let it be supposed that it is moving with a velocity v directly towards the orifice. This velocity is called 'velocity of approach' and the discharge through the orifice is increased. The energy possessed by the water can, theoretically, raise it to a height $\frac{v^2}{2g}$ or h. This is called the head due to the velocity of approach, and it must be added to the hydrostatic head. Practically, for reasons which will be given below, a head nh has to be added, n being 1·0 or less. The formula thus becomes

$$V=c_v\sqrt{2g(H+nh)} \quad . \quad . \quad . \quad (8).$$

If the fluid moved without resistance, a velocity v in any direction, and not only toward the orifice. could be utilised in increasing the head and the discharge, but practically the only useful component of the velocity is that parallel to the axis of the orifice.

In the case of an orifice in a thin wall (Fig. 6), the jet attains a minimum cross-section at AB, whose distance from the edge of the orifice is about half the diameter of the orifice, or half the least diameter if the orifice is of elongated form. This minimum section is called the 'vena contracta.' The ratio of its sectional area a' to the area a of the orifice is called the 'co-efficient of contraction,' and is denoted by c_c: thus $a'=c_c a$. The mean value of c_c is about ·63. A vena contracta occurs with any kind of orifice having sharp edges, and c_c is probably about the same. For a bell-mouth $c_c=1{\cdot}0$.

The discharge of an orifice is

$$Q=a'V=ac_c c_v\sqrt{2gH}.$$

Let $c_c c_v=c$. Then c is the 'co-efficient of discharge' and

$$Q=ac\sqrt{2gH} \quad . \quad . \quad . \quad (9).$$

Or when there is velocity of approach

$$Q=ac\sqrt{2g(H+nh)} \quad . \quad . \quad . \quad (10).$$

The value of c for orifices in thin walls averages about ·61, and for bell-mouthed orifices ·97. It does not usually vary much with the head. Generally the values of c_v, c_c, and c are not very greatly affected by the shape and size of an orifice nor by the amount of head. Generally c is better known than c_v or c_c, and it is also of far more importance.

When an orifice has a head of water on both sides it is said to be 'submerged' or 'drowned,' and H in the formula is the difference between the two heads. Thus for any orifice Q or O (Fig. 9), the head is BW. It has nothing to do with the actual depth of the orifice below AB. If an orifice is partly submerged it must be divided into two parts, and only the lower part treated as submerged. If the water-level at Y is higher than at X, as it may be when XUY is a stream whose size is not very great relatively to that of the orifice, the head is BX and not BW.[1] It is the pressure at X and not at Y

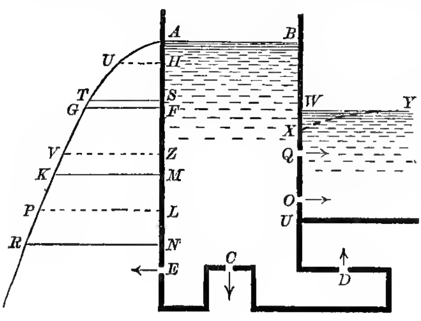


Fig. 9.

[1] Smith's *Hydraulics*, chap. iii.

that affects the discharge from the orifice. The rise from X to Y is owing to the stream being in 'variable flow' (art. 10).

When an orifice is in a horizontal plane, or when it is submerged, formulæ 7 to 10 apply, no matter what the size of the orifice may be. When an orifice is in a vertical or inclined plane the theoretical velocity of each horizontal layer of water is $\sqrt{2gH}$, where H is the head over that layer. When the vertical height between the upper and lower edges of the orifice is small compared to the head, the mean velocity in the orifice is practically that at its centre of gravity. If an orifice extends from M to N (Fig. 9), its centre being L, it is clear that, the curve KR being nearly straight, LP is practically the mean of all ordinates from M to N. But with an orifice HZ, whose centre is F, the protuberance of the curve UV causes the mean ordinate to fall short of that at F, and a correction has to be applied depending on the shape of the orifice and the ratio of its depth to the head over its centre.

6. Flow over Weirs.—Unless the contrary is stated, it will be assumed that all weirs have vertical side-walls, such forming in practice the vast majority. The remarks just made regarding the protuberance of the curve apply *a fortiori* to a weir. Let M (Fig. 9) be the level of the crest of a weir. Let $AM=H$ and $AS=\frac{4H}{9}$. The mean of all the velocities from A to M is represented by ST.[1] Thus the theoretical velocity V is $\sqrt{2g\frac{4H}{9}}$ or $\frac{2}{3}\sqrt{2gH}$. The practical formula is

$$Q=\frac{2}{3}\,cl\sqrt{2g}\,H^{\frac{3}{2}} \quad . \; . \; . \quad (11)$$

where l is the length of the crest, H the head on the crest, and c is a co-efficient of discharge whose value for sharp-edged[2] weirs averages about ·62, and for others varies greatly according to the form of the weir. With increase of head the co-efficient increases in some cases and decreases in others. It is not usual to give a separate formula for finding v or to divide c into c_v and c_c, but roughly these are about the same for sharp-edged weirs as for sharp-edged orifices. If there is velocity of approach the formula is

$$Q=\frac{2}{3}\,cl\,\sqrt{2g}\,(H+nh)^{\frac{3}{2}} \quad . \; . \; . \quad (12)$$

where n is 1·0 or more, and h, as for orifices, is $\frac{v^2}{2g}$, v being the velocity of approach.

[1] For proof see chap. iii. art. 19.

[2] In this paragraph 'sharp-edged' means 'in a thin wall.'

When the water on the downstream side of the weir or 'tail water' rises above its crest (Fig. 10), the weir is said to be 'submerged' or 'drowned' instead of being 'free.' The discharge of AB is found by the ordinary weir formulæ, equations 11 and 12. The discharge of BC is considered as being that of a submerged orifice BC under a head AB, and is found by equation 9 or 10. The tail-water level should be observed at L, see remarks concerning submerged orifices (art. 5), but is often observed at M. The co-efficients used allow for the contraction of the stream.

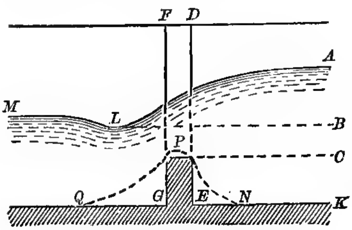


FIG. 10.

If instead of a weir there are lateral contractions, $FGED$, the above equation can still be used, the length l in equation 11 or 12 and the area a in equation 9 or 10 being measured in the contracted part.

In the case, for instance, of a stream in flood the fall AB may be small compared to BC in the case of a weir or to BK in the case of a contracted channel. In such cases equation 9 or 10 alone is used, and generally 10, since there is usually considerable velocity of approach. The co-efficients for such cases are not always accurately known. See also art. 19.

7. Concerning both Orifices and Weirs.—With all kinds of apertures small heads are troublesome, not only because of the difficulty in measuring them exactly, but because complications occur, and the co-efficients are not properly known.

At a weir the water-surface always begins to fall at a point A (Fig. 11) situated a short distance upstream of the weir. Hence, whatever the crest and end contractions may be, there is always surface contraction. The angular spaces between the wall and the bed and sides of the channel are occupied by eddies. The fall in the surface begins where the eddies begin. From this point the section of the stream proper or forward-moving water diminishes, its velocity and momentum increase, and the increased surface-fall is necessary to give the increased momentum (art. 10). A similar fall occurs upstream of an orifice, though it may only be perceptible when the orifice is near the surface.

The section where the eddies begin will be termed the 'approach section.' It is here that the head should be measured and the velocity of approach observed or calculated, but when, as often happens with a weir, and generally with an orifice, the surface upstream of A is nearly level, the head may be observed either at A or upstream of it. It must not be observed downstream of A. In some of the older observations on weirs the head was measured from D to C instead of from A to E, but the co-efficients thus obtained are more variable, and it is very difficult in practice to observe the water-level at D with accuracy. The section for velocity of approach may be shifted either way from AB provided its area is not appreciably altered.

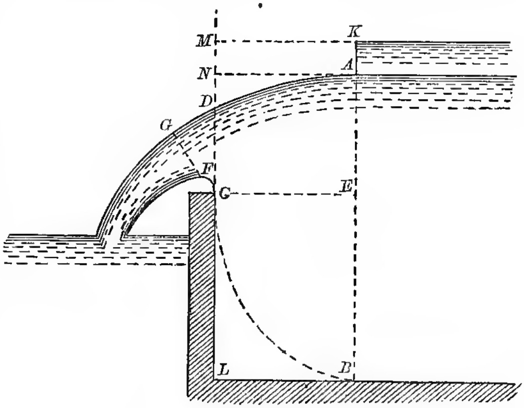


FIG. 11.

The velocity of approach, v, is the discharge, Q, of the aperture divided by the area, A, of the approach section. If water enters a reservoir in such a manner as to cause a defined local current towards the aperture, the sectional area of the current may be estimated or observed, and this area, not that of the whole cross-section of the reservoir, used for determining the velocity of approach. If the axis of an aperture is oblique to the direction of the approaching water, the component of the velocity of the latter parallel to the axis of the aperture may be taken to be the velocity of approach. Equations 8, 10, and 12 cannot be solved directly because, until Q or V is known, v and h are unknown. It is impossible to find v by direct observation, in the case of a proposed structure or unless the water is actually flowing, and even then it is not a convenient process. The usual procedure is to estimate a value for v, calculate h, solve equation 10 or 12, divide by A, and thus find a corrected value for v. If this differs much from the value first assumed, it can be substituted and Q calculated afresh. Velocity of approach has very little effect when the area of the approach section is about fifteen times that of the smallest section of the stream issuing from the aperture, that is for a sharp-edged aperture nine or ten times the area of the aperture, and for a bell-mouthed orifice fifteen times the area of the orifice. In a weir the height of the aperture is to be considered AE, not DC.

In order that the contraction may be complete the margin must be clear for a distance from the aperture extending in all directions to about three times the least dimension of the aperture. Any further extension has no effect. If the ratio of the width of the clear margin to the least dimension of the aperture is reduced to 2·67 and 2·0, the discharge is increased by only about ·16 and ·50 per cent. respectively, so that practically a ratio of 2·75 is sufficient and will be so regarded. In a weir the length of crest is usually the greater dimension, and the least dimension is then the head AE.

Another condition which is essential for complete crest contraction is that air shall have free access to the space under the issuing stream. In an aperture in a thin wall with complete contraction air usually has free access unless the tail water rises very nearly to the crest or lower edge, when its surging may shut out the air. In a weir with no end contractions the width of the channel, both upstream and downstream of the weir, is, very likely, the same as the length of the crest, and air will be excluded unless openings in the sides of the downstream channel are provided to admit it. Any want of free admission of air causes the sheet of water to be pressed down by the air above it, the contraction is reduced and various complications may occur. It is also necessary for complete contraction that the edges be perfectly sharp. Any rounding increases the discharge.

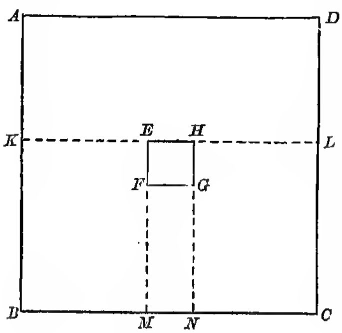


Fig. 12.

In Figs. 12 and 13 $ABCD$ is the boundary of the minimum clear margin necessary to give full contraction, supposing $EFGH$ to be an orifice, $KBCL$ the boundary supposing it to be a weir, and $FMNG$ supposing it to be a weir with no end contractions. In Fig. 13 $EH = EF \times 20$. The ratios of the areas within these boundaries to those of the apertures are 42·25, 24·38, and 3·75 in Fig. 12, and 8·29, 4·78, and 3·75 in Fig. 13. It is thus clear that of the two conditions, namely, sufficiency of the marginal area to give full contraction and sufficiency of the area of the approach section to give a negligible velocity of approach, one does not necessarily imply the other. The two matters must be kept distinct. An elongated aperture, especially a weir, is most likely to have a high velocity of approach and a square aperture, especially an orifice, to have incomplete contraction. Even when the area of the approach

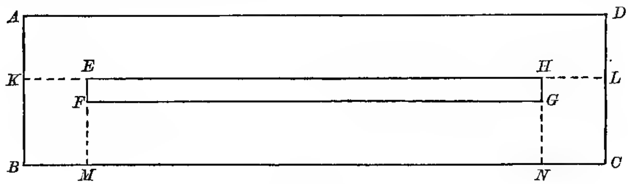


FIG. 13.

section is very large, it may allow of incomplete contraction in a portion of an aperture if unsymmetrically situated.

The co-efficients for apertures in thin walls are known with more exactness than for others, but they are best known for orifices when the contraction is complete, and for weirs either when it is complete on all three sides or complete at the crest and absent at the sides. The co-efficient n for velocity of approach is not very accurately known. Hence very high velocities of approach are objectionable where Q has to be accurately computed from assumed co-efficients, but when v is not very high, that is, when the area A is more than three times that of the smallest section of the issuing stream, Q depends very little on n.

The fall in the surface upstream of an aperture, the rise CF due to crest contraction in a sharp-edged weir, and the effect of velocity of approach greatly complicate the theoretical discussion of weir formulæ.

SECTION III.—FLOW IN CHANNELS

8. Definitions.—The 'border,' or 'wet border,' B, of a stream is the perimeter of its cross-section, omitting, in the case of an open stream, the surface width. The 'hydraulic radius,' R, also called in the case of an open stream the 'hydraulic mean depth,' is the sectional area A divided by the border. Thus $R=\frac{A}{B}$. The flow of a stream is 'uniform' when the mean velocities at successive cross-sections are equal; that is, when the areas of the cross-sections are equal. Otherwise the flow is 'variable.' A pipe is uniform when all its cross-sections are of equal area. The flow in such a channel must be uniform when it is flowing full. An open channel is uniform when it has a constant bed-slope and a uniform cross-section. The flow in such a channel is uniform when the water-surface is parallel to the bed, but otherwise it is variable. The 'inclination' or 'surface-slope' of an open stream is the 'fall' or difference between the water-levels at any two points divided by the horizontal distance between them. The 'virtual slope' or 'virtual inclination' of a pipe is the difference between the levels of two points in the hydraulic gradient divided by the horizontal distance between them.

9. Uniform Flow in Channels.—When a stream flows over a solid surface the frictional resistance is independent of the pressure, and approximately proportional to the area of the surface, and to the square of the velocity. Thus, if f is the resistance for an area of one square foot at a velocity of one foot per second, the resistance for an area A and a velocity V is nearly fAV^2. The value of f increases with the roughness of the surface.

In the case of a uniform stream, open or closed, $ACDB$ (Fig. 14), the second term on the right in equation 5 (p. 11) vanishes, and the loss of head h in a length L is equal to the fall in the surface or in the hydraulic gradient. In an open stream the pressures on the ends AC, BD of the mass of water are equal, and the accelerating force is that component of its weight which acts parallel to its axis or $WAL\frac{h}{L}$. On the assumption that the resistance is entirely due to friction between the stream and its channel, the resistance is approximately $fLBV^2$. Since the motion is uniform this is equal to the accelerating force, or

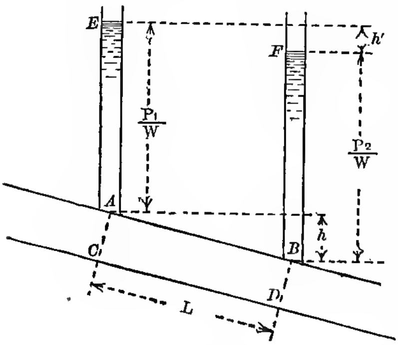


Fig. 14.

$$V^2 = \frac{W}{f} \cdot \frac{A}{B} \cdot \frac{h}{L}.$$

But $\frac{A}{B} = R$ and $\frac{h}{L} = S$, the surface-slope of the stream. Let

$\frac{W}{f}=C^2$. Then $$h=\frac{V^2L}{C^2R} \quad . \quad . \quad . \quad (13),$$
or $$V=C\sqrt{RS} \quad . \quad . \quad . \quad (14)$$
where C is a co-efficient. In the case of a uniform pipe the pressures on the ends have to be taken into consideration, but the resulting equation is the same, S being the hydraulic gradient EF. For if P_1 and P_2 are the pressures at A and B, the resultant pressure on the mass $ACDB$, resolved parallel to its axis, is $A(P_1-P_2)$ or $WA\left(\frac{P_1}{W}-\frac{P_2}{W}\right)$ or $WA(h'-h)$. The component of the weight parallel to the axis is as before WAh. These two together are WAh'. Equation 14 is the usual formula for uniform flow in streams. It is known as the 'Chézy' formula. Obviously the co-efficient C is greater the smoother the channel. The formula for the discharge is
$$Q=AC\sqrt{RS} \quad . \quad . \quad . \quad (15).$$

The theoretical proof just given takes no account of the resistances due to the internal motions of the fluid, nor of the facts that the velocities at all the different points in the cross-section differ from one another, that the mean velocity V of the whole is greater than the mean velocity v of the portions in contact with the border, and that the frictional resistance may not be exactly as V^2, nor even as v^2. Practically, it is found that the co-efficient C depends not only on the nature of the channel, but on R and S. The co-efficient increases with R; that is, generally with the size of the stream. It depends also to some extent on S, and perhaps on other factors which will be mentioned. It increases with S in pipes of the sizes met with in practice, and in open streams of small hydraulic radius. The value of C varies generally between 40 and 120 for earthen channels, and between 80 and 160 for clean pipes. The chief difficulty with all kinds of channels consists in forming a correct estimate of the value of C. The difficulty is the greater because the roughness of a particular channel may be altered by deposits or other changes.

Let an open stream of rectangular cross-section have a depth of water D, width W, and velocity V. Let W be great relatively to D, then R is practically equal to D and the fall in a length L is $\frac{V^2L}{C^2D}$. Let other reaches of the same stream have equal lengths, but widths $2W$, $3W$, etc., the longitudinal slopes being flatter, so that D is the same in all. The velocities will be

$\frac{V}{2}$, $\frac{V}{3}$, etc., and the losses of head will be $\frac{V^2L}{4C^2D}$, $\frac{V^2L}{9C^2D}$, etc. The total loss of head in two reaches of widths W and $3W$ is $\frac{V^2L}{C^2D}(1+\frac{1}{9})$. The loss of head in two reaches, each of width $2W$, will be $\frac{V^2L}{C^2D}(\frac{1}{4}+\frac{1}{4})$. Thus, the loss of head in a reach of length $2L$ and width $2W$ is less than half the loss in an equal length of the same mean width, but in which the width is W for half the length and $3W$ for the other half. If the streams compared have circular or semicircular sections the difference is still greater. Thus, in conveying a given discharge to a given distance, the advantage as regards fall is on the side of uniformity in velocity.

10. Variable Flow in Channels.—When the flow is variable, the loss of head from resistances is the same as in a uniform stream, that is $\frac{V^2L}{C^2R}$, provided the change of section is gradual and the length L short, so that the velocity and hydraulic radius change only a little, say by 10 per cent., V and R being their mean values. Then, from equation 5 (p. 11), the fall in the surface or hydraulic gradient in the length L is

$$h=\frac{V^2L}{C^2R}-\frac{V_1^2-V_2^2}{2g} \quad . \; . \; . \; (16)$$

where V_1 and V_2 are the velocities at the beginning and end of the length L. The equation may be written

$$V=C\sqrt{R}\sqrt{\frac{h+h_v}{L}} \quad . \; . \; . \; (17)$$

where $h_v=\frac{V_1^2-V_2^2}{2g}$. This is the equation for variable flow in streams. It is the same as equation 14 $\left(\text{since } S=\frac{h}{L}\right)$ with the addition of the quantity h_v, which is introduced because of the change in the *vis viva* of the water. The quantity V_1^2 is the square of the means of all the different velocities in the cross-section. It ought strictly to be the mean of the squares. In a case which was worked out, it was found to be 3·3 per cent. in excess. But a nearly equal error occurs with V_2. The quantity h_v thus represents the change of *vis viva* without appreciable error.

If the section of the stream is decreasing, V_1 is less than V_2, h_v is negative, and V is less than it would be in a uniform stream with the same values of R and S. Or, V being the same, the fall h in the surface, or in the hydraulic gradient, is greater than in a uniform stream. This is because work is being 'stored' in the water as its velocity increases. If the section is increasing V_1 is greater than V_2, h_v is positive, and V is greater than in a uniform stream, or V being the same, h is less. Work is being 'restored' by the water. There may even be a rise in the surface or line of hydraulic gradient instead of a fall.

Consider any stream AE (Fig. 15) in which the sectional areas A and E are equal and the velocities therefore equal, and let the area D be not more than 10 per cent. greater than C. Make C' and C'' each equal to C. Evidently the quantities h_v for the lengths AC', $C''E$ will be equal, but of opposite signs, and the total fall in the surface in $AC'+C''E$ will be the same as if the flow were uniform and the section of the stream were an average between the sections at A and C'. The same is true of the length $C'C$ and of CC''. It does not matter whether the fluctuations in section are due to changes in the width or in the depth, or both. The formula $V=C\sqrt{RS}$ therefore applies to a variable stream AE if the velocities at both ends of it are equal and the fluctuations moderate, but evidently it does not apply any the better to a short length of such a stream in which the velocities at the ends are not equal. Evidently in the stream AE, S varies from point to point. It is greater as A is less. S in the formula must be got from the total fall, and C suited to the average section.

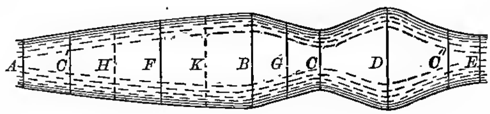


Fig. 15.

Now let the fluctuations be so great that the reaches must be subdivided before the equation can be applied to them. Make F equal to G. The fall in $C'F+GC$ is the same as in a uniform stream of section H. The fall in $FB+BG$ is the same as in a uniform stream of section K. The total fall in $C'C$ is the same as the sum of the falls in two uniform streams of sections H and K. This total fall is (art. 9) greater than that in a uniform stream, having a section equal to the mean of H and K. It will also be seen in section v. that if there are any abrupt changes the falls at the contractions are by no means counterbalanced by the rises at the expansions. Thus a variable stream is less efficient than a uniform stream of the same mean section, or in other words, it must have a greater total fall in order to carry the same discharge.

This and the result arrived at in article 9 are analogous to other mechanical laws. Uniformity in speed is best, slight fluctuations are unimportant, but great, and especially abrupt, fluctuations give reduced efficiency.

It is clear that the formula $V = C\sqrt{RS}$ applies to the case last considered if a suitable value is given to C and S is the slope deduced from the total fall. It even applies approximately to a stream in which the two end velocities are not equal, provided the length is considerable, so that h_v is small relatively to h. It applies to such a case still more nearly if the value assigned to C is such as to take account of the change in the end velocity, C being greater than for uniform flow if V increases and less if it decreases. It may not always be easy to say how much C should be altered in such a case, but it may still be highly convenient to use the formula in generalising regarding such a stream, for instance in comparing the discharges for two different water-levels or stages of supply in an open stream. Thus the formula for uniform flow applies either exactly or nearly to a vast number of cases met with in practice in which more or less approximate uniformity of flow exists.

11. Concerning both Uniform and Variable Flow.—Pipes are nearly always of approximately uniform section, and the flow in them nearly uniform, but the sections are seldom exactly equal. Open channels are sometimes nearly uniform and, if there is no disturbing cause, the flow is nearly uniform. But in both cases much confusion and error have been caused by applying the formula for uniform flow to variable streams of short lengths, or, supposing the short length to be uniform, by carrying the slope or hydraulic-gradient observations into variable reaches.

Owing to a change, for instance a change of slope, or of section, or a weir, in a uniform open stream, the water may be 'headed up' (Fig. 16) or 'drawn down' (Fig. 17) for a great distance, AB, upstream of the point of change. In these cases the surface-slope AB differs from the bed-slope, and the flow is variable although the channel is uniform. Heading-up is also known as 'afflux' or 'back-water.' In all such cases the water-surface AB, which would, if the upstream reach had continued without any change, have followed the line BC, has to accommodate itself to

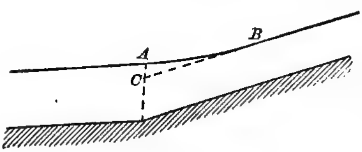


FIG. 16.

the downstream level at A, and assumes a curve such that the surface-slope changes in the opposite manner to the sectional area. Downstream of A the flow is uniform. In uniform closed channels the section of the stream cannot vary, and if from any cause the gradient-level at any point is altered, the change of slope runs back to the commencement of the pipe.

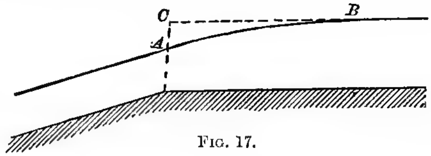


FIG. 17.

In the absence of any disturbing cause, that is when the flow is uniform throughout, it is obvious from equations 14 and 15 that in an open stream an increase of discharge is accompanied by a rise of water-level and *vice versa.* The same is the case in a variable stream. In uniform flow in an open stream, the dimensions and slope of the channel being known, the discharge can be found if the water-level is given and *vice versa.* The surface-slope is the same as the bed-slope. In variable flow the surface-slope may be very different from the bed-slope, and it is necessary to know the water-levels at two points in order to find the discharge, or to know the discharge and the water-level at one point in order to find the water-level at the other point.

A large stream, whether in an open or closed channel, has an advantage over a small one both in sectional area and in velocity. For as A increases R usually increases, and with it C. If the slopes are equal Q is much greater for the larger stream. If Q is the same for both, S is much less, that is the loss of head is less, for the larger stream. This applies to variable as well as to uniform streams. A fire-hose of diameter D is fitted at its end with a tapering 'nozzle' whose least diameter d is perhaps $\frac{D}{3}$, so that the velocity of the issuing jet is nine times the velocity in the hose. If the hose were made of diameter d the loss of head in it would be greatly increased, and more pressure would be required to drive the water through it. The size is limited by convenience in handling. If part of the hose stretches under pressure, so that the flow is variable, there is a gain all the same. Again, let Fig. 16 represent an irrigation distributary with discharge Q, the bed-slope downstream of A being the same as upstream, so that BC is the water-level. To supply water to high ground near A a weir may be made, raising the surface to BA, and enabling a discharge q to be drawn off at A, whereas a small branch made for this purpose from *B*, with a slope such as *BA*, might discharge hardly any water.

The theoretical proof (art. 1) regarding the variation of pressure with depth depended on the assumption that the velocities at all points in a cross-section were equal. Though they are not equal, it is found in practice that the law holds good.

12. **Relative Velocities in Cross-section.**—The velocity at any point in a straight uniform stream flowing in a channel is, generally speaking, greater the further the point is removed from the border. The border retards the motion of the water next to it, and the retardation is thus communicated to the rest of the stream. In a pipe of square or circular section the velocity is greatest at the axis, and thence decreases gradually to the border. In an open channel the form of cross-section varies greatly in different streams, and the distribution of the velocities varies with it. The distribution of velocities in the cross-section of a variable stream, provided the section of the channel changes gradually, is practically the same as if the flow were uniform. The distribution depends on the form of the section, and is not likely to be appreciably affected by the fact that the whole velocity is slowly changing. In all cases the velocity changes more rapidly near the border (probably very rapidly quite close to the border, but observations cannot be made there) and less rapidly towards the centre of the stream. Thus all velocity curves are convex downstream. Nothing in this article relates to the velocities at or near to abrupt changes of any kind.

13. **Bends.**—In flow round a bend the distribution of velocities is modified, the line of greatest velocity being shifted, by reason of the centrifugal force, towards the outer side of the bend, and all the velocities on the outer side being increased while those on the inner side are reduced. The loss of head from resistance in a bend is greater than in the same length of straight channel. The additional resistance is chiefly caused by work done in redistributing the velocities consequent on the transfer of the maximum line from its normal to its new position, and in the fresh redistribution after the bend is passed. This fresh redistribution cannot be effected instantaneously, so that the normal distribution is not restored till some distance below the termination of the bend. Besides these resistances it is probable that wherever the distribution is abnormal, no matter whether any redistribution is in actual progress or not, the resistance is greater, owing to the high velocities near the border on the outer side of the bend.

For a given channel and given radius of bend the total resistance or loss of head caused by the bend is not proportional to its length because, however long it may be, the redistribution has to be effected only twice. If the lower half of a bend is reversed in position, thus forming two curves, the loss of head in the whole bend is greater than before, because the redistribution of velocities has now to be effected in the opposite direction, doubling the work of this kind done before. No abnormal distribution of velocities occurs upstream of a bend. The laws regarding bends, both in pipes and open channels, are imperfectly known. Recent experiments on pipes tend to show that, for a given angle subtended by a bend, the actual radius of the bend is, down to a certain limit, of no great consequence. The only bend which has any considerable effect is a fairly sharp one. A succession of such bends may have great effect. Flow round a bend may be either uniform or variable. If in a sharp bend in an open channel the section of the stream is the same as in the straight reaches, the surface gradient must be greater, and there will be heading-up—though probably slight—in the upstream reach.

SECTION IV.—CONCERNING BOTH APERTURES AND CHANNELS

14. Comparisons of different cases.—The difference between the case of an aperture and that of a channel depends on the nature of the work done. It is a difference of degree and not of kind. In flow through a small orifice in the side of a large reservoir a mass of water which is at rest has a velocity impressed on it. The motive-power is the pressure of the water due to the head, and the work done consists almost entirely in imparting momentum to the water, friction and resistance being unimportant. In uniform flow in a channel a mass of water slides, under the influence of gravity, with a constant velocity. The motive-power is that component of the weight of the water which acts parallel to the surface or line of gradient, and the work done consists in overcoming friction and the resistance caused by internal movements. No fresh momentum is imparted. These are the two extreme cases. In flow through some kinds of apertures there are considerable resistances, and in variable flow in channels much of the work may consist in the imparting of momentum. The two extreme cases thus merge one into the other.[1] Most cases of

[1] Fig. 10, p. 16, may be regarded as a case of variable flow.

abrupt changes in channels, dealt with in articles 17 to 21, occupy an intermediate position.

Comparing channels or apertures which entirely surround the flowing stream with those which leave the water-surface free, it will be found that the latter are far more elastic than the former. In the case of the pipe *GEF* (Fig. 5, p. 9) and the orifice *C* (Fig. 9, p. 14), if it is desired to double the discharge, it is necessary to quadruple the head or the hydraulic gradient. In either case a very great rise in the water-level *AB* is required. But for a weir, since Q is roughly as $H^{\frac{3}{2}}$, in order to double Q it is only necessary to increase H by some 60 per cent. For an open channel with vertical sides the discharge—recollecting that C increases with R—is doubled by increasing the depth about 50 per cent. The above comparisons do not of course take exact account of variations in the co-efficients. For an open channel with sloping sides the discharging power may vary very greatly for a quite moderate change of water-level. When the changes in the conditions governing the flow are slight, so that the co-efficient is practically unaltered, the changes in the discharge are as follows: a change of 1 per cent. in the head over an orifice or in the slope of a channel changes the discharge ·5 per cent.; a change of 1 per cent. in the head on a weir or in the sectional area of a stream changes the discharge 1·5 per cent.

A 'module' is an arrangement by which it is sought to ensure a constant discharge of water from a fluctuating source of supply. Generally it is a machine which automatically alters the size or position of an aperture as the water-level varies. Some modules are imperfect, and in such cases, having regard to the preceding paragraph, it is clearly best that the water to be delivered should pass through an orifice or pipe, and the surplus over a weir or through an open channel. In Foote's module (Fig. 18) a gate *E*, regulated at intervals by hand, causes the water-level in the canal at *C* to be nearly constant, and higher than at *D*. By an orifice *F* water flows into the tank *FA*, and on to the branch *AB*, the surplus passing over a

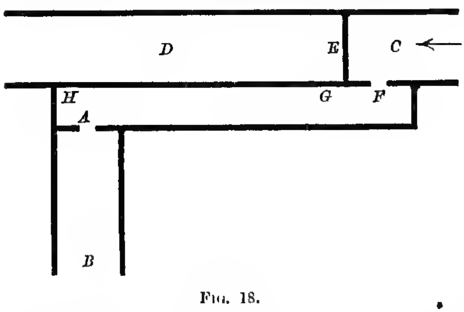


FIG. 18.

weir *GH*. The regulation is better the longer the weir, but it would be improved by so arranging the gate *E* that the water would flow over it instead of under it.

Even if the water in a canal is steady, an outlet consisting of an orifice of fixed size will not, if submerged, give a constant discharge if the branch channel is liable to be altered. If it is enlarged, its water-level falls, and thus the head at the outlet is increased. The limit is not reached until there is a free fall.

15. **Special Conditions affecting Flow.**—The condition of water, as for instance its temperature or the amount of suspended matter which it contains, has in some cases an effect on the flow. A rise in the temperature of water probably causes an increase in the discharge, while an increase in the suspended matter causes, for flow in channels, a decrease; but it seems that appreciable changes in the discharge are caused only by great changes in the conditions, and scarcely even then unless the channels or apertures are small and the velocities also low.

At very low velocities the nature of flow in pipes is essentially different from that at ordinary velocities. For any given pipe there is a certain 'critical velocity.' For velocities lower than this the motion is in parallel filaments, V varies nearly as S and as R^2 and increases with the temperature of the water. When the velocity rises to the critical amount, a very rapid or even sudden change occurs, the motion becoming first sinuous and then eddying. The following formulæ and figures are approximations. Experiments have been few. For any pipe the critical velocity, V_c, is inversely as R the radius of the pipe. At 0° Cent. it is, for a 1-inch pipe, about ·47 feet per second, for a 12-inch pipe ·04 feet per second. At 100° Cent. the figures are ·07 and ·006, or little more than $\frac{1}{7}$th of the above. Let V – lower than V_c – be the mean velocity in a pipe. Then $V = 361D^2S(1 + \cdot 0337T + \cdot 000221T^2)$, where D is the diameter of the pipe. If V_o is the velocity of the central filament, $V_o = 2V$, and the velocity, U at any radius R, is $V_o\left(1 - \frac{r^2}{R^2}\right)$. The kinetic energy of the water instead of being slightly in excess of $\frac{V^2}{2g}$ (art. 10) is $\frac{V^2}{g}$. If ordinary turbulent motion is artificially produced, stream-line motion re-establishes itself when the disturbing cause is removed. For any pipe there is also a 'higher' critical velocity. At 0° Cent. it is, for a 1-inch pipe, 2·95 ft. per second, for a 12-inch pipe ·246 ft. per second. At 100° Cent. the figures are ·45 and ·037. At the higher critical velocity stream-line motion can exist, but a small disturbance upsets it, and once upset it is not likely to re-establish itself.

The subject of critical velocities is not of much practical importance because the velocity in an ordinary pipe or channel is above V_c, or if it falls as low as V_0 the discharge becomes a matter of little consequence.

16. Remarks.—The solution of a numerical question in Hydraulics by means of formulæ may be either direct or indirect. When the conditions are given and the discharge, say, is to be found, it is only necessary to look out the proper co-efficient and apply the formula. But frequently the problem is inverted and consists in finding a suitable set of conditions to give a particular result. This is especially the case when channels or structures have to be designed. In many cases a direct solution cannot be obtained by inverting the formula, either because its form is unsuitable—an instance of this has been given in article 7—or because the co-efficients are not known until the conditions are determined. It is often necessary to obtain an indirect solution by assuming a certain set of conditions, calculating the discharge or other quantity sought, and, if it is not what is desired, making alterations in the assumed conditions and calculating afresh. In order to facilitate calculations which would otherwise become very tedious, numerous working tables are given. By their use work is vastly reduced.

Both in apertures and channels the co-efficients in the formulæ vary more or less as above stated. Various attempts have been made to modify the formulæ (putting for instance H^m, R^n, S^p, instead of $H^{\frac{1}{2}}$, $R^{\frac{1}{2}}$, $S^{\frac{1}{2}}$) in such a way as to make the co-efficient constant. Such formulæ either have a restricted range or else the functions of H, R, and S involved are very inconvenient. It is far better to adhere to the simple indices in common use and to accept the variations in the co-efficients.

Although for discharge computation one should avoid complex conditions such as incomplete contraction, small heads, high velocity of approach, or variability of flow, yet in practice an engineer is frequently compelled to accept such conditions, and some attention will be given to methods of dealing with them.

In many of the more complicated cases (such as some considered in the following section and in chap. vii.) it may be difficult to arrive at any exact results by calculation, but it may still be most useful to recognise the existence of the phenomena referred to, and to take note of their general effects.

SECTION V.—ABRUPT AND OTHER CHANGES IN A CHANNEL

17. Abrupt Changes.—Any change in a channel, whether of sectional area or direction, and whether or not there is a bifurcation or junction, which is so sudden as to cause contraction or eddies is called an abrupt change. At an abrupt change the first term on the right in equation 5 (p. 11) is omitted. It would be small because of the small length of stream considered; and owing to the stream being bounded partly by eddies and changing rapidly in form, it would be difficult to assign values to the quantities R and C. The second term only is used. Thus the formulæ are analogous to, or identical with, those for apertures. In fact abrupt changes include submerged weirs and (in certain respects which will be specially noted) other apertures.

At abrupt changes there are special losses of head, owing to work being expended on eddies. The length and violence of the eddies at an enlargement are much greater than at a corresponding contraction (Figs. 3 and 4, p. 5), and the loss of head is consequently much greater. At a contraction the pressure at K, L is slightly greater, and in the case of an open stream the water-level slightly higher than in the flowing stream. These remarks apply also to orifices and weirs with which there is velocity of approach. At an expansion the conditions are the reverse. The loss of head at an abrupt change of any kind is most important when the velocity is high; it can seldom be calculated with exactness, and often can only be roughly estimated.

18. Abrupt Enlargement.—At an abrupt enlargement (Fig. 4) the loss of head due to the enlargement can be found theoretically by assuming that the intensity of pressure on $A'C$, $B'D$ is the same as at $A'B'$. Let V_1, A_1, be the velocity and sectional area at AB, P_1 the pressure on its centre of gravity, and V_2, A_2, P_2, similar quantities at EF. The force $A_2(P_2-P_1)$ causes the velocity to be reduced from V_1 to V_2. In a short time, t, the fluid $ABFE$ comes to $A'B'F'E'$. Since the momentum of $A'B'FE$ is unchanged the change of momentum in the whole mass is the difference between that of $ABB'A'$ and that of $EFF'E'$, and that is

$$WQt\left(\frac{V_1}{g}-\frac{V_2}{g}\right),$$

where W is the weight of a cubic foot of water and Q is the discharge per second. This change of momentum is equal to the impulse $A_2(P_2-P_1)\ t$, therefore

$$A_2(P_2-P_1)=\frac{WA_2V_2(V_1-V_2)}{g}$$

or
$$\frac{P_2-P_1}{W}=\frac{V_2(V_1-V_2)}{g}.$$

But $\frac{P_1-P_2}{W}$ is the fall h in the surface or line of gradient, therefore from equation 5 (p. 11)

$$h'+\frac{P_2-P_1}{W}=\frac{V_1^2-V_2^2}{2g},$$

subtracting the preceding equation from this

$$h'=\frac{V_1^2-V_2^2-2V_1V_2+2V_2^2}{2g}=\frac{(V_1-V_2)^2}{2g} \quad . \; . \; . \; (18),$$

or the loss of head is the head due to the relative velocity of the two streams. In order to simplify the calculation it has been assumed that the stream flows horizontally, that is, that the centres of gravity of the sections AB, EF are at one level, but the loss of head due to the enlargement is the same in any case. The pressure in the eddy has been found to be really less than in the jet, so that the assumption made is incorrect; and the formula has been found in practice to give incorrect results for small pressures and velocities, but for other cases it is fairly accurate.

Equation 18 is of the same form as the equation giving the loss by shock, in a case of impact of inelastic solid bodies; and the loss of head due to an abrupt enlargement is often called 'loss by shock,' though there is not really any shock, the stream always expanding gradually.

If there were no loss of head in the length AE there would be a rise of $\frac{V_1^2-V_2^2}{2g}$ in the surface or hydraulic gradient. In a pipe the loss of head $\frac{(V_1-V_2)^2}{2g}$ is always much less than $\frac{V_1^2-V_2^2}{2g}$, and there is actually a rise whose amount is approximately

$$\frac{V_2(V_1-V_2)}{g} \quad . \; . \; . \; (18\text{A}).$$

This proof is usually given only for a pipe, but it clearly applies to an open stream if there is no rise in the surface. If there is a rise the pressure on the wave QR, supposing Fig. 4 to be a vertical section, is not P but P_a (the atmospheric pressure), and the loss of head is greater than $\frac{(V_1-V_2)^2}{2g}$. Moreover, the section usually changes not only in size but in form, and the redistribution of the velocities absorbs more work. The rise in the water-level is thus generally slight, and it cannot usually be calculated accurately.

When an enlargement is immediately succeeded by a contraction so as to cause a deep recess, the water in the recess has little or no forward motion, and the flow is practically the same as if the recess did not exist.

19. Abrupt Contraction.—At an abrupt contraction in a pipe (Fig. 3) it is necessary, if exact results are required, to calculate the sectional area at the vena contracta *EF* and find the velocity V_2 at that section. Then, V_1 being the velocity at *ST*, the fall in the hydraulic gradient, due to increase in the velocity head from *ST* to *EF*, is $\frac{V_2^2 - V_1^2}{2g}$, but some head is lost owing to friction and to the eddies at *K*, *L*. The expansion of the stream from *EF* to *MN* causes loss of head, which may be calculated as explained in the preceding article. The case of an open stream is analogous, but the whole fall due to loss of head and increase of velocity head is considered together (art. 6) and equation 10 (p. 14) is used.

A particular case of abrupt contraction occurs when a stream issues from a reservoir. There is a fall in the surface or hydraulic gradient. Most likely the velocity of approach is negligible. If so the fall, in the case of a pipe, can be calculated without finding the area *EF* (chap. v. art. 1), and, if not, the above procedure can be adopted. For an open stream equation 10 is to be used.

At a local contraction the channel contracts and expands again, but not necessarily to the same size. For an open channel equation 10 is used. For a pipe there are various empirical formulæ for local narrowings, all involving the factor $\frac{V^2}{2g}$ (chap. v. art. 6).

20. Abrupt Bends, Bifurcations, and Junctions.—An abrupt bend (Fig. 19) is called an 'elbow. The contraction causes a local narrowing of the stream. It has been found in small pipes that, with an elbow of 90°, the head lost is very nearly $\frac{V^2}{2g}$

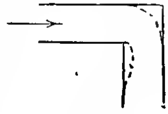


FIG. 19.

Judging from analogy and from observation it is probable that this is nearly true for any pipe and also for an open stream. For elbows of other angles the relative loss of head is known for small pipes (chap v. art. 6), and it may be assumed that for other channels it is roughly the same.

At a bifurcation (Figs. 20 and 21) the stream entering the branch may be regarded as flowing round a bend whose outer

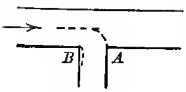


Fig. 20.

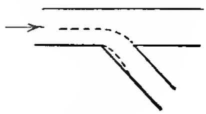


Fig. 21.

boundary is shown by the dotted lines. In the main channel below the branch there is an enlargement (art. 18). Let θ be the angle made by the centre lines of the branch and of the main channel upstream of it. When θ is 90° or thereabouts the whole head due to the velocity is lost, and there is a fall in the surface or hydraulic gradient of the branch of about the same amount as there would be if it issued from a reservoir. But if V is high the absence of contraction at A does not compensate for the excessive contraction at B, and the fall is increased, or the discharge of the branch diminished. When θ exceeds 90° the component of V resolved parallel to the axis of the branch may be regarded as velocity of approach, the discharge being increased accordingly. It is not known for what angle the velocity of approach compensates for the greater contraction as compared with that in the case of a reservoir. The angle differs with the velocity and probably with the width of the branch, and is perhaps generally not much

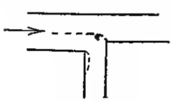


Fig. 22.

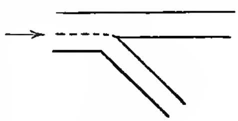


Fig. 23.

greater than 90°. By the arrangement shown in Figs. 22 and 23, the losses of head both in the branch and in the main stream are reduced, and that in the branch is not relatively altered by a high velocity. If

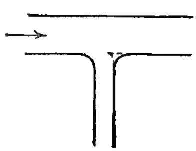


Fig. 24.

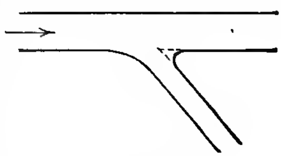


Fig. 25.

the branch is 'bell-mouthed' (Figs. 24 and 25) the loss of head in it is somewhat reduced, and it is further reduced by filling in the portions shown in dotted lines, thus doing away with eddies.

Figs. 20 to 25 represent junctions if the stream is supposed to flow in the directions opposite to those of the arrows. The losses of head are very much the same as in the corresponding cases of bifurcations.

21. **Concerning all Abrupt Changes.**—The 'limits' of an abrupt

change are those of the peculiar local flow caused by it. The upstream limit is, in Fig. 4, at *A'B'*, in Fig. 3, just as with a weir and certain kinds of orifices (art. 7), at *ST*. In the other cases it is where the eddying or curvature begins. In all cases eddies exist in the stream itself for some distance downstream of an abrupt change. The downstream limit is where these eddies have become reduced. They may not cease altogether for a long distance.

In the reach downstream of an abrupt change the flow, except for eddying and probably disturbance of the relation to one another of the various velocities in the cross-section, is normal, and the water-surface or hydraulic gradient takes the level suited to the discharge just as if no abrupt change existed. Within the limits of the abrupt change there occurs the fall or rise discussed in the three preceding articles. Thus the level of the surface or hydraulic gradient at the downstream limit of the abrupt change governs that at the upstream limit, and this again affects the slope in the upstream reach in the manner indicated above (art. 11). But the distribution of the velocities in the upstream reach is normal.[1] There is nothing to affect it until the abrupt change actually begins. (Cf. also Bends, art. 13.) Thus, at all changes, whether of sectional area or direction of flow, and whether strictly abrupt or not, the effect on the hydraulic gradient or slope is wholly upstream, but eddies and disturbance of the velocity relations are wholly downstream.

It follows that discharge observations in which the mean velocity of the whole stream is to be deduced from observations taken, say, in the centre only, should not be made within a considerable distance downstream of an abrupt change, but may be made a short distance upstream of it.

Any alteration which makes a change less abrupt reduces the loss of head. This has been seen in considering bends, elbows, and bifurcations. Regarding changes of section an instance would be the rounding of the edges of the weir in Fig. 10, p. 16, or the addition of long slopes upstream and downstream. It has been seen (art. 10) that in a short channel which gradually alters in section and then reverts to its former section, the gain of head is equal to the loss. In an open channel there will be a slight local hollow in the surface or a protuberance on it. The hollow can often be seen over a submerged weir which has gradual slopes. In any case the loss of head is negligible if the change is gradual, and especially if it is free from angularities.

[1] Having regard to the altered cross-section. See art. 12.

Section VI.—Movement of Solids by a Stream

22. Definitions.—When flowing water transports solid substances by carrying them in suspension, they are known as 'silt.' Water also moves solids by rolling them along the channel. The weight of silt present in each cubic foot of water is called the 'charge' of silt. Silt is chiefly mud and fine sand; rolled material is sand, gravel, shingle, and boulders. When a stream obtains material by eroding its channel, it is said to 'scour.' When it deposits material in its channel, it is said to 'silt.' Both terms are used irrespective of whether the material is carried or rolled. Material of one kind may be rolled and carried alternately.

23. General Laws.—It is well known that the scouring and transporting power of a stream increases with its velocity. Observations made by Kennedy prove that its power to carry silt decreases as the depth of water increases.[1] The power is probably derived from the eddies which are produced at the bed. Every suspended particle tends to sink, if its specific gravity is greater than unity. It is prevented from sinking by the upward components of the eddies. If V is the velocity of the stream and D its depth, the force exerted by the eddies generated on one square foot of the bed is greater as the velocity is greater, and is, say, as V^n. But, given the average charge of silt, the weight of silt in a vertical column of water whose base is one square foot is as D. Therefore the power of a stream to support silt is as V^n and inversely as D. Kennedy found that for the heavy mud mixed with fine sand found in the rivers of Northern India—except in their low stages—where they debouch from the Himalayas,

$$V = {\cdot}84 D^{\cdot 64} \quad . \; . \; . \quad (19)$$

This equation is not exact. It is impossible to construct a theoretical equation which shall include both suspended and rolling matter, because the proportions in which they exist are not known.

A stream of given velocity and depth can only carry a certain charge of silt. When it is carrying this it is said to be 'fully charged.' In this case, if there is any reduction in velocity, or if any additional silt is by any means brought into the stream, a deposit will occur (unless there is also a reduction of depth) until

[1] *Min. Proc. Inst. C.E.*, vol. cxix.

the charge of silt is reduced again to the full charge for the stream. The deposit may, however, occur slowly, and extend over a considerable length of channel.

The full charge is affected by the nature of the silt. The specific gravity of mud is not much greater than that of water, while that of sand is about 1·5 times as great. The particles of sand are larger. If two streams of equal depths and velocities are fully charged, one with particles of mud and the other with particles of sand, the latter will sink more rapidiy and will have to be more frequently thrown up. They will form a smaller proportion of the volume of water.

It is sometimes supposed that the inclination of the bed of a stream, when high, facilitates scour, the material rolling more easily down a steep inclined plane. The inclination is nearly always too small to have any appreciable direct effect on the rolling force. In fact the bed is generally more or less undulating, and the movement may be either uphill or downhill. The inclination of the surface of the stream of course affects its velocity, and this is the real factor in the case.

It has sometimes been said that increased depth gives increased scouring power, because of the increased pressure, but this is not so. The increased pressure due to depth acts on both the upstream and downstream sides of a body. It is moved only by the pressure due to the velocity.

To what degree the addition of a charge of silt to a pure stream affects its velocity is not known. It is not likely that it has any appreciable effect.

If a stream has power to scour any particular material from its bed, it has power to transport it; but the converse is not usually true. If the material is hard and compact the stream may have far more difficulty in eroding it than in transporting it.

If a stream is not fully charged, it tends to become so by scouring its bed. A stream fully charged with mud cannot scour mud from its bed, but its power to roll solids is, perhaps, unaffected by its being charged with mud.

In the 'Inundation Canals,' so called because they flow only when the rivers are in flood, fed from the rivers of Northern India, the silt entering a canal usually consists of sand and mud. The sandy portion, or most of it, is deposited in the head reach of the canal, forming a wedge-shaped mass, with a depth of perhaps two or three feet at the head of the canal, diminishing to zero at a point a few miles from the head. Beyond this point the water, charged with mud and perhaps a little sand, usually flows for many miles without any deposit occurring, although there are frequent reductions in the velocity caused by the diminutions in the size of the stream as the distributaries are taken off, and sometimes also by reductions in the gradient. The absence of further deposits, inexplicable till the discovery of Kennedy's law, is due to the fact that the depth of water diminishes as well as the velocity. Many of the channels were constructed long ago by the natives, and they seem to have learned from experience to give the channels such widths that the depth of water decreases at the proper rate.

It is a common practice to so reduce the velocity of a stream that silting must take place. The object may be either to 'warp up' certain localities by silt deposit or to free the water from silt, and thus reduce the deposit in places further down. When the velocity of a stream is arrested altogether, as it practically is when a stream flows through a large reservoir, the whole of the silt will deposit if it has time to do so, that is, if the reservoir is large enough. Low-lying and marshy plots of ground may be silted up, and rendered healthy and culturable by turning a silt-bearing stream through them. In order to prevent deposit in the head of a canal the water may be made to pass through a 'silt-trap' or large natural or artificial basin, where the velocity is small, or the supply may be drawn from the upper layers of the river water (art. 24).

Silting and scouring are generally regular or irregular in their action according as the flow is regular or irregular, that is, according as the channel is free or not from abrupt changes and eddies. In a uniform canal fed from a river the deposit in the head of the canal forms a wedge-shaped mass, as above stated, the depth of the deposit decreasing with a fair approach to uniformity. Salient angles are most liable to scour, and deep hollows or recesses to silt. Eddies have a strong scouring power. Immediately downstream of an abrupt change scour is often severe.

Most streams vary greatly at different times both in volume and velocity and in the quantity of material brought into them. Hence the action is not constant. A stream may silt at one season and scour at another, maintaining a steady average. When this happens, or when the stream never silts or scours appreciably it is said to be in 'permanent régime.'

Waves, whether due to wind or other agency, may cause scour, especially of the banks. Their effect on the bed becomes less as the depth of water increases, but does not cease altogether at a depth of 21 feet, as has been supposed. Salt water possesses a power of precipitating silt.

24. Distribution of Silt Charge.—Since the eddies are strongest near the bed, the charge of silt must generally increase towards the bed, but the rate of increase varies greatly. Mud having a low specific gravity, the charge is probably nearly as great near the surface as elsewhere. Sand is heavy, and is oftener rolled than carried. When carried it is usually in much greater proportion near the bed. Materials, such as boulders, do not generally rise much above the bed. A perfectly clear stream may be rolling solids. The ratio of the silt-charge at the surface to that at the bed thus varies from 0 to 1. For a given kind of silt the rate of variation from surface to bed probably increases with the depth and decreases with the velocity. The distribution in any particular stream can only be ascertained by observation, or by experience of similar streams. It is a matter of great practical importance, as affecting the best bed-level for a branch taking off from the stream. The results of observations show considerable discrepancies, even when averaged, and individual observations very great discrepancies. In some rivers 10 to 17 feet deep the silt charge has been found to increase at the rate of about 10 per cent. for each foot in depth below the surface. In others, with depths ranging up to 16 feet, the silt charge at about three-fourths or four-fifths of the full depth has been found to bear to that near the surface, a ratio varying from $1\frac{1}{4}$ to 2.

SECTION VII.—HYDRAULIC OBSERVATIONS AND CO-EFFICIENTS

25. Hydraulic Observations.—It is frequently necessary in Hydraulic Engineering to observe water-levels, dimensions of streams, and velocities, and from these to compute discharges. The object of a set of observations may be either simply to ascertain, say, the discharge in a particular instance, or to find and record the co-efficients applicable to the case, so as to enable other discharges under similar conditions to be calculated. Observations of the latter class, when extensive, are usually termed 'Hydraulic Experiments.' A consideration of the instruments and methods adopted in Hydraulic Observations may be strictly a matter of Hydraulic Engineering, but it is necessary to include it in a general manner in a Treatise on Hydraulics, both because the principles involved in such work are closely connected with the laws of flow, and also in order that proper estimates may be formed of the errors which are possible and of the reliability of the results which have been arrived at by various observers.[1]

In making observations accurate measurements of lineal dimensions, depth, and water-levels are necessary, as well as accurate timing. The number and duration of the observations should be sufficient to eliminate the effects of the irregular motion of the water, and bring out the true average values of the quantities sought for. Owing to imperfections in these matters, or in the instruments used, errors of various kinds may occur. These are known as 'observation errors.' They may balance one another more or less, but are liable to accumulate in one direction in a remarkable manner. Care in observing, as well as sufficiency in the number of observations, are therefore essential points. An error in measuring length or time has, of course, a greater relative effect when the amount measured is small. In a channel the fall in the surface or hydraulic gradient is often a small quantity, and thus in slope observations the error is often large. With an aperture under a small head the error in observing it may be serious. It has been shown by Smith[2] that, even in the careful experiments made by Lesbros on orifices, the co-efficients were probably affected by such causes as the expansion and contraction of the long iron handles attached to the movable 'gates,' and to the bending, under great pressure, of the plates forming the orifices. Besides quantities which can be actually measured there are conditions which can be observed but may be overlooked, such as a slight rounding of a sharp edge, the clinging of some portion of the water to an aperture when it is supposed to be springing clear, or the occurrence of a deposit in a channel. Such matters not always very perceptible may have considerable effects on the flow.

Again, there are conditions which cannot be ascertained, and assumptions are made regarding them. It has, for instance, been assumed that a local surface-slope too small to be observed is the same as the observed slope in a great length, or that the diameter of a pipe, measured at only a few places, is constant throughout. Lastly, there are some things very difficult to describe, such as the degree of sharpness of an edge, or of roughness of a channel. Thus there is often, in accounts of experiments, a defective or erroneous description of the conditions which existed. This may be termed 'descriptive error.' In some cases it has been

[1] Details will be given in chap. viii.

[2] *Hydraulics*, chap. iii.

very great. Its effect is similar to that of observation error, and the line between the two cannot easily be drawn.

When the quantity whose law of variation is sought depends on several conditions which vary together, it is often difficult to determine the effect of the variation of any one condition alone. As far as possible observations should be made with only one condition varying at a time. Generally, observations at one site are kept distinct from those at other sites, but if the conditions of different sites are nearly similar, it is legitimate to combine observations at different sites. In such a case, care should be taken that the effect of any slight or accidental dissimilarity in the sites will not affect any one set of values, but will be distributed throughout all. It would, for instance, be undesirable to have all the low-water observations at one site and the high-water observations at another.

A series of observations containing a source of error may show results quite consistent with one another, and may be of great use in bringing out certain laws. The well-known weir experiments of Francis and of Fteley and Stearns give results which are consistent, and were for long accepted as practically correct; but when they are compared with the later results of Bazin certain discrepancies appear, and it is clear that one or the other set of experiments contains some error.

Detailed accounts of Hydraulic Experiments do not, of course, find a place in a textbook. References to the chief works on such experiments have already been given (p. 7), but special points will be noticed whenever necessary.

26. Co-efficients.—From the causes above stated the co-efficients, or other figures, arrived at by various observers frequently show grave discrepancies. This is especially the case with the older experiments. In the more recent ones the discrepancies have been reduced.

The 'probable errors' of co-efficients have in some cases been estimated by those who have investigated them. The meaning of this may be explained by an example which will be made to include all kinds of errors. Let a weir have a crest 1 foot wide, sharp edges, and a head of 1 foot. Suppose the co-efficient arrived at is ·600, and that it is estimated that the observation error may probably be 1 per cent. either way. Then 1 per cent. is the probable error, and the value of the co-efficient is as likely to be between ·606 and ·594 as to be outside of these limits. But there may also have been descriptive errors connected with, say, the width of the crest or sharpness of the edges, and the real probable error may be much greater than 1 per cent. Finally, if the co-efficient is applied to a weir, over which water is actually flowing, there may be again observation error in measuring the head. Sometimes these different errors balance one another, but sometimes, as before remarked, they all accumulate in one direction.

The co-efficients for different cases contain probable errors of very different amounts. For thin-wall apertures under favourable circumstances, the probable error is only about ·50 per cent. For channels and especially for pipes, owing chiefly to the causes above indicated (arts. 9 and 11), it may easily be 5 or 10 per cent.

Although in the above instance the final operation of observation introduces an additional error, complete observation is much better than calculation. If no co-efficient had been assumed at all, but the discharge of the stream carefully observed, as well as the head on the weir, then both the discharge and the co-efficient for that particular case would have been obtained in the best possible manner.

The results of individual experiments nearly always show irregularities, that is when plotted they do not give regular curves. The usual method is to draw a regular curve in such a manner as to average the discrepancies and correct the original observations. Most published co-efficients have been obtained in this manner.

When an experimenter obtains a series of co-efficients for any particular case, he often connects them by an empirical formula involving one or two constants. This has been done by Bazin and Kutter for open channels, and by Fteley and Stearns, Francis and Bazin for certain kinds of weirs. What the engineer really needs and uses is a table of the co-efficients, but the formulæ may be useful in finding a co-efficient when a table is not at hand, or in finding its value for cases intermediate between those given in the tables or outside the range of the observations. This last practice must, however, be adopted with caution and within narrow limits.

Further experiments are required in all branches of hydraulics. A feature in future experiments will no doubt be the increased use of automatic and self-recording methods.

The most recent observations generally command most confidence. Causes of error are constantly being studied and eliminated. Due weight is given to this consideration in the task—often difficult—of deciding what figures shall be adjudged to be the best.

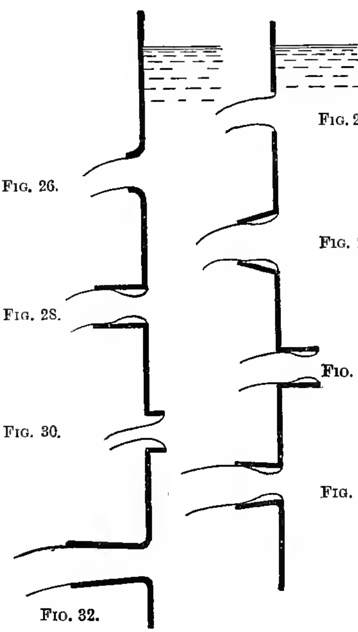
CHAPTER III

ORIFICES

[For preliminary information see chapter ii. articles 4, 5, 7, 14, and 15]

Section I.—Orifices in General

1. General Information.—The principal kinds of orifices or short tubes met with in practice, with their average co-efficients, are as follows:—



Sketch.	Description of Orifice or Tube.	Average Co-efficients for Complete Contraction.		
		c_c	c_v	c
Fig. 25A.	Orifice in thin wall,	·63	·97	·61
Fig. 26.	Bell-mouthed tube,	1·0	·97	·97
Fig. 27.	Convergent conical tube, . .	·98*	·96*	·94*
Fig. 28.	Cylindrical tube, .	1·0	·82	·82
Fig. 29.	Inwardly projecting cylindrical tube, .	1·0	·72	·72
Fig. 30.	Borda's mouthpiece, . .	·52	·98	·51
Fig. 31.	Divergent conical tube, . .	...	...	1·46†
Fig. 32.	Divergent tube with bell-mouth,	1·0‡	2·0‡	2·0‡

* For the smaller end of the tube and when angle of cone is 13°.
† For the smaller end when angle is 5° 6′.
‡ For the smallest section.

The co-efficients given, except for conical tubes, are approximate and average values, further details being given in the succeeding articles. The length of a tube must not exceed three times the diameter; otherwise the co-efficient is reduced, owing to friction, and the tube becomes a pipe. A tube generally has its axis horizontal, but may have it in any direction. If the lengths of the cylindrical tubes (Figs. 28 and 29) are reduced till the jet springs clear from the upstream edge, the co-efficients change to the values shown for Figs. 25A and 30. The length at which the change takes place may for a very great head be two diameters or more, but is generally less than one diameter. The cross-sections of all the tubes are supposed to be circular, but the co-efficients apply nearly to square sections and to others differing not greatly from circles and squares. Thus 'cylindrical' includes 'prismatic,' and similarly with the others. In the case of an elongated section, 'diameter' is to be understood as 'least diameter.'

For orifices up to a foot in diameter, metal edges filed sharp should be used, if full contraction is required. For larger orifices edges of wood, stone, or brick give fair accuracy. These remarks apply to all kinds of orifices in which the edges are supposed to be sharp, that is to all except bell-mouths, though with a convergent conical tube the effect of want of sharpness is probably small, the final contraction occurring outside the tube.

In the cases of the inwardly projecting tubes represented by Figs. 29 and 30, the tubes are supposed to be quite thin and their inner edges sharp.

The co-efficient of discharge does not generally alter much as the head varies, so that, neglecting the effect of velocity of approach, the discharge through a given orifice under different heads is nearly as $H^{\frac{1}{2}}$. In order to double the discharge H must be quadrupled. If the head is doubled the discharge is increased in the ratio of about 1·4 to 1.

To facilitate the working out of problems, the theoretical velocities corresponding to various heads are given in table i. V can be found from H or H from V.

2. Measurement of Head.—Upstream of an orifice there may be a vortex in the water, or, when the velocity of approach is high, a wave or heaping of water where it strikes the wall, and the head should be measured a short distance upstream from such vortex or wave. If the part of a reservoir adjoining an orifice is closed (Fig. 33) the head may be measured at R, but if the length of the closed portion is more than thrice its least diameter, it is necessary to find the loss of head in it, treating it as a pipe.

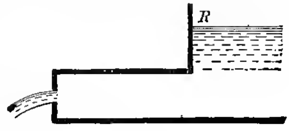


FIG. 33.

Smith states that for an orifice in a thin wall the head should probably be measured to the centre of gravity of the vena contracta. The matter seems to admit of no doubt, and the rule should apply to all kinds of orifices in which there is contraction. It is at the vena contracta and not elsewhere that the theoretical velocity is $\sqrt{2gH}$. In a bell-mouthed orifice in a horizontal wall the head would be measured to the 'discharging side' of the orifice, and the jet from an orifice in a thin horizontal wall issues under the same conditions, except that friction against the sides is removed. Under a small head the jet from an orifice in a thin vertical wall may drop appreciably in the distance PM (Fig. 34), and the true head, that at M, is not the same as at P, the centre of the orifice. Nearly all co-efficients have been obtained from orifices in vertical walls under considerable heads, so that it has made no difference how the head has been measured; but in applying these co-efficients to orifices in other positions the head should be measured to the vena contracta.

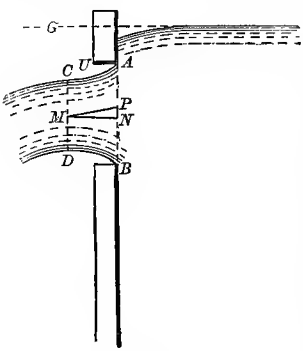


FIG. 34.

3. Incomplete Contraction.—The contraction in an orifice with a sharp edge may be partly suppressed by adding an internal projection AB (Fig. 35), extending over a portion of the perimeter of the orifice. The contraction is then said to be 'partial.' If the length AB is not less than 1·5 times the least diameter of the orifice, the co-efficients for orifices in thin walls are, according to Bidone—

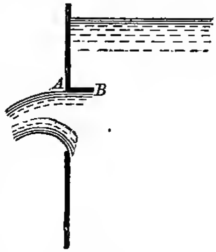


FIG. 35.

$$\text{For a rectangular orifice } c_p = c\left(1 + \cdot 152\frac{S}{P}\right) \quad . \quad . \quad . \quad (20),$$

$$\text{For a circular orifice } c_p = c\left(1 + \cdot 128\frac{S}{P}\right) \quad . \quad . \quad . \quad (21),$$

where c is the co-efficient of discharge for the simple orifice, P its perimeter, and S that of the portion on which the contraction is suppressed. Partial suppression may be caused by making one or more of the sides of an orifice flush with those of the reservoir. The above formulæ were obtained with small orifices and heads under six feet. They are not applicable when $\frac{S}{P}$ is greater than $\frac{3}{4}$ for a rectangle or $\frac{7}{8}$ for a circle. They are not quite reliable in any case, and especially when the orifice is elongated. With a rectangular orifice of length twenty times its breadth the suppression of the contraction on one of the long sides has been found to increase c by 8 to 12 per cent., whereas by the formula the increase should be 7·2 per cent.

The table on p. 56 shows that suppression of the contraction on 1, 2, 3, and 4 sides of an orifice 4 ft. square caused c to increase by about 4, 13, 28, and 56 per cent. respectively, the final result (c about ·95) being very much what would be expected.

If the contraction is suppressed on part of the perimeter, that on the remaining part increases, and this is what would be expected. The increase is, no doubt, most pronounced on the side opposite to the suppressed part, because the contracting filaments of water are no longer directly opposed by others.

In a bell-mouthed tube the contraction must be complete, whatever the clear margin may be. In all other cases decrease in the clear margin causes the contraction to be 'imperfect.' In chapter iv. (art. 3) some rules are given regarding the allowance to be made for imperfect contraction with weirs in thin walls. Considering them in connection with the above formulæ for partial contraction the figures shown in table ii. are arrived at. In this table S' is the length of the perimeter on which the clear margin is reduced, G the width of the margin in the reduced part, d the least diameter of the orifice, and c, c_i the co-efficients for the orifice with complete and incomplete contraction respectively. The table is meant for orifices in thin walls, but even for these it is only approximate. The table on page 56 deals with some other orifices with sharp edges. The above formulæ and figures apply to c_0 as well as to c, both probably altering in about the same proportion and c_v being constant. It may happen that the contraction is suppressed on one part of the perimeter of an orifice and imperfect on another part. Example 4, page 74, shows the method which may be adopted for such cases. When the contraction is either suppressed or very imperfect on nearly the whole perimeter the approximation becomes very doubtful.

When an orifice ·30 feet long and ·05 feet high was bisected by vertical brass sheets of various thicknesses, it was found that a very thin sheet had little or no effect either on c or on the jet, but a sheet ·04 feet thick increased c nearly 1 per cent., the jets, however, uniting a short distance from the orifice.[1]

4. Changes in Temperature and Condition of Water.—The results of some experiments by Smith, Mair, and Unwin respectively are shown in the following table :—[2]

Kind of Orifice.	Diameter.	Amount by which Temperature of Water was raised.	Effect on the Discharge.	Head.	Remarks.
	Inches.	Fahr.		Feet.	
Orifice in thin wall,	·24	82°	Decrease of 1½ per cent.	·56 to 3·2	In all cases the initial temperature of the water was normal, namely, 45° to 61° Fahr.
	·40	144°	Decrease of 1 per cent.	1 to 1·5	
	2·5	96°	Increase of ½ per cent.	1·75	
Bell-mouthed tube,	·40	110°	Increase of 3½ per cent.	1 to 1·5	
	1·5	115°	Increase of 2 per cent.	1·75	

It is clear that it requires a great change of temperature to cause an appreciable change in the discharge, and that the change is greater the smaller the orifice. The law governing the change is not clear. Smith considers that with a head of 10 feet a change of 50° in temperature probably has no appreciable effect for orifices of more than ·24 inch in diameter.[3]

Smith states that for small orifices (·05 foot and less in diameter, and with heads less than 1 foot) the discharge fluctuates considerably, and that this is perhaps due to unknown changes in the character of the water. With either larger heads or larger orifices

[1] Smith's *Hydraulics*, chap. iii.
[2] *Ibid.* and *Min. Proc. Inst. C.E.*, vol. lxxxiv.
[3] *Hydraulics*, chap. iii.

the uncertainty disappeared. It was not due to experimental error.

Smith also states as follows. Water containing clayey sediment may have a greater co-efficient because of its oiliness. Thick oil, though very viscous, has a greater co-efficient than water. When the water is in a disturbed condition, and approaches the orifice in an irregular manner, the jet may be ragged and twisted, but c is not affected appreciably. Greasy matter adhering to the edge of an orifice slightly reduces the discharge, if the diameter is ·10 foot or less, the reduction being due to the diminished size of the orifice.

5. **Velocity of Approach.**—The subject of velocity of approach is of more importance for weirs than for orifices, and a full discussion regarding it is given in chapter iv. (art. 5). In equations 8 and 10 (pp. 13 and 14) n may be taken to be 1·0, when the aperture is opposite that part of the approach section where the velocity is greatest—that is generally the central part and near the surface—and about ·80 when it is opposite a part where the velocity is lowest—that is near the side or bottom.[1] The method of solving the above equations has been stated in chapter ii. (art. 7). For an orifice with sharp edges, whenever velocity of approach has to be taken into account, there will very likely be imperfect contraction on some part of the perimeter, and c_i must be substituted for c.

Another method of procedure is to alter the forms of the equations. Since $h=\frac{v^2}{2g}=\frac{a'^2}{A^2}\cdot\frac{V^2}{2g}$ therefore equation 8 may be written $V^2=c_v^2\left(2gH+n\frac{a'^2}{A^2}V^2\right)$.

Whence $$V^2\left(1-c_v^2\cdot n\cdot\frac{a'^2}{A^2}\right)=c_v^2\cdot 2gH.$$

Or $$V=c_v\sqrt{2gH}\sqrt{\frac{1}{1-c_v^2 n\cdot\frac{a'^2}{A^2}}} \quad . \; . \; . \; (22).$$

And $$Q=c\cdot a\cdot\sqrt{2gH}\sqrt{\frac{1}{1-c_v^2 n\frac{a'^2}{A^2}}} \quad . \; . \; . \; (23).$$[2]

These can be solved directly. The quantity $\sqrt{\frac{1}{1-c_v^2 n\frac{a'^2}{A^2}}}$ is 'a co-efficient of correction' for velocity of approach. It may be denoted by c_a. Table iii. shows some values of $\frac{a'^2}{A^2}$ for different

[1] But there are then (art. 3, also pp. 18, 19) disturbing factors. Practically n is taken as 1·0.

[2] For other forms of this equation see chap. viii. art. 17.

values of $\frac{a'}{A}$, and it also shows the value of c_a and of the quantities leading up to it, for $c_v = \cdot 97$ and $n = 1\cdot 0$. For a bell-mouthed tube a' is simply the area of the discharging side of the tube and c_v is c. When $\frac{a'}{A}$ is less than $\frac{1}{3}$ a change in c_v or in n makes very little difference in c_a, and a mere inspection of the table will enable its proper value to be found. Thus the use of c_a simplifies matters. For other kinds of orifices c must be separated into its factors c_c and c_v, and a' found by multiplying a by c_c. But it will be seen from the examples (p. 72 *et seq.*) that the use of c_a may often be convenient. In all cases the use of c_a causes a little inaccuracy when $\frac{A}{a}$ is small. If greater accuracy is required c_a may be used for the first approximation only. Another form of c_a is $\sqrt{\frac{1}{1-c^2n\frac{a^2}{A^2}}}$, which would be very convenient for sharp-edged orifices, but there are so many values of c that extensive tables would be needed.

Let $cc_a = C$, then C is an 'inclusive co-efficient' and

$$Q = Ca\sqrt{2gH} \quad . \quad . \quad . \quad (24).$$

This formula is not convenient for general use, because it would be difficult to tabulate all the values of C for different kinds of orifices for various velocities of approach. But where it is desired to ascertain by experiment the co-efficients for any orifice, so as to frame a discharge table for that orifice alone, then equation 24 is by far the best and simplest to use.

If there are two orifices supplied from the same reservoir and situated not far apart, the discharge of each may be increased by the effect of the other, especially when both are in the same wall. In Bazin's experiments twelve orifices, each $8'' \times 8''$ nearly, and capable of being closed by gates, were placed side by side. The following values of the inclusive co-efficient C were found :—

Number of gates open :	1	2	3	4	5 or more.
Total co-efficient for all :	·633	·642	·646	·649	·650.

When one gate was raised two inches and the others were fully opened the co-efficients were as follows :—

Number fully open :	1	2	3	4	5 or more.
Co-efficient for the one partly open :	·650	·657	·660	·662	·663.

The contraction was not complete, the twelve orifices being in the end of a chamber only 18 feet wide. In order that two orifices in the same plane may have no effect on one another, it is probable that there should be no overlapping either of the minimum clear margins or of the minimum areas of approach sections requisite for full contraction and for negligible velocity of approach respectively (cf. chap. v. art. 2).

6. Effective Head.—The 'effective head' over an orifice is the head which would produce the actual velocity supposing c_v to be unity. If H and H_e are the actual and effective heads

$$V=c_v\sqrt{2gH}=\sqrt{2gH_e} \quad . \quad . \quad . \quad (25).$$

If $H-H_e=H_r$, then H_r is the head wasted in overcoming resistances. Let $\frac{H_r}{H_e}=c_r$, then c_r is the 'co-efficient of resistance,' or ratio of the wasted to the effective head.

$$\text{Since } 1+c_r=\frac{H_e+H_r}{H_e}=\frac{H}{H_e}.$$

$$\text{And from equation 25 } \frac{H}{H_e}=\frac{1}{c_v^2}.$$

$$\text{Therefore } c_r=\frac{1}{c_v^2}-1 \quad . \quad . \quad . \quad (26).$$

If there is velocity of approach $H+nh$ must be put for H in the foregoing. The following table shows the values of c_r for different values of c_v. The head wasted is only a small percentage of the effective head, when c_v is high, but it may be more than the effective head when c_v is low.

$c_v=$·995	·99	·98	·97	·95	·90	
$c_r=$·010	·020	·041	·063	·111	·233	
$c_v=$·85	·82	·80	·75	·72	·715	·70
$c_r=$·384	·489	·563	·778	·929	·956	1·049

The equation $V=\sqrt{2gH_e}$ gives the actual velocity for an orifice referred to an imaginary water-surface situated H_r feet below the actual surface (Fig. 40), but the equation will not apply to another similar orifice in the same reservoir at a different level, because H_r will not have the same value.

7. Jet from an Orifice.—The jet of water from an orifice retains its coherence for some distance and then becomes scattered. With an orifice in a thin wall, not circular and not in a horizontal plane, and with a head not very great compared to the size of the orifice, a phenomenon called 'inversion of the jet' occurs. The section of the jet is at first nearly of the shape of the orifice, but afterwards spreads into sheets perpendicular to the sides of the orifice. Those portions of the jet which issue under different heads behave somewhat similarly to separate jets, which, if two of them meet obliquely, spread into a sheet perpendicular to the plane containing them. This expansion into sheets reaches a limit, and the jet contracts again to nearly the form of the orifice, but if its coherence is retained it again throws out sheets in directions bisecting the angles between the previous sheets. This is probably due to surface tension or capillarity. The fluid is enclosed in an envelope of constant tension, and the recurrent form of the jet is due to vibrations of the fluid column.[1]

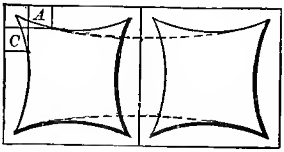


FIG. 36.

Fig. 36 shows the cross-sections of jets from two square orifices, the orifices being supposed to be far apart. At a corner the two streams A and C in contracting interfere with one another, and some fluid is forced towards the corner. The full line in Fig. 37 shows the form next assumed, and the dotted line that assumed subsequently. The dotted lines in Fig. 36 show the form of jet where the two squares are joined to form a rectangular orifice.

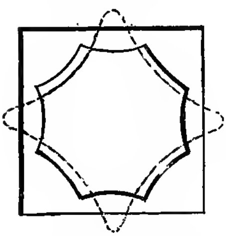


FIG. 37.

Let H_e be the effective head over an orifice. Then if the jet issues vertically upwards and H is not great, it rises to a height very nearly equal to H_e. It then expands on all sides (Fig. 38) and scatters. Let x be the head, measured from the plane AB, over any cross-section of the jet, and y the diameter of the jet at the cross-section. The velocity of the jet is very nearly $\sqrt{2gx}$ and its sectional area is as y^2. But since the discharges at all cross-sections are equal the velocities are inversely as the sectional areas. Therefore if d is the diameter of the jet at the vena contracta where the velocity is $\sqrt{2gH_e}$

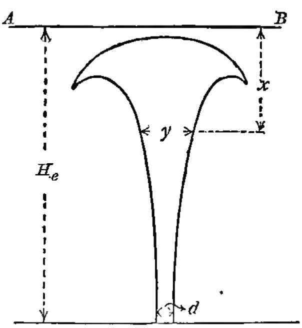


FIG. 38.

[1] *Encyclopædia Britannica*, ninth edition, Article 'Hydromechanics.'

$$\frac{y^2}{d^2}=\frac{\sqrt{2gH_e}}{\sqrt{2gx}}=\left(\frac{H_e}{x}\right)^{\frac{1}{2}}$$

Or $$y=d\left(\frac{H_e}{x}\right)^{\frac{1}{4}} \quad . \quad . \quad . \quad (27).$$

Theoretically y should be infinite when $x=0$, but practically the jet breaks up and scatters. The velocity of the jet decreases uniformly; that is, decreases by equal amounts in equal periods of time. When the head is great the jet does not retain its coherence long enough to rise to the height H_e.

A body of water issuing from an orifice in a direction not vertical describes, like any other projectile, a curve which, if the resistance of the air is neglected, is a parabola with a vertical axis and apex upwards. If the jet issues with velocity V, and at an angle θ with the horizon (Fig. 39), the equation to the parabola, as given in Dynamical Treatises, is

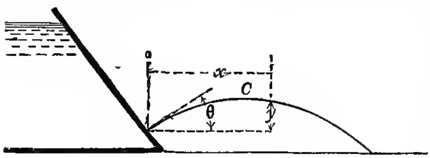


FIG. 39.

$$y=x\tan\theta-x^2\frac{g\cdot\sec^2\theta}{2V^2} \quad . \quad . \quad . \quad (28)$$

where y is the height of any point above the orifice corresponding to any horizontal distance x. The maximum value of y, that is the height of the point C above the orifice, is $\frac{V^2}{2g}\sin^2\theta$. If $y=0$

$$x=\frac{2V^2}{g}\cdot\frac{\tan\theta}{\sec^2\theta}=\frac{V^2}{g}\sin(2\theta) \quad . \quad . \quad . \quad (29).$$

This gives the range of the jet on a horizontal plane passing through the orifice. If $\theta=45°$, $x=\frac{V^2}{g}$. This is the maximum range, and in this case the maximum height is $\frac{V^2}{4g}$.

If the jet issues horizontally (Fig. 40) equation 28 becomes

$$y=x^2\frac{g}{2V^2}=\frac{x^2}{4H_e} \quad . \quad . \quad . \quad (30),$$

and the range of the jet on a horizontal plane H' feet below the orifice is

$$x=2\sqrt{H_eH'} \quad . \quad . \quad . \quad (31).$$

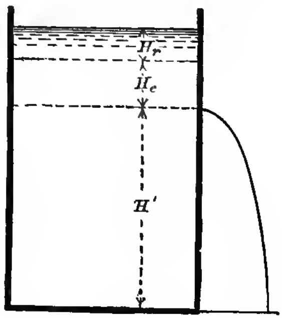


FIG. 40.

The range is a maximum when $H_e=H'$, or, for a plane passing through the bottom of a reservoir, when the orifice is slightly below mid-depth. (See also Nozzles, art. 16.)

Section II.—Orifices in Thin Walls

8. Values of Co-efficient.—The co-efficient c is best known for circular orifices. It is greater the smaller the orifice. It increases for small heads. Smith concluded that, with a great head, c was about ·592 for orifices of all sizes. This is disproved by the later and very careful experiments of Judd and King (*Engineering News*, 27th Sept. 1906) and Bilton (*Min. Proc. Inst. C.E.*, vol. clxxiv.). Some of their figures are as follows :—

Diam. of orifice,	·75 in. (B.)	·75 in. (J. & K.)	2 in. (J. & K.)	2·5 in. (J. & K.)
Head 4 feet,	·613	·609	·608	·596
Head 8 feet,	·613	·610	·608	·596
Head 92 feet,		·615	·608	·596

Bilton concludes that for each size of orifice there is a 'critical head' H_c which is greater the smaller the orifice and never exceeds 4 feet. For heads greater than H_c, c remains constant. For an orifice of a given size some observers regularly obtain lower values of c than others. Any slight rounding of the edge increases c, especially with a small orifice, and this fact tends to discredit any specially high figures. But there may be errors in measuring v_a or the diameter of the orifice or the volume discharged. Experiments made in 1898 by Bovey, Farmer, and Strickland give values of c for ·5-inch,[1] 1-inch, and 2-inch orifices generally about ·010 less than those obtained by Bilton and by Judd and King. The causes of the discrepancies may have been any of those just mentioned.

A complete set of values of c as arrived at by Bilton—and now accepted—for circular orifices is given in table iv. When the critical head is reached the co-efficient is underlined. The figures for heads which are very small, relatively to the size of the orifice, are not quite reliable. This is chiefly owing to the difficulty of observing H exactly. Bilton concludes that c is the same in whatever direction the jet issues, that it is practically the same for all circular orifices having diameters of 2·5 inches or more, and that for smaller orifices it so increases as to become 1·0 for an indefinitely small orifice. (*Cf.* table v.)

Barnes[2] arrives at figures which are in excess of Bilton's by some 1 per cent. for diameters of 1 inch to 2·5 inches, but he does

[1] Shown in table vii.
[2] *Hydraulic Flow Reviewed.*

not take account of Judd and King's figures for the 2·5-inch orifice. A few experiments show that c may continue to decrease for diameters greater than 2·5 inches, and figures for three larger orifices are included in table iv. H_c for these orifices is not exactly known, but c is practically constant for heads greater than 1·42 feet. For very large orifices in vertical planes H_c must obviously exceed 1·42 feet. The head 1·42 feet for the three larger orifices comes within the range of table x., but the values of c given in table iv. are to be used with the ordinary formula without correction. (Chap. ii. art. 5, p. 15.)

With square orifices, the streams A and C (Fig. 36) by interfering with one another prevent complete contraction occurring in the corner. Few experiments have been made, but Smith concludes that for a square orifice c is about ·005 greater than for a circular orifice of the same diameter and under the same head. See notes to table iv.

For a triangular orifice c is about ·007 greater than for a square of the same area. This is doubtless because, the angles being more acute than those of a square, the suppression of contraction in them is still greater.

Regarding rectangular orifices other than squares, c can be compared with that for a square whose side is equal to the short side of the rectangle. Tables vi. and vii. show values of c arrived at respectively by Fanning and Bovey. Fanning's figures showing c as increasing for great heads seem to be slightly inaccurate. The experiments considered by him did not include heads greater than 23 feet and only a few of these. The figures in table vi. above the thick horizontal lines are the uncorrected co-efficients. Fig. 36 and the text below it show that the jet from a rectangular orifice is greater, relatively to the size of the orifice, than for a square, and that the relative size will go on increasing as the orifice is lengthened. Since, for considerable heads, c is probably the same for all large square orifices, it would be expected that for a rectangular orifice c in table vi. would depend only on the shape of the orifice, *i.e.* it would be the same for the $4' \times 1'$ as for the $1' \times \cdot 25'$ rectangle. It will be seen that this is not far from being the case, but that the figures for the greater heads are hardly in excess of those for the corresponding square orifices. The same seems to be the case in table vii. (*Cf.* variation of figures for "orifice" in table on p. 68.)

As might be expected, c is not altered appreciably by turning an orifice about its axis into a fresh position. See remarks in table vii.

The manner in which c varies for orifices of different sizes and shapes is the opposite to what it would be if the friction of the orifice had any appreciable effect. The smaller the orifice, and the greater its deviation from a circle, the greater is the ratio of the border to the sectional area, but the greater the co-efficient.

9. Co-efficients of Velocity and Contraction.—The co-efficient c_v is, for small heads, about the same for orifices in thin walls as for bell-mouthed orifices (art. 14). It was found by Judd and King to be ·996 for the smaller orifices and ·999 for the larger, the heads ranging from 7 to 92 feet. It is usual to find c_v by observing the range of the jet on a horizontal plane (art. 7)—though the resistance of the air may cause some slight error—and to find c_c by dividing c by c_v. Judd and King, however, measured the velocity of the jet by means of a Pitot tube (chap. viii. art. 14), and they measured the diameter of the jet at the vena contracta by micrometer callipers. The resulting values of c_c and c_v agreed well.

Diameter of orifice	= ·75 in.	1 in.	1·5 in.	2 in.	2·5 in.
c_c	= ·613	·612	·605	·608	·596

The distance from the plane of the orifice to the point where the jet attained its minimum section was $·8D$ for the ·75-inch and $·5D$ for the 2·5-inch orifice (D being the diameter of the orifice), and the jet thereafter continued to have the same section. Bazin found the section, after the vena contracta had been passed, to continue to contract, but very slightly.

10. Co-efficients for Submerged Orifices.—All the co-efficients above mentioned are for cases in which the orifice discharges into air. Table viii. shows the results found by Smith for drowned orifices, the downstream water being ·57 feet to ·73 feet above the centre of the orifice. The co-efficients are less by about 1 per cent., or for small sizes 3 per cent., than for similar orifices discharging into air. The cause may perhaps be the formation of eddies, and the friction of the jet against the water surrounding it.

The following co-efficients (C) were obtained by Stewart.[1] The tubes had sharp upstream edges. They were of wood and fixed in a 10-foot channel, with margin at each side 3 ft., at bottom 2·9 ft., at top about 2 ft. $\frac{G}{d}$ (average) was only ·68 ft., so that the contraction was not complete. It was wholly suppressed on one or more sides, as noted in column 1 of the table, by adding curved approaches.

11. Remarks.—If an orifice in a thin wall is in a surface not

[1] *Engineering News*, 9th Jan. 1908.

SUBMERGED ORIFICES AND TUBES 4 FEET SQUARE.

Suppressions.	H in Ft.	Length of Tube in Ft. and Class of Orifice.							Remarks
		·31	·62	1·25	2·5	5	10	14	
		Thin Wall.		Intermediate.		Cylindrical Tube.			
Nil	·05	63	·65	·67	·77	·81	·82	·85	In this group $A=9{\cdot}2a'$ and $c=C$. In other groups $c < C$. In final group $A=5{\cdot}6a$ and c some 2·5 per cent $< C$.
	·10	·61	·63	·65	·72	·76	·78	·80	
	·20	·61	·63	·65	·71	·77	·79	·81	
	·25	·61	·63	·65		·78	·81	·83	
	·30	·61	·64	·66		·80	·83	·85	
Bottom	·05	·67			·74	·81		·85	In every column C reaches a minimum value as H increases. It increases again when H is further increased. Similarly with other groups.
	·10	·64			·70	·77		·80	
	·20	·63			·69	·78		·82	
	·25	·63				·79			
	·30	·64							
Bottom and one side	·05	·74			·77	·83		·86	For the 2·5-foot tube the suppressions produce no effect. Pressure of air surrounding jet (art. 12) probably increased.
	·10	·69			·72	·79		·81	
	·20	·68			·71	·80		·83	
	·25	·68				·81			
	·30	·69							
Bottom and two sides	·05	·83			·77	·88		·89	·88, ·83, ·85 Values of C for the 14-foot tube when a cross bulkhead was added at tail end. Ordinarily no tail bulkhead existed, and back eddies formed along sides of tube.
	·10	·77			·72	·83		·84	
	·20	·77			·71	·84		·86	
	·25	·78				·85			
	·30	·79							
All four sides	·05	·95			·94	·94	·93	·93	It is only in this group that suppression of contraction much affects the cylindrical tubes. For cylindrical tubes in general see art. 12.
	·10	·93			·91	·90	·89	·89	
	·20	·95			·92	·91	·91	·91	
	·25	·97				·93			
	·30	·98							

plane, the co-efficient will be greater or less than for a plane surface, according as the surface is concave or convex towards the reservoir.

In some districts in America, where water is sold for mining purposes, the quantity taken is measured by orifices. The 'Miner's Inch' is a term which often means the quantity of water discharged by an orifice 1 inch square, in a vertical thin wall, under a head of $6\frac{1}{2}$ inches. In this case, if c is taken at ·621, Q is 1·53 c. ft. per minute; but the head is not always the same, and the orifices used are of many different sizes, generally much larger than a square inch: the Miner's Inch is then some fraction of the total discharge, and its value in c. ft. per minute varies from 1·20 to 1·76. The Miner's Inch is, in fact, a name with local varieties of meaning. The wall containing the orifice is often made of 2-inch plank, and the chief practical point to be noted is, that with a small orifice, or a very long orifice of small height, not only is exactness of size more difficult to attain, but there may be a chance of the orifice acting as a cylindrical tube, and giving a greater discharge than intended. Before the discharge of the orifice can be known, the size, shape, head, degree of sharpness, thickness of wall, width of clear margin, and velocity of approach must all be known.

Section III.—Short Tubes

12. Cylindrical Tubes.—In a cylindrical tube (Fig. 41) the jet contracts, but it expands again, fills the tube, and issues 'full bore.' The sectional area at GK is, as in a simple orifice in a thin wall, about ·63 times the area at LM, but the velocity at GK is greater than $\sqrt{2gH}$, and the discharge through the tube is greater than that from an orifice of area LM. When the flow first begins, the air in the spaces NG, KO is at the atmospheric pressure, and the discharge is not greater than that from an orifice LM. The action of the water exhausts the air and produces a partial vacuum. Let p be the pressure in NG, KO. The pressure in the jet GK is also p. The pressures at QR and ST are P_a. Let V, v be the velocities at GK and QR. The loss of head from shock between GK and QR (equation 18, p. 32) is $\frac{(V-v)^2}{2g}$. Then from equation 5, p. 11, if the tube is horizontal,

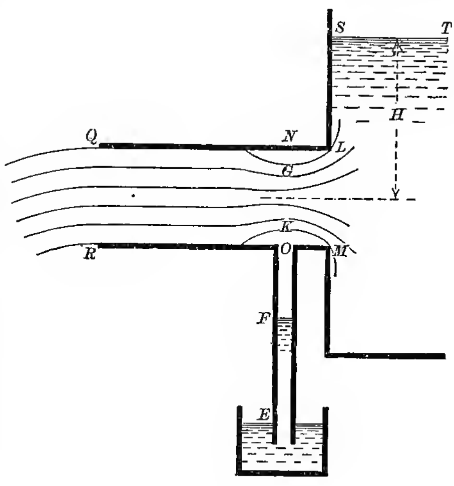


Fig. 41.

$$H+\frac{P_a}{W}=\frac{p}{W}+\frac{V^2}{2g} \quad . \quad . \quad . \quad \text{(A)}$$

And $$H+\frac{P_a}{W}=\frac{P_a}{W}+\frac{v^2}{2g}+\frac{(V-v)^2}{2g} \quad . \; . \; . \; (B).$$

But $$v=\cdot 63V \text{ and } V-v=\cdot 37V.$$

Therefore from (B) $$H=\frac{V^2}{2g}\left\{(\cdot 63)^2+(\cdot 37)^2\right\}=\frac{\cdot 534V^2}{2g}$$

Or $$V=\sqrt{\frac{2gH}{\cdot 534}}=\sqrt{\frac{2gH}{\cdot 73}}=1\cdot 37\sqrt{2gH}.$$

Practically there is some loss of head between LM and GK, and actually

$$V=1\cdot 30\sqrt{2gH} \quad . \; . \; . \; (32),$$
$$v=\cdot 63V=\cdot 82\sqrt{2gH} \quad . \; . \; . \; (33).$$

Also from (A) $$H+\frac{P_a}{W}=\frac{p}{W}+\frac{V^2}{2g}$$
$$=\frac{p}{W}+(1\cdot 30)^2H.$$

Therefore $$\frac{P_a}{W}-\frac{p}{W}=\cdot 69H \quad . \; . \; . \; (34),$$

Or the pressure at GK is less than the atmospheric pressure by $\cdot 69WH$. The result is nearly the same if the tube is not horizontal, provided H is large relatively to the length of the tube. If c_c is not exactly ·63, or if the actual loss of head differs from that assumed, the above results are somewhat altered. With a great head the vacuum becomes more perfect, the contraction, owing to the diminished pressure on the jet, less complete, and the figures 1·30 and ·69 are reduced. For moderate heads they are found to be about 1·32 and ·75.

If holes are made at N, O, water does not flow out but air enters, and the discharge of the tube is reduced. If a sufficient number of holes are made, or if the whole tube and reservoir are in a vacuum, or if the tube is greased inside, so that water cannot adhere to it, the discharge is no greater than for a simple orifice. If the holes are made at a greater distance from LM than about 1½ diameters the discharge is unaffected. If a tube is added communicating with a reservoir E, the water for ordinary heads rises to a height $EF=\cdot 75H$, and if the height EO is less than this, water will be drawn up the tube and discharged with the jet. This is the crudest form of the 'jet pump.' The height to which water can be pumped, even if the vacuum is perfect, is limited to 34 feet. The discharge of the tube is reduced by the pumping. With a great head the quantity $\cdot 75H$ may exceed 34 feet, but in no case can the difference of pressures exceed that due to 34 feet.

The co-efficient of discharge for a cylindrical tube, like that for a simple orifice, increases as the head and diameter decrease. The approximate values are given in table ix., but the number of observations made has not been great. For large tubes see p. 55.

The co-efficient for a tube ACG or $ACGE$ (Fig. 42), CD being $AB \times \cdot 79$, has been found to be the same as for a simple cylinder.

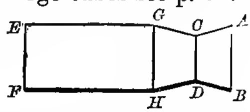


FIG. 42.

13. Special forms of Cylindrical Tubes. —If the tube projects inwards (Fig. 43) the contraction and loss of head by shock are greater than in the preceding case, and if the edge of the tube is sharp the co-efficients c_v and c are reduced to about $\cdot 72$. This is because some of the water comes from the directions AB and CD. For small tubes see table v.

When the length AC (Fig. 44) is so short that the jet does not again touch the tube, it is known as Borda's mouthpiece. For

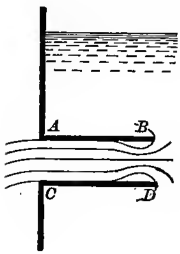


FIG. 43.

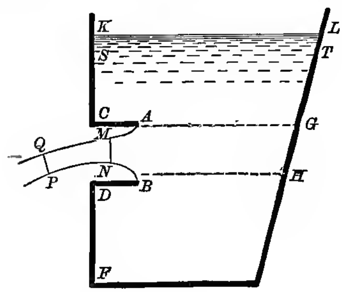


FIG. 44.

small heads AC is about half of AB. The co-efficient c_v is about the same as for a simple orifice, but the contraction is greater. It is the greatest that can be obtained by any means. The value of c_c is $\cdot 52$ to $\cdot 54$. That of c is $\cdot 51$ to $\cdot 53$, and it does not vary much. The jet also retains its coherence longer than those from other kinds of orifices.

The co-efficient for Borda's mouthpiece can be found theoretically. The velocity of the fluid along the sides of the reservoir FD, SC, which in most orifices is considerable, is here negligible. Thus the pressures on all parts of the reservoir are taken to be the simple hydrostatic pressures, and they all balance one another except the pressure on GH, which, resolved horizontally, is $Wa\left(H+\frac{P_a}{W}\right)$. The horizontal pressure on $AMNB$ is $P_a a$. The difference between the two is WaH. In a short time t let the water between KL and MN come to $STQP$. Its change of horizontal momentum is the difference between the horizontal momenta of $KSTL$ and of $MNQP$, and that is the horizontal momentum of $MNQP$, since $KSTL$ has no horizontal momentum. This change of momentum is caused by the force WaH. Equating the impulse and momentum,

$$WaHt = WQt\frac{V}{g} = Wc_c aVt\frac{V}{g}.$$

Therefore $$H = c_c\frac{V^2}{g}.$$

Let $$V^2 = 2gH.$$

Then $$H = \frac{V^2}{2g} = c_c\frac{V^2}{g}.$$

Or $$c_c = \tfrac{1}{2}.$$

When a tube is placed obliquely to the side of the reservoir (Fig. 45) the co-efficient is about $c - \cdot 0016\,\theta$ where θ is the number of degrees in the angle made by the axis of the tube with a line perpendicular to the side of the reservoir, and c is the co-efficient for the tube when θ is 90° (Neville).

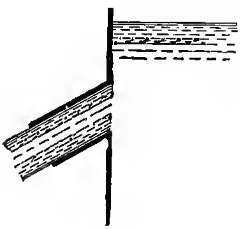


Fig. 45.

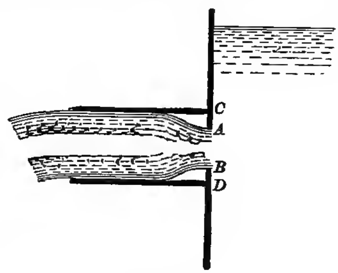


Fig. 46.

For a cylinder with a thin diaphragm at its entrance (Fig. 46) the following co-efficients are given by Neville. They apply only when the tube is filled, which it will be if not too long nor too short.

Ratio of Area. AB to area CD.	Co-efficient of Discharge for CD.
·0	·000
·1	·066
·2	·139
·3	·219
·4	·307
·5	·399
·6	·493
·7	·587
·8	·675
·9	·753
1·0	·821

14. Bell-mouthed Tubes.—A simple bell-mouthed tube (Fig. 8, page 12) is made of the shape of the jet issuing from an orifice in a thin wall. The length BE is half the diameter AB, and the curves AC, BD have a radius of 1·30 times AB. This makes $CD = \cdot 80 \times AB$. The edges at A and B must be rounded and not left sharp. Weisbach found the following co-efficients for small bell-mouthed tubes:—

Head in feet:	·61	1·64	11·48	55·77	337·93
Co-efficients (c_v and c):	·959	·967	·975	·994	·994

This form of tube is often used as a mouthpiece for pipes to prevent loss of head by contraction. If the tube is not carefully made according to the above description c will probably not exceed ·95. For tubes of square cross-section 1 foot in diameter resembling bell-mouths co-efficients of ·94 and ·95 have been found.

15. Conical Converging Tubes.—In a conical converging tube (Fig. 47) the stream contracts on entering and again on leaving the tube. The co-efficients vary with the angle of the cone, but c_v is always greater than for a cylinder. The following table shows the co-efficients found by Castel for a tube whose smaller diameter was ·61 inch, and its length 2·6 times the smaller diameter. The co-efficients have reference to the smaller end of the tube. As the angle of the cone increases c_c diminishes and c_v increases. Their product c is a maximum for an angle of 13° 24′. The co-efficients were found to be independent of the head.

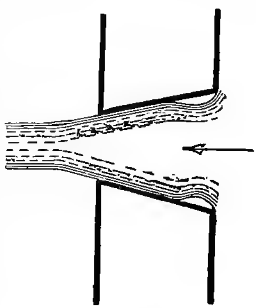


FIG. 47.

Angle of cone = 0° 0′	1° 36′	4° 10′	7° 52′	10° 20′	13° 24′	16° 36′	21° 0′	29° 58′	40° 20′	48° 50′
c_c = 1·000	1·000	1·002	·998	·987	·983	·969	·945	·919	·887	·861
c_v = ·830	·866	·910	·931	·950	·962	·971	·971	·975	·980	·984
c = ·829	·866	·912	·929	·938	·946	·938	·918	·896	·869	·847

If the angles at the entrance are rounded off so as to form a bell-mouth, c is increased by about ·015.

The following have also been found :—

Cross-section of Tube.	Head in Feet.	Smaller end of Tube.	Larger end of Tube.	Length of Tube.	Angle of Convergence.	c.
Circle	300	1·20 in. diam.	4·20 in. diam.	10 ins.	17°	1·00
Circle	2·7	1·21 „ „	1·50 „ „	·92 „		·934
Circle	1·8	2·17 „ „	2·75 „ „	7·67 „	4° 20′	·903
			3·50 „ „		10°	·898
			5·0 „ „		20°	·888
			9·83 „ „		45°	·864
Rectangle	9·6	·44 ft. × ·62 ft.	2·4 ft. × 3·2 ft.	9·59 ft.	11° 38′ and 15° 18′	·976 to ·987

Conical converging tubes are used to obtain a high velocity, but the above tables show that the velocity is not generally greater than for a bell-mouthed tube. The angle is usually 10° to 20°. A cylindrical tip is sometimes added, its length being about $2\frac{1}{2}$ times its diameter. In the case shown above, with a head of 300 feet, the jet did not touch the cylinder. If the tube projects inwards into the reservoir the co-efficient is reduced, but is greater than for an inwardly projecting cylinder. Conical tubes (Fig. 48) are used in India at canal falls for delivering streams of water on to wheels for driving mill-stones. There is loss of head both at the entrance and at the bend. The loss would be reduced by using a bell-mouth and a curve.

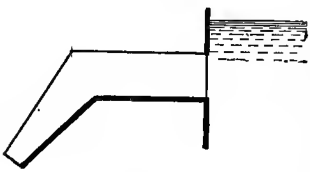


FIG. 48.

16. **Nozzles.**—In order to give a high velocity to the stream issuing from a hose-pipe a nozzle is applied to its extremity. Figs. 49 and 50 show 'smooth nozzles,' and Figs. 51 and 52

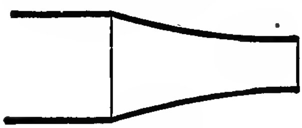


FIG. 49.

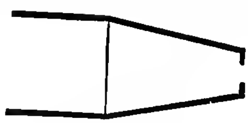


FIG. 50.

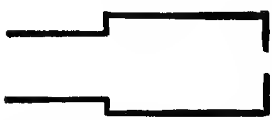


FIG. 51.

FIG. 52.

two forms of 'ring nozzle.' The diameter, d, of the orifice is usually about one-third of the diameter, D, of the pipe, and the length of the nozzle six to ten times d. Experiments with nozzles have been made by Ellis, Freeman, and others.[1] The pressure, p, at the entrance to the nozzle being measured by a pressure-gauge, the head on the nozzle is $\frac{p}{W}$. The following co-efficients have been found for the smooth nozzles, the pressure being 15 to 80 lbs. per square inch.

Diameter of orifice	$=\frac{3}{4}$ in.	$\frac{7}{8}$ in.	1 in.	$1\frac{1}{8}$ in.	$1\frac{1}{4}$ in.
c_v	$=\cdot983$	·982	·976	·972	·971

For the ring nozzle c is for Fig. 51 about ·74, and for Fig. 52, where a Borda's mouthpiece is added, about ·52. In both cases c_v is about the same as for smooth nozzles.

To allow for velocity of approach, since $\frac{D}{d}=3$, therefore $\frac{A}{a}=\frac{D^2}{d^2}=9{\cdot}0$. From table ii., noting that c_v is greater than ·97, it is clear that c_a is about 1·01, and the true co-efficient c must be increased 1 per cent. to give the inclusive co-efficient C.

The following table shows the vertical heights attained by jets from nozzles in experiments made by Ellis. It will be seen that the height of the jet is greater for the smooth nozzle than for the ring. It is also greater the larger the diameter of the nozzle, and this may be due to the jet longer retaining its coherence.

Vertical Heights of Jets from Nozzles.

Pressure in pounds per square inch.	Pressure-head in feet.	1-inch Nozzle.		$1\frac{1}{4}$-inch Nozzle.		$\frac{3}{4}$-inch Nozzle.
		Smooth.	Ring.	Smooth.	Ring.	Smooth.
10	23	22	22	23	22	...
20	46	43	42	43	43	...
30	69	62	61	63	63	59
50	115	94	92	99	95	92
70	161	121	115	129	123	113
100	230	148	136	164	155	133

The total height to which the jet remains serviceable as a fire-stream is less than that to which the scattered drops rise, the former height being about 80 per cent. of the latter for small

[1] *Transactions American Society of Civil Engineers*, vol. xxi.

heads and 60 or 70 per cent. for greater heads, but it is difficult to say exactly to what height the stream is serviceable. The heights given in the above table are the total heights. Many kinds of nozzles have been tried, but with none of them does the stream remain clear, polished, and free from spraying up to the end of the first quarter of its course. Such a stream can be obtained for a pressure of 5 or 10 lbs. per square inch, but not for a good working pressure.

17. Diverging Tubes.—With a conical diverging tube (Fig. 53) the jet contracts on entering and expands again. With a tube having an angle of 5°, smaller diameter 1 inch, and length $3\frac{1}{2}$ inches, the co-efficient of discharge for the smaller end was ·948; but with a tube having an angle of 5° 6′ and a length of nine times the smaller diameter, a co-efficient of 1·46 was found. The case is similar to a cylindrical tube. If the angle exceeds 7° or 8° the jet may not fill the tube, and the co-efficient is then reduced. If the angle is further increased, the jet does not touch the tube, and the case becomes an orifice in a thin wall.

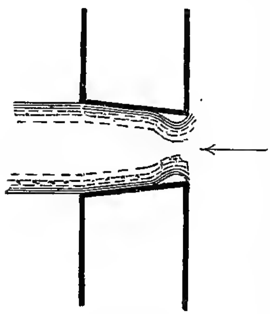


Fig. 53.

If the tube projects inwards into the reservoir the co-efficient is reduced, but is greater than for an inwardly projecting cylinder. If the length of the tube is now reduced so that the jet does not touch the tube, the co-efficient is greater than ·51, the value for Borda's mouthpiece, and becomes about ·61 if the taper is increased till the case becomes a simple orifice.

A compound diverging tube (Figs. 54 to 60) consists of a converging or bell-mouthed tube with an additional length in which the tube expands again. If there are no angularities no head is lost by shock. The case is similar to that of a cylindrical tube. The pressure at the discharging end of the tube being P_a, the pressure at the neck is less because of the higher velocity.

The following table contains information regarding various diverging tubes. It is clear that the co-efficient increases with the ratio of expansion (column 5) and decreases as the taper (column 6) increases, the highest co-efficients being obtained with high ratios of expansion and gentle taper. With a mean taper of 1 in 13·7 the limit seems to be reached when the ratio of expansion is 3·15, but with a taper of 1 in 5·33, not till the ratio is 5·0.

A negative pressure in the neck is impossible (chap. ii. art. 1), but if the vacuum there were perfect the pressure would be zero and the velocity would be $\sqrt{2g\left(H+\frac{P_a}{W}\right)}$ or $\sqrt{2g(H+34)}$. By making H small the discharge could be increased enormously, but practically the vacuum is always imperfect, and at a certain point the water ceases to fill the tube at the discharging end. The maximum co-efficient ever obtained is 2·43.

The remarks regarding pumping action made under cylindrical tubes apply equally to diverging tubes. In a vacuum or with a greased tube the discharge from a diverging tube is no greater than from the mouthpiece alone, and the same may be the case with a great head, the stream passing the expanding portion without touching it.

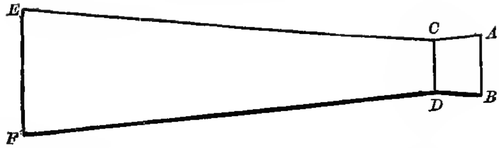


Fig. 54.

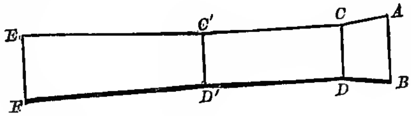


Fig. 55.

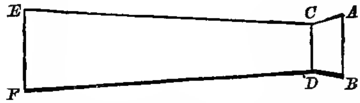


Fig. 56.

In Figs. 54, 55, and 56 $AB = 1{\cdot}5$ in., $CD = 1{\cdot}21$ in., $AC = {\cdot}92$ in.

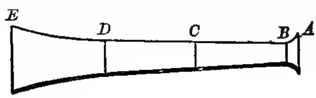


Fig. 57.

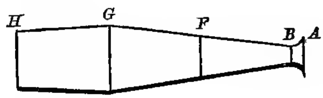


Fig. 58.

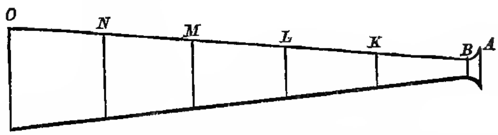


Fig. 59.

In Figs. 57, 58, and 59 AB is a bell-mouthed tube with diameter at $B = \frac{3}{8}$ in. All the other segments except DE (Fig. 57) are conical, and each is 2 in. long.

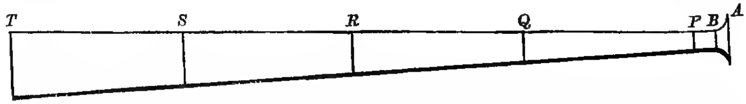


Fig. 60.

In Fig. 60 the piece AB has a cycloidal curve and BP is cylindrical. The other pieces, each 1 ft. long, are conical, but the angle of the cone is least for PQ and increases for each successive piece.

The tubes were submerged. The head varied from ·1 ft. to 1·5 ft., the co-efficient generally increased with the head (probably because the vacuum was more complete), the values 2·08 and 2·43 with the tube AS being for heads of ·13 ft. and 1·36 ft. respectively. But for a head of 1·39 ft. the co-efficient was 2·26.

(1)	(2)	(3)	(4)	(5)	(6)
Reference to Figure.	Tube.	Co-efficient for Smallest Diameter.	Smallest Dia-meter.	Ratio of Diameter at Discharging End to Smallest Diameter.	Taper of Tube, or Rate at which Diameter increases.
			Inches.		
Fig. 54	AE	1·40	1·21	2·48	1 in 5·5
,, 55	AE	1·38	1·21	1·24	1 in 14·1
,, 55	$AC+C'E$	1·43	1·21	1·24	1 in 14·1
,, 56	AE	1·57	1·21	1·59	1 in 9·1
Fig. 57	AC	1·52	·375	1·58	1 in 9·1
	AD	1·78	·375	2·17	1 in 9·1
	AE	1·87	·375	3·83	1 in 5·6 (mean)
,, 58	AF	1·69	·375	2·33	1 in 4·0
	AG	1·79	·375	3·67	1 in 4·0
	AH	1·79	·375	3·33	1 in 6·6 (mean)
,, 59	AK	1·88	·375	2·0	1 in 5·33
	AL	2·03	·375	3·0	1 in 5·33
	AM	2·07	·375	4·0	1 in 5·33
	AN	2·09	·375	5·0	1 in 5·33
	AO	2·09	·375	6·0	1 in 5·33
Fig. 60	AQ	1·48 to 1·60	1·22	1·42	1 in 23·3
	AR	1·98 to 2·16	1·22	2·30	1 in 15·1 (mean)
	AS	2·08 to 2·43	1·22	3·15	1 in 13·7 (mean)
	AT	2·05 to 2·39	1·22	4·0	1 in 13·1 (mean)

In Fig. 54 $EF = 3$ in. $CE = 9·75$ in.

In Fig. 55 $EF = 1·5$ in. $CC' = 3·0$ in. $C'E = 4·1$ in.
$CD = C'D'$

In Fig. 56 $EF = 1·92$ in. $CE = 6·5$ in.

In Fig. 57 Diameters at C, D, E are $\frac{19}{32}$ in., $\frac{13}{16}$ in., $1\frac{7}{16}$ in.

In Fig. 58 Diameters at F, G, H are $\frac{7}{8}$ in., $1\frac{3}{8}$ in., $1\frac{1}{4}$ in.

In Fig. 59 Diameters at K, L, M, N, O are $\frac{3}{4}$ in., $1\frac{1}{8}$ in., $1\frac{1}{2}$ in., $1\frac{7}{8}$ in., $2\frac{1}{4}$ in.

In Fig. 60 Diameters at B, P are 1·22 in., and at Q, R, S, T 1·74 in., 2·81 in., 3·85 in., 4·90 in.

Co-efficients for Sluices, etc.

Kinds of Aperture.	Description.	Width of Opening.	Height of Opening.	Co-efficient.	Head.
Sluice,[1] .	Shown in Fig. 61.	2·0 ft.	1·31 ft. to ·10 ft.	·61 to ·69 (averages)	·33 ft. to 9·8 ft. over upper edge.
	As above, but with boards *CF* or *DE* added.	Do.	Do.	·64 to ·70 (averages)	Do.
Do., .	In woodwork 1·77 ft. thick at bottom, and ·87 ft. elsewhere.	4·265 ft.	1·7 ft.	·625	6 ft. to 14 ft. over centre.
		Do.	·39 ft.	·803	
Iron gates,[2] Bari Doab Canal, India.	Working in grooves in the masonry heads of distributaries.	4 ft. to 10 ft.	3 ft. to 2 ft.	·72 to ·78 (averages)	·25 ft. to 4·8 ft.
Orifice,[3] .	Shown in Fig. 62.	·5 ft.	·5 ft.	·593	·5 ft. over upper edge.
	1-inch plank placed against a 6-inch space between two 2-inch planks.	1·0 ft.	,,	·607	
		1·5 ft.	,,	·615	
		2·0 ft.	,,	·621	
		2·5 ft.	,,	·626	

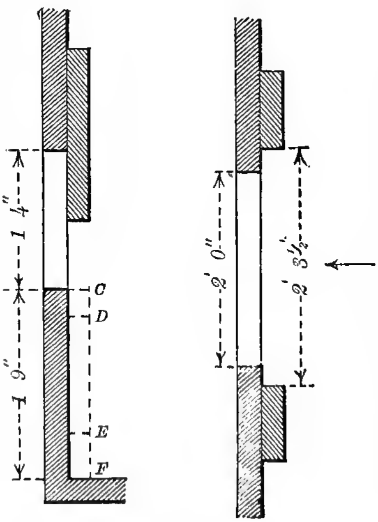


Fig. 61

[1] The smaller values of *c* occurred with the greater height of opening. For any given height of opening *c* varied as the head changed, being generally greatest for a head of about 1 ft.

[2] The co-efficient includes the allowance for velocity of approach, which was considerable. There was no contraction at the bottom and sides. The openings were generally submerged. *C* increases as *H* decreases, and it also increases with the size of the opening.

[3] The co-efficient varies in a similar manner to that for an orifice in a thin wall.

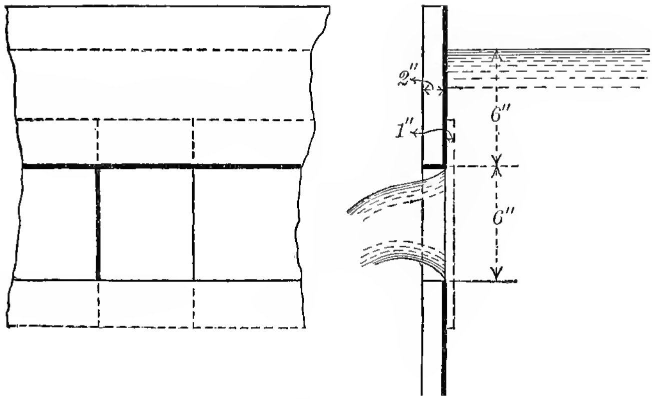


FIG. 62.

SECTION IV.—SPECIAL CASES

18. Sluices and other Apertures.—A sluice is an orifice provided with a gate or shutter. Generally there are adjuncts which complicate the case and render the co-efficient uncertain. When the gate is fully open the case may approximate to that of an orifice in a thin wall. When it is nearly closed the case may resemble that of a prismatic tube. Where accuracy is required the co-efficient must be determined experimentally. It may have any value from ·50 to ·80, or even outside these limits. The preceding table shows some values. Sometimes when a thick gate is lifted the flow tends to force it down again, especially when it is raised slightly. This is probably due to the formation of a partial vacuum under the gate.

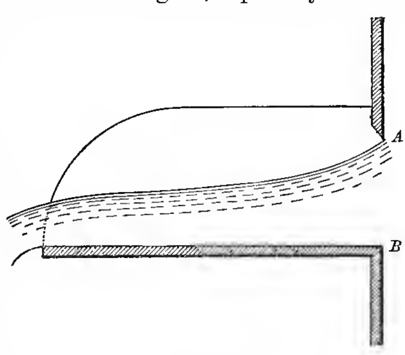


FIG. 63.

If the sides and lower edge of an orifice are produced externally so as to form a 'shoot' (Fig. 63) the co-efficient c may be greatly altered. The air has access to the issuing stream, so that reduction of pressure in the vena contracta cannot take place, as in a cylindrical tube. On the other hand the friction of the shoot has to be overcome. When the head is more than two or three times the height AB the discharge of the shoot may be nearly the same as that of the simple orifice, but otherwise it is reduced. For an orifice 8 inches by 8 inches with H_1 $4\frac{1}{2}$ inches the addition of a horizontal shoot 21 inches long reduced c from ·57 to ·48. With a horizontal shoot 10 feet long the following co-efficients have been found,[1] the orifice being ·656 feet wide. H_1 and H_2 are the heads over the upper and lower edges of the orifice.

$H_2 - H_1$.	H_1 in feet.						Remarks.
	·066	·164	·328	·656	1·64	9·84	
feet.							
·656	·48	·51	·54	·57	·60	·60	Full contraction.
·164	·49	·58	·62	·63	·63	·61	
·656	·53	·55	·57	·59	·61	·61	Lower edge of orifice flush with bottom of reservoir.
·164	·59	·61	·63	·65	·65	·65	

19. Vertical Orifices with small Heads.—Let $ACDB$ (Fig. 64) be a bell mouthed orifice. The equations for orifices of different forms are found by integration. An orifice is supposed to be divided into an infinite number of horizontal layers. The discharge of any layer is $c_v \sqrt{2gH} \cdot l dH$ where H is the head over the layer, l its length in the plane of the orifice, and dH its thickness. For a rectangular orifice

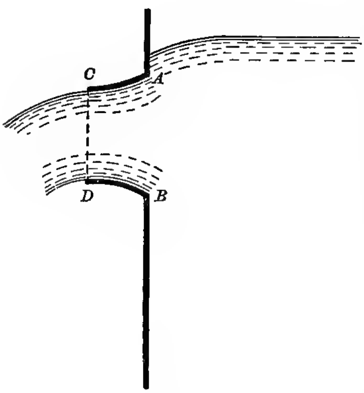


Fig. 64.

$$Q = c_v l \sqrt{2g} \int_{H_t}^{H_b} H^{\frac{1}{2}}\, dH$$

$$= \tfrac{2}{3} c_v l \sqrt{2g} (H_b^{\frac{3}{2}} - H_t^{\frac{3}{2}}) \quad . \; . \; . \quad (35),$$

where H_t and H_b are the heads at C and D respectively. The discharge is the difference between the discharges of two weirs

[1] Morin's *Hydraulique*, second edition, pp. 36 and 37.

with crests at C and D respectively, and no contraction. For a triangle whose base is upward and horizontal and of length l

$$Q=\tfrac{2}{3}c_v l\sqrt{2g}\left(\tfrac{2}{5}\cdot\frac{H_b^{\frac{5}{2}}-H_t^{\frac{5}{2}}}{H_b-H_t}-H_t^{\frac{3}{2}}\right)\quad . \; . \; . \; (36).$$

For the same triangle with base downwards and horizontal

$$Q=\tfrac{2}{3}c_v l\sqrt{2g}\left(H_b^{\frac{3}{2}}-\tfrac{2}{5}\cdot\frac{H_b^{\frac{5}{2}}-H_t^{\frac{5}{2}}}{H_b-H_t}\right)\quad . \; . \; . \; (37).$$

For a trapezoidal orifice, the lengths of whose upper and lower sides are l_t and l_b respectively, these sides being horizontal, the equation is obtained from equation 35 with 36 or 37. It is

$$Q=\tfrac{2}{3}c_v\sqrt{2g}\left\{l_bH_b^{\frac{3}{2}}-l_tH_t^{\frac{3}{2}}+\tfrac{2}{5}(l_t-l_b)\frac{H_b^{\frac{5}{2}}-H_t^{\frac{5}{2}}}{H_b-H_t}\right\}\quad . \; . \; . \; (38).$$

For a circle whose radius is R and H the head over its centre

$$Q=c_v\pi R^2\sqrt{2gH}\left(1-\frac{1R^2}{32H^2}-\frac{5}{1024}\cdot\frac{R^4}{H^4}-\frac{105}{65,536}\cdot\frac{R^6}{H^6}-\text{etc.}\right)\ldots(39).$$

If velocity of approach has to be allowed for nh must be added to each of the heads in equations 35 to 39. Thus equation 35 becomes

$$Q=\tfrac{2}{3}c_v l\sqrt{2g}\{(H_b+nh)^{\frac{3}{2}}-(H_t+nh)^{\frac{3}{2}}\}\quad . \; . \; . \; (40).$$

In every case the discharge calculated by the above equations is less than that obtained with the same co-efficient by equation 9 or 10, p. 14, but owing to the much greater simplicity of these last, it is better to use them, and to multiply the result by a second co-efficient to correct the error. These 'co-efficients of correction,' c_k, are given in table x.[1] In this table D is the height, measured vertically, between the upper and lower edges of the orifice C and D (Fig. 64), and the head in column 2 is that over a point halfway between these edges. This, in the case of triangular or semi-circular orifices, is not the head over the centre of gravity of the orifice,[2] but this latter head must be used in equation 9 or 10. The correction required is practically negligible when $H=2D$. It is greatest when $H=\cdot50D$, that is when the upper edge of the orifice is at the surface, which of course it never can be exactly.

All the above equations apply to orifices with sharp edges, but they ought to be applied to the vena contracta. Not only is D less for CD (Fig. 34, p. 45) than for AB, but H is greater because of the fall PN which the jet undergoes between AB and CD. Thus the ratio in column 2 of table x. is always greater for

[1] Smith's *Hydraulics*, chap. ii.

[2] The distance of the centre of gravity of a semicircle from its diameter is ·4244 of the radius.

CD than for AB. The co-efficients for orifices in thin walls, those which are above the horizontal lines in the columns of table vi., have however been obtained by applying the above equations to the orifice AB, and for such orifices the co-efficients should be so used, or if equation 9 or 10 is used, c_k should be taken with reference to AB. But for a sluice, cylindrical tube, or other aperture for which some other co-efficient c is to be employed, the correct method is to ascertain c_c and c_v, obtain the approximate dimensions of the jet, and find the fall PN by equation 31 (p. 52). This has been done for some square orifices, and the results utilised by adding column 1 to table x. For any entry in this column the corresponding entry in column 2 gives the approximate figure for the jet, and the value of c_k (to be applied to the result found by equation 9 or 10) is that in column 3. For a rectangle whose horizontal side l is less than D, the vena contracta is nearer to the orifice, the fall PN is less, and the contraction of the jet in a vertical direction less, so that the figures in column 1 approach nearer to those in column 2. When l is less than $\cdot 5D$ column 1 is not needed.

The co-efficients for vertical orifices under small heads are not well determined. The smallness of the margin on the upper side of the orifice tends to produce incomplete contraction there and to increase c; but, on the other hand, there is a fall in the water-surface upstream of the orifice, the head is measured above the fall, and this, according to Smith, reduces c. A vortex may also be formed, and possibly it may penetrate the orifice and reduce c. For the above reasons the corrections are of use chiefly for large orifices. They could, for instance, be applied to Stewart's co-efficients (page 56) for cases of free—not submerged—discharge.

With an orifice in a horizontal plane under a small head the proportion of water approaching axially is reduced and the contraction is probably increased, except with bell-mouths. The co-efficients for such cases having nearly all been obtained for orifices in vertical planes, are not likely to apply correctly to others, even if the head is measured to the vena contracta.

The matter in this article refers to cases where H is small compared to the orifice. If, in addition, H is actually small, the difficulties attending such cases (chap. ii. art. 7) are added.

Examples

Example 1.—Water enters the condenser of a steam-engine at the sea-level from a reservoir whose water-surface is 10 feet above the injection orifice. The pressure in the condenser is 3 lbs. per square inch. Find the theoretical velocity of flow into the condenser.

The atmospheric pressure in the reservoir is 14·7 lbs. per square inch. The resultant pressure is thus 11·7 lbs. per square inch or 1685 lbs. per square foot. This is equivalent to a head of $\frac{1685}{62\cdot4}=27$ feet. The total effective head is therefore 37 feet. From table i. the velocity is 48·7 feet.

Example 2.—Find the discharge from a circular bell-mouthed tube, 1 foot in diameter, situated in the middle of the end of a horizontal trough of rectangular section, 2 feet wide and 2 feet deep.

The head is 1 foot. From the table in article 14 c_v is probably ·96. From table x. the co-efficient of correction for small heads is ·992. A is 4 square feet and a' is ·7854 square feet. $\frac{A}{a'}=\frac{4}{\cdot7854}=5\cdot01$. From table iii. the co-efficient of correction for velocity of approach is 1·02. From table i. $\sqrt{2gH}=8\cdot02$. Then $Q=\cdot96\times8\cdot02\times\cdot785\times\cdot992\times1\cdot02=6\cdot12$ cubic feet per second.

Example 3.—A culvert 3 feet long, consisting of a semicircular arch of 1 foot radius resting on a level floor, has to pass a discharge of 9 c.ft. per second. There is a free fall downstream. What will be the water-level upstream?

From table ix. c may be taken to be ·80. Also $a=2\times\cdot785=1\cdot57$ square feet.

To obtain an approximate solution

$$Q=9=\cdot80\sqrt{2gH}\times1\cdot57\ \therefore\ \sqrt{2gH}=\frac{9}{\cdot80\times1\cdot57}=7\cdot17.$$

From table i. $H=\cdot80$, or the water will be ·80 foot above the centre of gravity of the aperture or ·22 foot above the crown of the arch.

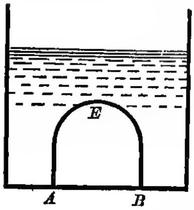
The contraction, supposed to be complete elsewhere, is nearly absent at the crown, and may be taken to be suppressed on one fourth of the perimeter, thus (table ii.) making

$$c=\cdot80\times1\cdot04=\cdot832.$$

In table x. $D=1\cdot0$ foot, and the head over the centre of the orifice is $\cdot22+\cdot50=\cdot72$ foot or $\cdot72D$. This corresponds to $\cdot86D$ for the vena contracta, and the figure in column 8, differing, no doubt, hardly at all from column 4, is ·989.

The above two corrections are 4 per cent. plus and 1 per cent. minus, so that Q is really 3 per cent. more than assumed. To make it right deduct 6 per cent. from H, which will thus be $\cdot80\times\cdot94=\cdot752$ foot, that is, the water is ·18 foot above the crown.

Example 4.—For the culvert shown in the annexed diagram (2 feet wide and 5 feet long), let there be an open approach channel 4 feet wide, with vertical walls and floor level with that of the culvert. Find the discharge when the upstream head is 1 foot above the crown of the arch, and the downstream head 6 inches above it.



In this case there is incomplete contraction on all sides, and also velocity of approach. From example 3, $a=3{\cdot}57$ square feet; $A=12{\cdot}0$ square feet; $P=4{\cdot}0+3{\cdot}14=7{\cdot}14$ feet; $S=2{\cdot}0$ feet. If the contraction were complete on AEB, c_p would be (art. 3) about $\cdot 80\times(1+\cdot 152\times\frac{2}{7})=\cdot 80\times 1{\cdot}043=\cdot 834$. The average margin on AEB is about 1·30 feet. Therefore $\dfrac{G}{d}=\dfrac{1{\cdot}30}{2}=\cdot 65$, and $\dfrac{S'}{P}=\dfrac{5{\cdot}14}{7{\cdot}14}=\cdot 75$. From table ii. $\dfrac{c_i}{c}=1{\cdot}035$ about. Therefore

$$c_i=\cdot 834\times 1{\cdot}035=\cdot 863.$$

The head is ·5 foot, and as the orifice is wholly submerged no correction for small head is needed. From table i. $\sqrt{2gH}$ is 5·67. $Q=\cdot 863\times 5{\cdot}67\times 3{\cdot}57=17{\cdot}47$ cubic feet per second.

To allow for velocity of approach by the usual method,

$$v=\frac{17{\cdot}47}{12}=1{\cdot}46 \text{ feet per second. Let } n=1{\cdot}0.$$

From table i. $h=\cdot 033$, $H+h=\cdot 533$. From table i. $V=5{\cdot}87$. Then $Q=\cdot 863\times 5{\cdot}87\times 3{\cdot}57=18{\cdot}08$ cubic feet per second.

To allow for velocity of approach by a co-efficient of correction, for the contracted section c_v is (art. 12) about 1·30, and $c_c=\dfrac{\cdot 863}{1{\cdot}30}=\cdot 664$. Therefore $a'=3{\cdot}57\times\cdot 66=2{\cdot}36$ square feet, and $\dfrac{A}{a'}=\dfrac{12{\cdot}0}{2{\cdot}36}=5{\cdot}09$. From table iii., noting that c_v is about 1·30 instead of ·97, and that the figures in column 3 are to be increased, c_a is about 1·03, that is, 3 per cent. must be added to 17·47, making 17·99 cubic feet per second.

Note.—Further examples may be obtained by taking cases analogous to some of those in examples of chap. iv.

Table I.—Heads and Theoretical Velocities. (Art. 1.)

For a head greater than 10 feet divide the head by 100 and take ten times the corresponding velocity. Thus for a head of

120 feet the velocity is 87·9, or ten times the velocity given for a head of 1·2 feet. For a velocity over 25 divide it by 10 and multiply the corresponding head by 100. The same methods can be adopted to facilitate interpolations. Thus for $H = \cdot 032$ look out 3·2.

In the first fifteen entries the heads correspond to certain definite velocities. These entries may be useful in cases of velocity of approach. After that the velocities correspond to definite heads.

H	*V*	*H*	*V*	*H*	*V*	*H*	*V*	*H*	*V*	*H*	*V*
·0022	·38	·13	2·89	·45	5·38	·84	7·35	2·3	12·2	6·2	20·0
·0025	·40	·135	2·95	·46	5·44	·85	7·40	2·4	12·4	6·3	20·1
·0027	·42	·14	3·00	·47	5·50	·86	7·44	2·5	12·7	6·4	20·3
·0030	·44	·145	3·05	·48	5·56	·87	7·48	2·6	12·9	6·5	20·5
·0033	·46	·15	3·11	·49	5·62	·88	7·53	2·7	13·2	6·6	20·6
·0036	·48	·155	3·16	·50	5·67	·89	7·57	2·8	13·4	6·7	20·7
·0039	·50	·16	3·21	·51	5·73	·90	7·61	2·9	13·7	6·8	20·9
·0042	·52	·165	3·26	·52	5·79	·91	7·65	3	13·9	6·9	21·0
·0045	·54	·17	3·31	·53	5·85	·92	7·70	3·1	14·1	7	21·2
·0049	·56	·175	3·36	·54	5·90	·93	7·74	3·2	14·3	7·1	21·3
·0052	·58	·18	3·40	·55	5·95	·94	7·78	3·3	14·5	7·2	21·5
·0056	·60	·185	3·45	·56	6·00	·95	7·82	3·4	14·8	7·3	21·6
·0066	·65	·19	3·50	·57	6·06	·96	7·86	3·5	15·0	7·4	21·8
·0076	·70	·195	3·55	·58	6·11	·97	7·90	3·6	15·2	7·5	21·9
·0087	·75	·20	3·59	·59	6·17	·98	7·94	3·7	15·4	7·6	22·1
·01	·80	·21	3·68	·60	6·22	·99	7·98	3·8	15·6	7·7	22·2
·015	·98	·22	3·76	·61	6·28	1	8·02	3·9	15·8	7·8	22·4
·02	1·13	·23	3·85	·62	6·32	1·05	8·22	4	16·0	7·9	22·5
·025	1·27	·24	3·93	·63	6·37	1·1	8·41	4·1	16·2	8	22·7
·03	1·39	·25	4·01	·64	6·42	1·15	8·60	4·2	16·4	8·1	22·8
·035	1·50	·26	4·09	·65	6·47	1·2	8·79	4·3	16·6	8·2	23·0
·04	1·60	·27	4·17	·66	6·52	1·25	8·97	4·4	16·8	8·3	23·1
·045	1·70	·28	4·25	·67	6·57	1·3	9·15	4·5	17·0	8·4	23·2
·05	1·79	·29	4·32	·68	6·61	1·35	9·32	4·6	17·2	8·5	23·4
·055	1·88	·30	4·39	·69	6·66	1·4	9·49	4·7	17·4	8·6	23·5
·06	1·97	·31	4·47	·70	6·71	1·45	9·66	4·8	17·6	8·7	23·6
·065	2·04	·32	4·54	·71	6·76	1·5	9·83	4·9	17·7	8·8	23·8
·07	2·12	·33	4·61	·72	6·81	1·55	9·98	5	17·9	8·9	23·9
·075	2·20	·34	4·68	·73	6·86	1·6	10·2	5·1	18·1	9	24·1
·08	2·27	·35	4·75	·74	6·91	1·65	10·3	5·2	18·3	9·1	24·2
·085	2·34	·36	4·81	·75	6·95	1·7	10·5	5·3	18·5	9·2	24·3
·09	2·41	·37	4·87	·76	6·99	1·75	10·6	5·4	18·7	9·3	24·4
·095	2·47	·38	4·94	·77	7·04	1·8	10·8	5·5	18·8	9·4	24·6
·10	2·54	·39	5·01	·78	7·09	1·85	10·9	5·6	19·0	9·5	24·7
·105	2·60	·40	5·07	·79	7·13	1·9	11·1	5·7	19·2	9·6	24·8
·11	2·66	·41	5·14	·80	7·18	1·95	11·2	5·8	19·3	9·7	24·9
·115	2·72	·42	5·20	·81	7·22	2	11·3	5·9	19·5	9·8	25·0
·12	2·78	·43	5·26	·82	7·26	2·1	11·7	6	19·6	9·9	25·2
·125	2·84	·44	5·32	·83	7·31	2·2	11·9	6·1	19·8	10	25·4

Table II.—Imperfect and Partial Contraction for Large Rectangular Orifices in Thin Walls. (Art. 3.)

$\frac{S}{P}$	Values of $\frac{G}{d}$.						Remarks.
	3	2·67	2	1	·5	0	
	Approximate Values of $\frac{c_i}{c}$.						
·25	1	1·000	1·002	1·006	1·015	1·04	If $\frac{G}{d}$ is not the same at all parts of the border of the orifice its mean value is to be taken. The figures for $\frac{G}{d}=1$ and ·5 are only approximations. As $\frac{G}{d}$ approaches zero c_i increases rapidly.
·50	1	1·001	1·003	1·013	1·030	1·13	
·75	1	1·001	1·004	1·019	1·045	1·28	
1	1	1·002	1·006	1·025	1·060	1·56	

Table III.—Co-efficients of Correction for Velocity of Approach. (Art. 5.)

($c_v = ·97$. $n = 1·0$.)

(1)	(2)	(3)	(4)	(5)	(6)
$\frac{A}{a'}$	$\frac{a'^2}{A^2}$	$c_v^2 n \cdot \frac{a'^2}{A^2}$	$1 - c_v^2 n \cdot \frac{a'^2}{A^2}$	$\sqrt{1 - c_v^2 n \cdot \frac{a'^2}{A^2}}$	$\frac{1}{\sqrt{1 - c_v^2 n \cdot \frac{a'^2}{A^2}}}$ or c_a.
1·3$\dot{3}$	·5625	·529	·471	·687	1·456
1·5	·4444	·418	·582	·763	1·311
2	·2500	·235	·765	·875	1·143
2·5	·1596	·150	·850	·922	1·072
3	·1111	·104	·896	·947	1·056
5	·0400	·038	·962	·981	1·019
10	·0100	·010	·990	·995	1·005
15	·0044	·004	·996	·997	1·003
20	·0025	·0024	·9976	·999	1·001

Table IV.—Co-efficients of Discharge for Circular Orifices in Thin Walls. (Art. 8.)

Head.	Diameter of Orifice in Inches.									
	·25	·50	·75	1	1·5	2	2·5	6	9	12
Feet.										
·17	·683	·663	...	...	...	...	...	...	...	...
·25	·680	·657	·646	·640	...	...	...	...	...	...
·5	·669	·643	·632	·626	·618	·612	·610	...	...	...
·75	·660	·637	·623	·619	·612	·606	·604	...	...	...
1	·653	·636	·618	·612	·606	·601	·600	...	...	...
1·42	·645	·624	·614	·608	·603	·599	·598	·597	·594	·592
1·5	·643	·623	·613							
1·83	·638	·621								
2	·637									
2·5	·635									
3·75	·629									

For a square orifice add ·005 to the above figures for same diameter and head.

The first five lines of the table on page 56 show that c for an orifice 4 feet square averaged about ·614 under low heads. This value is consistent with the above figures. It was increased by perhaps ·024 because of incomplete contraction, but it may have been decreased owing to the submergence of the orifice.

Table V.—Co-efficients of Discharge[1] for Sharp-edged Re-entrant Tubes. (Art. 13.)

Diameter (Inches)	·125	·250	·375	·50	·75	1·0	1·5	2·0	2·5
Co-efficient	·91	·87	·85	·83	·81	·79	·77	·76	·75

The length of tube was in each case 2·5 diameters. The heads were ·5 ft. and upwards. The co-efficient showed no tendency to vary with the head.

As in the case of orifices in thin walls, c tends to become 1·0 for an indefinitely small orifice.

[1] Bilton's co-efficients (*Min. Proc. Inst. C.E.*, vol. clxxiv.).

TABLE VI.—CO-EFFICIENTS OF DISCHARGE FOR RECTANGULAR ORIFICES, ONE FOOT WIDE, IN THIN WALLS. (Art. 8.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Head.	Height of Orifice in Feet.							
	·125	·25	·50	·75	1	1·5	2	4
Feet.								
·2	·634							
·3	·634	·632						
·4	·633	·632	·621					
·5	·633	·632	·619	·615				
·6	·633	·632	·619	·613	·610			
·8	·633	·632	·618	·612	·606	·630		
1	·632	·632	·618	·612	·605	·624		
1·25	·631	·632	·618	·611	·604	·624	·632	
1·5	·630	·631	·618	·611	·604	·619	·627	
2	·629	·630	·617	·610	·605	·617	·628	
2·5	·628	·628	·616	·610	·605	·615	·627	·645
3	·627	·627	·613	·610	·605	·613	·619	·637
4	·624	·624	·614	·609	·605	·611	·616	·630
6	·615	·615	·609	·604	·602	·606	·610	·618
8	·609	·607	·603	·602	·601	·602	·604	·610
10	·606	·603	·601	·601	·601	·601	·602	·604
20	·607	·604	·602	·601	·601	·601	·602	·605
30	·609	·604	·603	·602	·601	·602	·603	·605
40	·611	·606	·604	·603	·602	·603	·605	·607
50	·614	·607	·605	·604	·602	·603	·606	·609

TABLE VII.—CO-EFFICIENTS OF DISCHARGE FOR SMALL ORIFICES (area ·196 square inch) IN THIN WALLS. (Art. 8.)

Head.	Equilateral triangle, base upward.	Square * with sides vertical.	Circular.	Rectangle with long side horizontal.		Remarks.
				4 to 1 †	16 to 1 ‡	
Feet.						* With diagonal vertical c is about ·0014 greater.
1	·636	·627	·620	·643	·664	† With long side vertical c is about ·0014 less.
2	·628	·620	·613	·636	·651	‡ With long side vertical c is about ·0005 less for heads up to 10 feet, and about ·0005 more for the greater heads.
4	·623	·616	·608	·629	·642	
6	·620	·614	·607	·627	·637	
10	·618	·612	·605	·624	·633	
14	·618	·610	·604	·622	·630	
20	·616	·609	·603	·621	·629	

Table VIII.—Co-efficients of Discharge for Submerged Orifices in Thin Walls. (Art. 10.)

Head.	Size of Orifice in Feet.				
	Circle ·05 ft.	Square ·05 ft.	Circle ·1 ft.	Square ·1 ft.	Rectangle ·05 ft. × ·3 ft.
Feet. ·5	·616	·620	·602	·609	·622
1	·610	·615	·602	·606	·622
1·5	·607	·612	·601	·605	·621
2	·604	·609	·600	·604	·620
2·5	·603	·608	·599	·604	·619
3	·602	·607	·599	·604	·618
4	·601	·607	·599	·605	...

Table IX.—Co-efficients of Discharge for Cylindrical Tubes. (Art. 12.)

Head.	Diameter of Tube in Inches.			
	·25	·50	1	3
Feet. ·5	·84	·83	·82	...
2	·83	·82	·81	·80
22	...	...	·80	·80

TABLE X.—CO-EFFICIENTS OF CORRECTION FOR VERTICAL ORIFICES WITH SMALL HEADS. (Art. 19).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Head over centre of square orifice with sharp edges.	Head over centre of bell-mouthed orifice or of vena contracta for sharp-edged orifice.	Rectangle.	Circle or semicircle with diameter vertical.	Triangle with base upward.	Triangle with base downward.	Semicircle with diameter upward.	Semicircle with diameter downward.	Remarks.
	·50D	·943	·960	·924	·979	·937	·965	The co-efficients have not been worked out in detail for triangles and semicircles, but can be easily estimated from the figures given in the first and tenth lines. When the head is greater than D the co-efficients for orifices of all shapes are nearly equal.
	·52D	·950	·965					
	·55D	·957	·970					
	·60D	·966	·975					
·52D	·70D	·976	·982					
·64D	·80D	·982	·987					
·78D	·90D	·986	·990					
·92D	1·0D	·989	·992					
1·13D	1·2D	·992	·994					
1·44D	1·5D	·995	·997	·996	·998	·996	·997	
	2·0D	·997	·998					
	2·5D	·998	·999					
	3·0D	·999	·999					
	4·0D	·999	1·000					

CHAPTER IV

WEIRS

[For preliminary information see chapter ii. articles 4, 6, 7, 14, and 15]

SECTION I.—WEIRS IN GENERAL

1. General Information.—The following statement shows a few typical kinds of weirs, and gives some idea as regards the co-efficients. Further co-efficients will be given in subsequent articles, and from them the values for many cases occurring in practice can be inferred, but the varieties of cross-section are innumerable, the co-efficients vary greatly, and generally can only be found accurately by actual observation. When the length, l, of a weir is great relatively to H, it makes little difference whether there are end contractions or not.

To ensure complete contraction iron filed sharp should be used for the upstream edges with small heads. For heads of over a foot planks or masonry may be used.

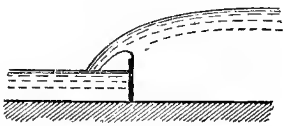
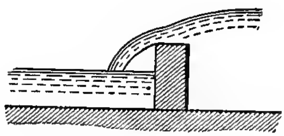
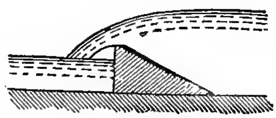
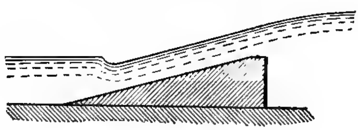
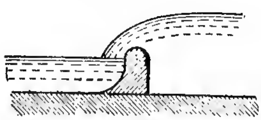
Since the inclusive co-efficient C increases with H, it follows that when there is velocity of approach Q increases faster than $H^{\frac{3}{2}}$. If H is doubled Q is about trebled. To double the discharge H must be multiplied by 1·5. If a given volume of water passes in succession over two similar weirs, one of which is three times as long as the other, the head on it will be half that on the other. If a volume of water, passing in succession over two weirs, alters, the heads on both will alter in nearly the same ratio. These rules are only approximate, and when there is no velocity of approach they are somewhat modified. To facilitate calculations the values of $H^{\frac{3}{2}}$ corresponding to different values of H are given in table xi.

Smith states that with low heads such as ·2 foot the discharge may be affected by a change in the temperature of the water of 30° Fahr. If the water is disturbed by waves or eddies the discharge is probably reduced, unless 'baffles' are used [1] to calm it.

In the sheet of water passing the edge of a weir in a thin wall

[1] Or grids. They should not be so near to a weir or orifice as to interfere with the flow of approach.

VARIOUS KINDS OF WEIRS AND THEIR CO-EFFICIENTS.

Type of Weir.	Dimensions of Weirs for which Co-efficients are quoted. Height.	Top Width.	Upstream Slope.	Down-stream Slope.	Co-efficient C for Head of 1 foot.	Manner in which Co-efficient varies as Head increases.
	Feet.	Feet.				
 FIG. 65. Thin Wall.	1·64				·67	Increases slowly
 FIG. 66. Flat top, vertical face and back.	2·46	1·31	vertical	vertical	·54	Increases rapidly
 FIG. 67. Steep back and sloping face.	1·64	·33	2 to 1	vertical	·75	Increases
 FIG. 68. Steep face and sloping back.	1·64	·33	vertical	5 to 1	·61	Increases
 FIG. 69. Rounded.	1·64				·85	Increases

These weirs are some of the types used by Bazin in his experiments. There were no end contractions. The co-efficient C includes the allowance for velocity of approach.

the velocity is greatest at the lower side, but with a broad-topped weir the friction on the top reduces the velocities nearest the weir. In every case the initial horizontal velocity of the whole sheet may be taken to be $\frac{2}{3}\sqrt{2gH}$, and the path of the sheet calculated as for orifices (chap. iii. art. 7). Fig. 70 shows a separating weir as used for water-supplies of towns. After heavy rain the water is discoloured and H is great, so that the sheet falls as shown and the water is conveyed to a waste channel. At other times the water falls into the opening K and is conveyed to the service reservoirs. The velocity at the ends of a weir is generally less than elsewhere, and it increases up to a point distant about $3H$ from the ends. The pressure in the water passing over the crest of a weir is less than that due to the head.

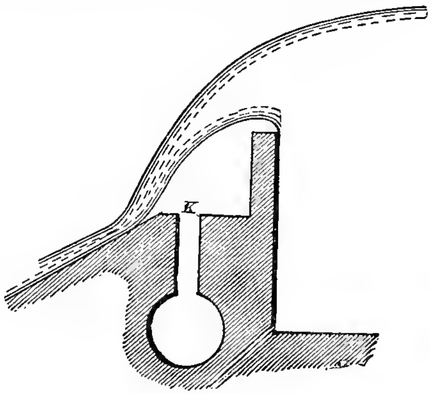


FIG. 70.

The following statement shows the chief experiments on weirs in thin walls:—

Observer.	No. of Observations made.	Length of Weir.	Head. From	Head. To	Height of Weir.	State of Contraction.	Distance of Measuring Section from Crest of Weir.
		Feet.	Feet.	Feet.	Feet.		Feet.
Francis,	46	10	·6	1·6	4·6	Complete or nearly complete.	6·0
,,	19	10	·6	1·0	2·0		6·0
,,	6	4	·7	1·0	4·6 & 2·0		6·0
Smith,	12	2·6	·6	1·7	3·8		7·6
Lesbros,	21	1·77	·1	·6	1·8		11·5
Poncelet & Lesbros,	6	·66	·08	·7	1·8		11·5
Fteley & Stearns,	54	2·3 to 5	·15	·94	3·6	Variable.	6·0
Lesbros,	34	·66	·06	·7	1·8		11·5
Francis,	17	10	·7	1·0	4·6	End contractions absent.	6·0
Fteley & Stearns,	10	19	·5	1·6	6·6		6·0
,, ,,	30	5	·07	·8	3·2		6·0
Lesbros,	14	·66	·06	·8	1·8		11·5
Bazin,	295	6·56	·23	1·0	3·7 to ·8		16·4
,,	38	3·28	·23	1·3	3·3		16·4
,,	48	1·64	·23	1·8	3·3		16·4

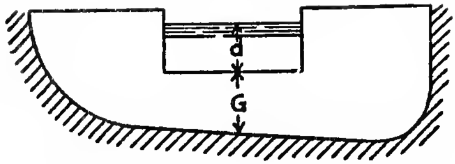


Fig. 70a.

2. Formulæ.—The ordinary weir formula (equation 11, p. 15) and the other formulæ deduced from it are defective in form. It has been said that the head ND (Fig. 71) ought to be taken into account, the discharge of the weir being considered to be that of an orifice whose bottom edge is C and top edge D. But a weir is not an orifice. The surface contraction makes the cases different. It is possible that the head H should be measured from F and not from C, and it is unlikely that $\frac{4}{9}H$ really represents exactly the head corresponding to the mean velocity. The case is really one of variable flow in a short channel, and it would probably be treated as such if it were practicable to observe the heads at D and F. As it is, shortcomings in the formula are made good by the values given to the co-efficients.

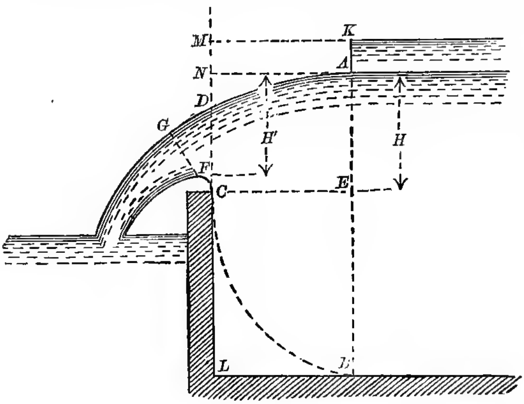


Fig. 71.

In all weir formulæ m can be written for $\frac{2}{3}c$, and this plan is adopted by Bazin; but c is the true co-efficient expressing the relation between the actual and the theoretical discharge, and it is desirable that c should be used both in formulæ and in tables. Since $\frac{2}{3}\sqrt{2g}=5\cdot35$ this figure can be used in calculations instead of $8\cdot02$, and multiplication by $\frac{2}{3}$ is thus unnecessary. The values of $\frac{2}{3}c\sqrt{2g}$ corresponding to different values of c are given in table xii. and denoted by K. They are the discharges per foot run over a weir with $H=1$ foot. Engineers frequently condense the formula by using K instead of c, but the value of c should not be lost sight of.

3. Incomplete Contraction.—From a comparison of the co-efficients obtained for various weirs in thin walls, Smith arrives at the formula

$$c_p = c\left(1 + \cdot 16\frac{S}{P}\right)$$

where c_p and c are the co-efficients for two equal weirs, one with partial and one with full contraction. P is the complete perimeter of the weir, that is $l+2H$, S the length of the perimeter over which the contraction is suppressed. This formula applies for heads ranging from ·3 foot to 1·0 foot; it is not exact, but may be used for finding co-efficients not otherwise known.

When the contraction is imperfect,[1] whether or not the margin is sufficient to give a negligible velocity of approach, the formula arrived at by Smith is

$$c_i = c\left(1 + x\frac{S'}{P}\right)$$

where c_i is the co-efficient for the weir with imperfect contraction, S' the length of its perimeter on which the contraction is imperfect, and x is as follows, d being the least dimension of the weir and G the width of the clear margin.

$\frac{G}{d}=$	3	2·67	2	1	·5	0
$x=$	0	·0016	·005	·025	·06	·16

When the contraction is imperfect over the whole perimeter $S'=P$, and when

$\frac{G}{d}=$	3	2·67	2	1	·5	0
the increase in c per cent. $=$	0	·16	·50	2·5	6	16

But when S is a very large fraction of P, or when $S'=P$ and $\frac{G}{d}$ is very small—that is, when there is not much contraction left except at the surface—the rules become of doubtful application.[2]

4. Flow of Approach.—Bazin observed some surface-curves for weirs 3·72 feet and 1·15 feet high, and for each weir with several heads ranging from ·5 feet to 1·5 feet. He finds y (Fig. 71)[3] to be in every case about $3H$, but the upper portions of the curves are so flat, especially for the lower heads, that it is impossible to say exactly where they begin. Observations made by Fteley and Stearns, with H nearly constant and different values of G, give results somewhat similar to Bazin's, but when G is less than H, y is

[1] For definitions of 'partial' and 'imperfect' see chap. iii. art. 3.

[2] Rounding of crest and sides may increase c some 20 per cent. When contraction is thus suppressed the surface contraction doubtless increases, *cf.* chap. iii. art. 3.

[3] $AN=y$.

about 2·5G. The above indicates the proper distance from the weir to the measuring section. In weirs with end contractions G', the distance of the end of the weir from the side of the channel must be used instead of G if it exceeds G. In a weir with a long sloping face Smith found y to be 40 feet with $H=7\cdot24$ feet.

The fall ND or F for weirs in thin walls is generally between $\frac{H}{10}$ and $\frac{H}{4}$. It is much greater with broad-topped weirs. In the above experiments with weirs in thin walls $\frac{F}{H}$ was found to be as follows:—

$G=$	3·56	1·7	·5	3·72	1·15	feet.
$H=$	·614	·606	·564	·5 to 1·5	·5 to 1·5	,,
$\frac{F}{H}=$	·148	·145	·114	·149	·143	
	Fteley and Stearns.			Bazin.		

Some other values are

$H=$	·68	·37	·20	·08 feet.	Poncelet and Lesbros, weirs in thin walls, full contraction, length ·66 foot.
$\frac{F}{H}=$	·08	·11	·15	·25	

And for flat-topped weirs

$H=$	·5	·1	·5	·1	·5	·1	feet.
$\frac{F}{H}=$	·27	·28	·29	·40	·64	·67	
Top width:	·5 inch.		2 inches.		3 inches.		

According to Smith F is somewhat greater in weirs with no end contractions than in others, and increases slightly with l.

Fteley and Stearns found that just upstream of a weir the pressure, at least near the bottom, is greater than at the same level further upstream. Generally the difference is nearly as h or $\frac{v^2}{2g}$, and it also increases as G decreases. It never exceeded the amount due to a head of ·03 foot, and was generally much less.

5. **Velocity of Approach.**—The ordinary formulæ for weirs with velocity of approach are

$$\left.\begin{aligned} Q &= \tfrac{2}{3}cl\sqrt{2g}\,(H+nh)^{\frac{3}{2}} \\ &= ml\sqrt{2g}\,(H+nh)^{\frac{3}{2}} \end{aligned}\right\} \quad . \; . \; . \; (41).$$

By using a variable co-efficient of correction c_a we obtain the inclusive co-efficients $C=cc_a$ and $M=mc_a$.

The formulæ with inclusive co-efficients are

$$\left.\begin{aligned} Q &= \tfrac{2}{3}Cl\sqrt{2g}\,H^{\frac{3}{2}} \\ &= Ml\sqrt{2g}\,H^{\frac{3}{2}} \end{aligned}\right\} \quad . \; . \; . \; (42).$$

For weirs in thin walls with complete contraction equation 42 is not ordinarily suitable, because while the values of c are known and tabulated those of C are not known, and if calculated for many different values of v would fill a formidable set of tables. But for other kinds of weirs C is often known as well as or better than c. In these cases, and also in cases where Q is to be measured for some particular weir, and the co-efficients ascertained and recorded, equation 42 is eminently suitable.[1]

Where c is not known the use of c_a renders the adoption of the indirect or tentative solution unnecessary in certain cases, and so saves trouble (see examples 1 and 5). It is not convenient to give a formula, as in the case of orifices (equation 22, p. 48), for calculating c_a, because equation 11 gives Q and not v. In order to find v it would be necessary to separate c into c_c and c_v, and these quantities are not properly known. Values of c_a have, however, been found by working out various cases, and are given in table xiii. for two values of c. Others can be interpolated if required. The excess of c_a above 1·0 is nearly as c^2, and for a given value of c nearly as n. The co-efficient c_a may be used either for solving ordinary problems or for obtaining values of C from c or M from m.

The inverse process of finding c from C or m from M is as follows:—

Since $Q=vA$,

Therefore from equation 42 $$\frac{v^2}{2gH}=\frac{M^2l^2H^2}{A^2}=M^2\frac{a^2}{A^2} \quad . \quad . \quad . \quad (42\text{A}).$$

$$\text{But } Q=ml\sqrt{2g}\left(H+n\frac{v^2}{2g}\right)^{\frac{3}{2}}$$

$$=ml\sqrt{2g}H^{\frac{3}{2}}\left(1+n\frac{v^2}{2gH}\right)^{\frac{3}{2}}.$$

Since the last term in the brackets is small compared to the first term, the expression in brackets is nearly equal to $1+\frac{3}{2}n\frac{v^2}{2gH}$.

Adopting this value and substituting from equation 42A

$$Q=ml\sqrt{2g}H^{\frac{3}{2}}\left(1+\frac{3}{2}nM^2\frac{a^2}{A^2}\right) \quad . \quad . \quad . \quad (43).$$

From equations 42 and 43

$$m=\frac{M}{1+\frac{3}{2}\,nM^2\frac{a^2}{A^2}} \quad . \quad . \quad . \quad (44).$$

It is of course impossible to observe either m or n directly. The

[1] Provided the bed is not liable to alter, see example 5.

observations give M directly, and either m or n can be found by assuming a value for the other. Generally m is assumed or deduced from its values for a similar weir with no velocity of approach, and n is then calculated. When the length of a weir is the same as the width of the channel of approach and G is the height of the weir: equation 44 becomes

$$m = \frac{M}{1+\frac{3}{2}nM^2\frac{H^2}{(G+H)^2}} \quad . \; . \; . \; (45),$$

and in this form is given by Bazin.

On the assumption that the effect of the energy due to the velocity of approach is the same as that of raising the water-level by a height AK (Fig. 71) equal to $\frac{v^2}{2g}$, the discharge is the same as that through an orifice with heads KA and KE, and the old form of equation was

$$Q = \tfrac{2}{3}cl\sqrt{2g}\left\{(H+h)^{\frac{3}{2}} - (h)^{\frac{3}{2}}\right\},$$

which is similar to equation 35, p. 70. This equation cannot be of the true theoretical form, chiefly because the original weir formula (equation 11, p. 15) is not so. It would, however, be right to use it, as the best attempt at a theoretical formula, if there were any advantage in doing so. But the last term $h^{\frac{3}{2}}$ is generally small and often minute, while the formula is more complicated than equation 12. The method of allowance for v is largely empirical, and it is better to use the more simple formula 12. With this formula n might be expected to be somewhat less than unity.

From article 7, chapter ii. it is clear that for weirs with velocity of approach the contraction may be either perfect or imperfect. When it is imperfect the increase of discharge is due partly to the energy of the water. represented by $\frac{v^2}{2g}$ and partly to reduced contraction due to smallness of the margin. The value of n from both causes combined has been found to be, for weirs in thin walls, from 1·0 to 2·5. Smith rightly separates the two causes, and, discussing various experiments, concludes that n should be 1·4 for weirs with full contraction, and 1·33 for weirs with no end contractions. The effect of reduced contraction, if any, was estimated separately, but the allowance made in the cases of weirs with no end contractions was not quite sufficient according to the rules given in article 3 above, so that n was a little overestimated, and Smith himself suggests that this may be so. Since Smith wrote, the results of Bazin's experiments on weirs with no end contractions have appeared. Owing to their general regularity and extent they are entitled to great weight. By analysing them on

Smith's principle it is found that n varies from ·86 to 1·37, and averages about 1·1. For moderate velocities of approach Q depends only a little on n (see table xiii.), and it is not worth while to give here the detailed analysis.[1] Bazin himself gives 1·54 as the mean value of n, but this includes the effect of reduced contraction. Both sets of experiments, namely Bazin's and those discussed by Smith, include high velocities of approach, the ratio $\frac{A}{a}$ being sometimes only 1·6. For weirs with full contraction the experiments discussed by Smith are not numerous, and his resulting figure 1·4 somewhat doubtful. It seems high in comparison with the others, and may be put at $1{\cdot}3\dot{3}$.

The variations in n, and especially its exceeding the value 1·0, are not easy to explain. A weir is usually in the centre of a channel, and the average deflection of the various portions of the approaching stream is then a minimum, especially if its greatest velocity is also in the centre, so that a large proportion of the water flows straight. In a weir so placed n will be a maximum; but this is no reason for its being greater than unity. The whole of the water, and not only the quickest water, has to pass over the weir. At the approach section the velocity distribution (chap. ii. art. 21) is normal. The total energies of the various portions of the stream may (chap. ii. art. 10) exceed the energy due to v, but the difference is probably only a few per cent., and nothing like 33 or even 20. Moreover, some little energy must be lost in eddies between the approach section and the weir. Thus in no case will the available energy appreciably exceed that due to $\frac{v^2}{2g}$. A high velocity of approach does not of itself reduce contraction. The high velocity occurs in the portion EB (Fig. 71) as well as in AE. With an orifice in the side of a reservoir a high velocity does not cause reduced contraction, but rather the contrary. The surface curves for weirs do not indicate any reduced surface contraction when v is high. Reduction of the clear margin is allowed for separately; and there are high values of n for cases in which the clear margin is ample.

It is probable that the deviations of n from unity are chiefly due to the incorrect form of the equation used. If a curved crest FC is added, the flow will not be appreciably affected, but the head will now be H' instead of H. The co-efficients of the two weirs must be such that $cH^{\frac{3}{2}}=c'H'^{\frac{3}{2}}$. Suppose A now reduced so that v becomes considerable, then $c(H+nh)^{\frac{3}{2}}$ must equal $c'(H'+n'h)^{\frac{3}{2}}$, and this occurs when $n'=n\frac{H'}{H}$. If c is ·60 and c' is ·80 (values likely to occur in practice), $\frac{H}{H'}=\frac{n}{n'}=1{\cdot}2$. Thus it

[1] It will be found in Appendix C.

can be seen how imperfections in the formula may cause n to change, and also that for a weir with a sharp edge n is greater than for a rounded weir.

The following values for n seem suitable for weirs situated in the centre of the stream :—

	Weirs with end contractions.	Weirs without end contractions.
Weir with sharp edge, .	$1 \cdot 3\dot{3}$	1·2
Rounded weir, . . .	1·1	1·0

For other kinds of weirs the value can be estimated. For a weir not in the centre a reduction can be made. When the edges are sharp, and the margin insufficient for complete contraction, an additional allowance for this must be made by the rules of article 3.

Section II.—Weirs in Thin Walls

6. Co-efficients of Discharge.—The chief experiments on weirs in thin walls, except Bazin's, have been analysed by Smith, who has prepared tables of the values of c at which he arrives, and his results somewhat condensed are shown in tables xiv. to xvi., but he notes that when H is less than ·2 foot the figures are not reliable. Those cases which are marked (?) Smith considered doubtful, owing to the absence of observations for such cases. For the others he gives the probable error as only ·3 per cent. It is of course known that end contractions reduce the discharge, and that their effect increases with H and decreases with l. Smith in his analysis considers all the experiments (except Bazin's) mentioned in article 1—those with and those without end contractions and those having various degrees of contraction—together, and to a certain extent infers one set of values from the other.

But further observations have been made by Stewart and Longwell (*Trans. Am. Soc. C.E.*, vol. lxxvi.) on short weirs with full or nearly full contraction. The weirs were only one foot high, and for this reason the figures, for the cases where H was highest, have been slightly reduced by Gourley and Crimp (*Min. Proc. Inst. C.E.*, vol. cc.). Their figures—in some cases again slightly altered so as to accord with the rules of art. 3—for weirs less than 3 feet long are shown in table xiv., and supersede Smith's figures, some of which he himself considered doubtful. For the 3-foot weir their figure for a 2-foot head is shown; for smaller heads their figures exceed Smith's by about ·004. The co-efficients in table xv., obtained from experiments by Castel, do not, for the smaller heads, accord with those of table xiv. and are probably incorrect. Such very short weirs are not important, measurements by orifices being better.

For weirs with no end contractions Bazin obtains figures differing from those of Smith. Smith's co-efficients attain a minimum as H increases and then increase, but Bazin's decrease as long as H increases. Smith's co-efficients increase as l decreases, but Bazin's are constant. The discrepancies are important because of the different laws which they indicate, and because of the high standard of accuracy obtainable with weirs in thin walls. The methods used for observing the head are described in chapter viii. article 6. Bazin's measuring section was (art. 1) 16·4 feet upstream of the weir. It has been suggested that the surface fall in this length caused an error. Calculations show that the error must have been inappreciable. Whether Bazin's weir had a length of 6·56 feet, 3·28 feet, or 1·64 feet, his values of c come out the same. Bazin considers that in Fteley and Stearns' experiments baffles were placed too near the weirs. Bazin's co-efficients are confirmed by experiments made by Rafter[1] and to some extent by experiments made at Wisconsin University.[1] They should be used for weirs 1·5 to 8 or 9 feet long, without end contractions. For longer weirs Smith's figures should be used.

The detailed values of Bazin's co-efficients given in table xvi. are, owing to Bazin's values of n not being accepted (art. 5), slightly higher for the greater heads than the values arrived at by Bazin himself. They accordingly differ less from Smith's figures. Bazin calculated c, or rather m, for heads ranging from ·16 to 1·97 feet, but his actual observations were within the range shown in table xvi. Bazin also gives a complete table of the values of M, and from it table xviii. giving values of C has been framed.

It has been found that when there are no end contractions the sheet of water after passing the crest of a weir tends to expand laterally, except when H is less than ·20 feet, and the side-walls have usually been prolonged downstream of the crest, openings for free access of air beneath the sheet being left. If the sides are not so prolonged c will be increased about ·25 per cent. when $H = \frac{l}{10}$, and more or less as H is more or less. It also appears that in such weirs moderate roughness of the sides of the channel has no appreciable effect on the discharge.

[1] *Hydraulic Flow Reviewed* (Barnes), Table 8.

Regarding triangular weirs in thin walls, observations have been made by Gourley and Crimp (*op. cit.*). They adopted a formula involving $H^{2\cdot47}$, but their figures enable c in the ordinary formula (equation 54, p. 111) to be calculated. The figures are given on p. 96. They confirm previous figures obtained by Thomson, by Barr (*Engineering*, vol. lxxxix. p. 473), and by Gaskell (*Min. Proc. Inst. C.E.*, vol. cxcvii.), and they are independent of the side slopes of the weir which varied from $\frac{1}{6}$ to 1 to 1 to 1.

7. Laws of Variation of Co-efficients.—The following laws, governing the variation of the co-efficient for complete contraction, are apparent:—

(1) For cross sections of similar shapes, *i.e.* a given ratio of l to H, c is less as the section is greater.

(2) In the short weirs the section is sometimes square, *i.e.* $l = H$ nearly. In these cases c tends to increase or become constant when H exceeds l.

(3) For the other rectangular weirs c decreases as H increases.

(4) For a triangular weir c is somewhat less than for a rectangular weir with the same values of l and H. The contraction in the acute angles is hindered (chap. iii. art. 8), but the surface contraction is probably increased because the surface stream has only narrower streams to hold it up.

Some of the laws are similar to those for orifices in thin walls, but the surface contraction in weirs creates a great distinction between the two cases.

8. Flow when Air is excluded.—With four weirs in thin walls, of heights 2·46 feet, 1·64 feet, 1·15 feet, and ·79 foot, further observations were made by Bazin, the access of air beneath the falling sheet being prevented by the closure of the openings which had been left for that purpose. The following statement shows the results noticed. The pressures under the sheets were observed, and the discharge was found to increase as the pressure decreased.

An interesting point for consideration is the conditions under which the different forms are assumed. This is stated by Bazin, and is shown in the above statement. With weirs not exactly similar to those of Bazin, it may be difficult to say when the various changes will occur, but it will at least be possible to foresee them and to take some account of them when they do occur. The occurrence of the form called 'drowned underneath' will obviously be affected by the condition of flow in the downstream reach. One lesson to be learnt is, that if complications are to be avoided and discharges accurately inferred the free access of air under the sheet is essential.

9. **Remarks.**—When discharges are to be measured by weirs those without end contractions may be easier to construct. For measuring the very variable discharge from a catchment area, a weir in a thin wall has been used with a central notch (Fig. 71A) which can deal with small discharges and so avoid very small heads (*Min. Proc. Inst. C.E.*, vol. cxciv.). It would seem best to construct *ab de* so as to have no contractions there. Otherwise when the water covers the whole crest, as in the figure, the central portion of the water is subject to contraction on *ab de*, but not on *bc ef*, and the co-efficient of the central portion must be doubtful. A triangular weir would probably be best if c were determined for large triangles.

Cippoletti's formula is $Q = 3{\cdot}367\ l\ H^{\frac{3}{2}}$ ($c = {\cdot}63$) and the weir is trapezoidal, the side slopes being $\frac{1}{4}$ to 1 and l being the bottom width. The sloping ends counteract the increasing effects of the end contractions as H increases, and c remains constant as long as H is not $> \frac{l}{3}$. It is not known that the formula is accurate when $l > 9$ feet or $H > 1{\cdot}5$ feet. When l is 3 feet or less, the formula is known to be accurate to within, say, 2 per cent., within the above limits, and to be inaccurate outside them. For instance, it gives results about 30 per cent. too small when $l = 1$ foot and $H = 2$ feet (*op. cit.*, vol. cxciii.).

Regarding trapezoidal weirs in general (Fig. 71B), let q_1 be the discharge of *abc* and q_2 that of *dbef* when they are separate and each has full contraction. Gourley and Crimp found (*op. cit.*) that, for *abeg*, $Q = q_1 + q_2$ to within 1 or 2 per cent. The length *be* varied from ·25 foot to 3 feet, the side slopes from $\frac{1}{5}$ to 1 to 1 to 1, and H from ·2 foot to 1 foot. For *cbeg* alone the discharge is probably q_2. When it is joined to *abc* contraction on *bc* ceases, but three acute angles—at *b* and *c*—are abolished. Thus for small trapezoidal weirs in thin walls with full contraction the special formula (art. 16) is not needed.

Experiments by Flinn and Dyer (*Trans. Am. Soc. C.E.*, vol. xxxii.) on trapezoidal weirs with lengths up to 9 feet and H up to 1·4 feet—side slopes $\frac{1}{4}$ to 1—show some fluctuations and are not accurate enough to test the above law further.

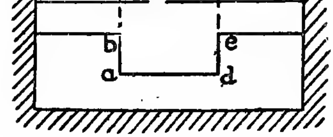


FIG. 71A.

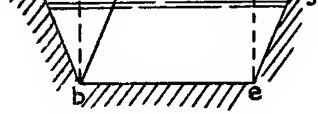


FIG. 71B.

Reference to Fig.	Name given to Case by Bazin.	Description of Case.	Conditions under which it occurs.	Effect on the Co-efficient of Discharge, C.
Fig. 72.	Adherent sheet.	Sheet in contact with weir and no air under it, or it may spring clear from the iron plate, enclose a small volume of air, and then adhere to the plank, or it may adhere to the top and bevelled edge and then spring clear, enclosing air as in the case following.	Under small heads.	C may possibly exceed that for a free sheet by 33 per cent.
Fig. 73.	Depressed sheet.	Air partly exhausted by the water and at less than atmospheric pressure. Water under sheet rises to higher level than that of tail water.	When case 1 does not occur, or when it occurs and H is increased. The change occurs abruptly.	C is higher than for free sheet, generally only slightly, but it may be 10 per cent. higher when it is on the point of assuming form 'drowned underneath.'
	Sheet drowned underneath.	Water under sheet rises to level of crest and all air is expelled.	When H is further increased so that H is not less than about $\cdot 4\,G$.	
Fig. 74.		(a) *Wave at a distance.*	When the fall $H+H_2$ is greater than about $\frac{3}{4}G$.	Value of $\frac{H}{G}$. — Value of $\frac{C'}{C}$. ·05 ... 1·22 ·40 ... 1·19 ·50 ... 1·13 ·60 ... 1·09 ·80 ... 1·04 1·00 ... 1·005 1·20 ... ·98 1·40 ... ·96 1·60 ... ·95 C is the co-efficient for a free sheet and C' for the case in question.
Fig. 75.		(b) *Wave covering foot of sheet.*	When the fall $H+H_2$ is not greater than about $\frac{3}{4}G$. For a given head H the greatest value of H_2 is $\frac{3}{4}G-H$.	The level of the tail water affects the discharge, and approximately $\frac{C''}{C}=\left(1\cdot05+\cdot15\frac{H_2}{H}\right)\ldots(46)$. See also article 13.

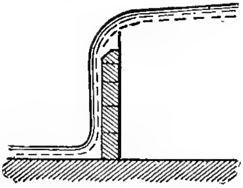


Fig. 72.

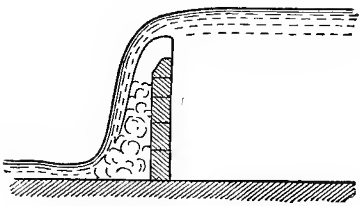


Fig. 73.

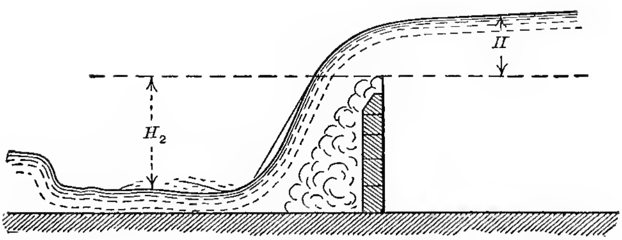


Fig. 74.

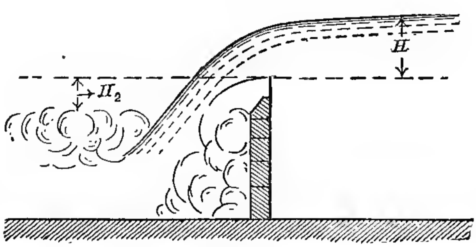


Fig. 75.

Francis found that end contraction might be allowed for by considering the length of the weir to be reduced by $\cdot 20H$, that is, by substituting $(l - \cdot 2H)$ for l in equation 11, page 15. He found that with the formula thus modified, the co-efficient, provided l is not less than $3H$ or $4H$, is nearly constant, its value being ·620 to ·624, and averaging ·623 for heads ranging from 5 to 19 inches. Results obtained by this formula are liable to differ by 1 or 2 per cent. from those of the ordinary formula with the co-efficients of table xiv. It is not known that the formula is correct when $l > 10$ feet. When $c = \cdot 623$, $Q = 3\cdot 33\ l\ H^{\frac{3}{2}}$.

In either formula—Cippoletti or Francis—velocity of approach can be allowed for (equation 12, p. 15). Both formulæ are useful attempts at simplification while adhering to simple indices. Further experiments may enable a Cippoletti weir to be designed with the sides curved, the slope altering as H increases so that c remains constant.

Co-efficients for Triangular Weirs in thin Walls. (Art. 6.)

$H = \cdot 1$	·2	·3	·4	·5	·6	·7	·8	·9	1 foot
$c = \cdot 616$	·605	·597	·591	·587	·584	·581	·579	·577	·575

Section III.—Other Weirs

10. Weirs with flat top and vertical face and back.—Generally the water at B (Fig. 76) holds back that upstream of it, and the discharge is less than for a weir in a thin wall under the same head. It is a sort of drowned weir, B being the tail-water level.[1] At A there is eddying water. When H is about $1\cdot 6W$ to $2W$—W being the top width—the sheet springs clear from the top, and the case becomes a weir in a thin wall. But if the sheet nearly touches at C (Fig. 77) the water gradually abstracts the air, and the sheet is pressed down, touches at C, and Q is slightly greater than for a weir in a thin wall. Table xvii. (prepared by Fteley and Stearns) shows the corrections to be applied to c, the co-efficient for weirs in thin walls, in order to give c_w the co-efficient for weirs with flat top and vertical face and back. The corrections apply

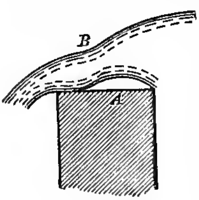


Fig. 76.

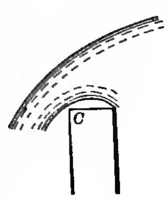


Fig. 77.

[1] See also art. 15.

strictly only to weirs without end contractions, but may be used for others.

Bazin made numerous observations on weirs of this kind, and his results are shown in table xix. Some observations made at Cornell University, in the United States of America, are included. Some of them contained sources of error. The method of observing the head (chap. viii. art. 6) admittedly caused error. Some of the figures were corrected after further observations and calculations (*Weir Experiments, Co-efficients and Formulæ*, R. E. Horton). As to those not so corrected, it was concluded that they were correct to within 6 per cent. They are marked (?) and are given as approximations and because other co-efficients for some of the heads are not available. These remarks refer also to tables xx., xxi., and xxii. The Cornell weirs are those 4 feet to 5·3 feet high. Results of experiments by the Geological Survey, U.S.A., on flat-topped weirs 11·25 feet high are also included. The various figures are consistent.

Bazin gives the following formula for obtaining C_w, the inclusive co-efficient for such weirs, from C, the co-efficient for a weir in a thin wall.

$$\frac{C_w}{C} = \cdot 70 + \cdot 185 \, \frac{H}{W} \quad . \quad . \quad . \quad (48).$$

The results given by this formula agree with the observed results generally within about 2 per cent., but for the widths of 6·56 feet, 2·62 feet, and 1·31 feet the error may be 3 or 4 per cent. They also agree with Fteley and Stearns' results within 1 or 2 per cent. When H was increased to about $2W$ the sheet sprang clear, but if H was gradually lowered the sheet remained clear till H was about $1{\cdot}6W$. Between these limits it was unstable. When the sheet springs clear the above formula of course is not needed. The thick lines in the table mark off the cases when H was less than $2W$. While H varies from $\frac{3W}{2}$ to $2W$, the ratio $\frac{C_w}{C}$ may change from ·98 to 1·07 if the sheet remains attached to the crest.

When air was excluded depressed and drowned sheets occurred under somewhat similar conditions to those with weirs in thin walls. Remarks regarding them are given in table xix. Their occurrence sometimes preceded and sometimes succeeded that of detachment of the sheet from the back or top of the weir, and rendered the conditions very complicated.

11. Weirs with sloping face or back.—Bazin's chief results for weirs of this class are given in tables xxi. and xxii., and the

Cornell results are included. Table xxi. contains the cases where the back of the weir was steep, so that the sheet generally sprang clear of it. Apparently no air openings were left, and the adherent depressed and drowned sheets often occurred. Table xxii. shows the cases where the back slopes gradually. In these last the stream flowing down the back is in uniform flow in an open channel.[1] Weirs of this kind with back slopes about 10 to 1 are used on some large canals in India and termed 'Rapids,' the profile of the water-surface being as sketched in Fig. 68, page 82. The flow at the crest is virtually that of a drowned weir. At the foot there is a standing wave (chap. vii. art. 11).

In weirs of these classes there are several variable elements. Pairs of cases in the tables can be compared in which only one element varied, so that its effect can be traced. By studying these cases and the tables generally it will be seen that C generally increases as the height of the weir decreases, as the top width of the weir decreases (but not so much for the greater heads), as the upstream slope is flattened, and as the downstream slope is made steeper.

12. Miscellaneous Weirs.—For a weir made of plank with a rounded crest of radius R the discharge with head H is about the same as for a weir in a thin wall with a head H'. The following table is given by Smith[2] :—

H.	Values of R.		
	·25 in.	·50 in.	1 in.
	Values of $H' - H$.		
·116	·006	·004	·003
·166	·014	·013	·015
·217	...	·021	·018
·284	·011	·029	·028
·351	·015	·028	·039
·41	·014	·028	·044
·49	·015	·030	·052

The chief results of the Bazin and Cornell observations on rounded weirs are given in table xx.

[1] But see art. 15. [2] *Hydraulics*, chap. v.

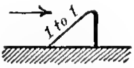


FIG. 78.

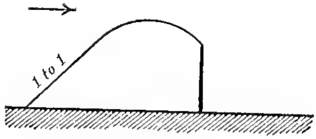


FIG. 78A.

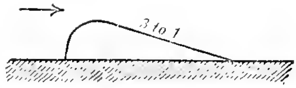


FIG. 78B.

For a weir formed entirely by lateral contraction of the channel, and having a crest length of 2 feet to 6 feet (Fig. 82, p. 107), c is ·65 to ·73 and C is ·70 to ·78, being greater for the larger sizes.

For a fall (Fig. 79) in which there is neither a raised weir nor a lateral contraction there is no local reduction of the approaching stream due to eddies or walls, and therefore no local surface fall of the kind ordinarily occurring. The surface curve is due to draw (chap. ii. art. 11). If the slope AB is not very steep the curve extends for a great distance.

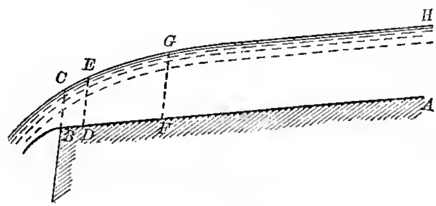


FIG. 79.

If V is the velocity at DE near to BC, then V is both the velocity of approach and the velocity in the weir formula, so that

$$V^2 = \tfrac{4}{9}c^2 2g\left(H + n\frac{V^2}{2g}\right)$$

or

$$V^2 = \tfrac{4}{9}c^2 2gH + \tfrac{4}{9}nc^2V^2$$

$$V^2(1 - \tfrac{4}{9}nc^2) = \tfrac{4}{9}c^2 2gH.$$

If $c = ·79$ and $c^2 = ·63$ and $n = 1·0$,

$$V^2 = \frac{·28}{1 - ·28}\ 2gH$$

$$V = ·62\ \sqrt{2gH}.$$

If the channel AB be supposed to be very smooth or steep the water-surface HG will be parallel to the bed, but there will always be a short length GC in which draw will occur. Falls of this kind occur at the ends of wooden troughs and shoots. They were used on one of the older of the great Indian canals, but the high velocity due to the draw caused such scour and damage that raised weirs had to be added.

SECTION IV.—SUBMERGED WEIRS

13. Weirs in Thin Walls.—The following statement shows the chief experiments which have been made.

Observer.	Length of Weir.	Upstream Head H_1.		Downstream Head H_2.		Height of Weir.
		From	To	From	To	
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
Francis, . .	11	1·0	2·3	·24	1·1	5·8
Fteley and Stearns,	5	·33	·81	·07	·80	3·2
Bazin, ...	6·56	·19	1·49	·79	1·26	·8 to 2·5

The weirs were all without end contractions. The level of the tail water was measured at M (Fig. 80), which is theoretically wrong;[1] the surging of this water renders exact measurements difficult. The co-efficients for submerged weirs are not, in most cases, well known, and exact results cannot be expected from them.

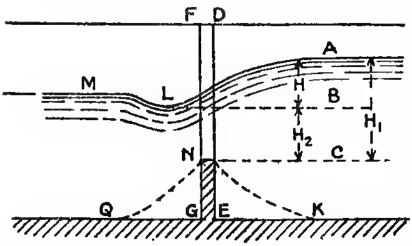


Fig. 80.

Let q_1 be the discharge through AB and q_2 through BC. Then

$$q_1 = \tfrac{2}{3} c_1 l \sqrt{2gH}.\ \ H \quad . \quad . \quad . \quad (49).$$

$$q_2 = c_2 l \sqrt{2gH}.\ \ H_2 \quad . \quad . \quad (50).$$

If c has the same value for both portions,

$$q = \tfrac{2}{3} cl \sqrt{2gH} \left(H + \frac{3H_2}{2}\right) \quad . \quad . \quad (51),$$

$$\left.\begin{aligned} \text{or } q &= cl \sqrt{2gH} \left(H_2 + \frac{2H}{3}\right) \quad . \quad . \quad . \\ \text{or } q &= cl \sqrt{2gH} \left(H_1 - \frac{H}{3}\right) \quad . \quad . \end{aligned}\right\} \quad (52).$$

The last two formulæ are those for an orifice having a height equal to the downstream head plus two-thirds of the fall. If there is velocity of approach $H + nh$ must be put for H and $H_1 + nh$ for H_1, but H_2 is left unaltered.

Francis makes $c_2 = \cdot 921 c_1$, that is, he multiplies H_2 in equation 52 by ·921. Smith, discussing the experiments of Francis and Fteley and Stearns, and reviewing a previous discussion by Herschel, substitutes ·915 for ·921 and recommends the formula—

$$q = c_s l \sqrt{2g\,(H+nh)} \left(\cdot 915\ H_2 + \frac{2\,(H+nh)}{3}\right) \quad . \quad . \quad . \quad (53).$$

This formula is for weirs in thin walls without end contractions: c_s is the co-efficient taken from table xvi. for the equivalent weir with a free fall (that is, the weir with a free fall giving the same discharge) and n is 1·33. The formula may be applied to weirs with end contractions and the same co-efficients used if $l - \cdot 2H_1$ be substituted for l.

If Q is the discharge for a free weir, and if H_1 remains constant while the tail water is raised by some cause operating in the

[1] See chap. ii. art. 6.

downstream reach, Q decreases very slowly till H_2 is about $\frac{H_1}{2}$. The discharge through AB is the same as before, while the velocity in BC is altered in the ratio $\sqrt{\frac{H+\frac{1}{2}H_2}{H}}$. The relative discharges are as follows, c being constant and velocity of approach being supposed to be negligible :—

$\frac{H_2}{H_1}=$	·00	·25	·$\dot{3}\dot{3}$	·50	·$\dot{6}\dot{6}$	·75.
or $\frac{H_2}{H}=$	·00	·$\dot{3}\dot{3}$	·50	1·0	2·0	3·0;
$\frac{q}{Q}$ (equation 52) $=$	1·00	·974	·953	·88	·77	·69
$\frac{q}{Q}$ (equation 53) $=$	1·00	·945	·933	·84	·71	·61.

Practically, this law is somewhat modified. Let it be supposed that for the free weir there is ample access of air. As the tail water rises above the crest the air is shut out. The under side of the sheet springs up to a somewhat higher level than the crest, but the surging of the tail water shuts out the air almost at once. The sheet of water is pressed down, and the discharge instead of decreasing increases a little. Practically it remains nearly constant during a certain rise of the tail water and then decreases. If the air passages become obstructed just before the tail water rises to the crest level, Q will begin to increase then, but this does not necessarily occur. Neither equation 52 nor 53 takes account of the increase in discharge when the tail water rises above the crest. If the air was shut out from the commencement, Q begins to decrease as soon as the tail water begins to rise. See equation 46, page 94.

Bazin uses the simple weir formula $q=\frac{2}{3}C_d l\sqrt{2g}H_1^{\frac{3}{2}}$ (where C_d is the inclusive co-efficient for the drowned weir and H_1 the upstream head) and finds the ratio $\frac{C_d}{C}$, C being the inclusive co-efficient for the 'standard weir,' 3·72 feet high with a free fall and with the same head H_1. His results are as follows :—

$\frac{H_2}{G}$ or Ratio of Down-stream Head to Height of Weir.	$\frac{H}{G}$ or Ratio of Fall in Water to Height of Weir.												
	·05	·10	·15	·20	·25	·30	·35	·40	·45	·50	·60	·70	E*
	Ratio $\frac{C_a}{C}$.												
·0	1·05	1·05	1·05	1·05	1·05	1·05	1·05	1·05	1·05	1·05	1·05	1·05	1·06
·05	·84	·93	·96	·98	1·00	1·01	1·01	1·02	1·02	1·03	1·03	1·04	1·05
·10	·74	·85	·90	·94	·96	·97	·98	·99	1·00	1·01	1·02	1·02	1·04
·15	·68	·80	·86	·90	·92	·94	·96	·97	·98	·99	1·00	1·01	1·03
·20	·64	·76	·82	·87	·90	·92	·94	·95	·96	·98	·99	1·00	1·02
·30	·58	·70	·77	·82	·86	·88	·90	·92	·94	·95	·98	·99	1·00
·40	·54	·66	·74	·79	·82	·85	·88	·90	·92	·93	·96	·98	·99
·60	·50	·61	·69	·74	·78	·81	·84	·87	·89	·90	·93	·96	·97
·80	·47	·58	·66	·71	·75	·79	·82	·84	·87	·89	·92	·94	·95
1·00	·45	·57	·64	·69	·74	·77	·80	·83	85	·87	·91	·94	·94
1·20	·44	·55	·63	·68	·72	·76	·79	·82	·84	·87	·90	·93	·93
1·50	·43	·54	·61	·67	·71	·75	·78	·81	·84	·86	·89	·92	·92

* This column shows $\frac{C_a}{C}$ when the tail water is below the crest, and the standing wave is at a distance (art. 8).

Actually the ratio $\frac{C_a}{C}$ is somewhat different with the weirs of different heights for the same values of $\frac{H_2}{G}$ and $\frac{H}{G}$, but the error in the figure given is usually only 1 or 2 per cent., except for very small values of $\frac{H_2}{G}$ and $\frac{H}{G}$, and in these cases the ratio is always uncertain. The values 1·05 in the first line of the table agree with the figure obtained by equation 46 (p. 94), when $H_2=0$. If, for any given weir, G is supposed to be 1·0, the above figures show $\frac{q}{Q}$ for various values of H and H_2. In this case, for a given value of $\frac{H}{H_2}$, the figures are high when H is high. This is due to velocity of approach, the standard weir having been high.

Bazin's figures may be compared with those given on page 101. Take for instance the cases where $H_2=2H$.

$\frac{H}{G}=$	·70	·50	·30	·10	·05	The figures on p. 101 are ·77 and ·71.
$\frac{H_2}{G}=$	1·40	1·00	·60	·20	·10	
$\frac{C_a}{C}=$	·92	·87	·81	·76	·74	

Again for the case where $H_2=\frac{H}{3}$.

$\frac{H}{G}=$	·70	·45	·15	The figures on p. 101 are ·974 and ·945.
$\frac{H_2}{G}=$	·23	·15	·05	
$\frac{C_a}{C}=$	1·00	·98	·96	

In the above case, where $\frac{H}{G}=\cdot 70$, $\frac{H_1}{G}=2\cdot 10$ and $\frac{a}{A}$ or $\frac{H_1}{G+H_1}=\frac{2\cdot 1}{3\cdot 1}$. The excessive velocity of approach accounts for the high value of $\frac{C_a}{C}$.

Bazin found that when H is reduced to about ·16G or ·21G, the sheet, instead of plunging beneath the surface (Fig. 75), suddenly assumes the form shown in Fig. 80 (which he terms the 'undulating' form, there being generally waves near M), but this does not affect the co-efficient. If H is now gradually increased, the undulating form remains till H is about ·28G or ·29G, but is unstable or liable at any moment to revert to the other form.

14. **Other Weirs.**—The results of Bazin's observations on weirs of other kinds are shown in the following table. Instead of giving the co-efficient ratios Bazin gives the equivalent heads. The conditions of flow are complicated in such cases, and formulæ can probably apply only with the co-efficient varying to a great extent. The height H_2', to which the tail water can rise before it begins to affect the discharge, varies greatly for different weirs. For a weir in a thin wall it is very small, and it is largest for weirs with flat tops. For the weir No. 5 in the table H'_2 was $\frac{5}{6}H_1$. For weirs with a sharp top it was minus, zero, and plus for downstream slopes of 1 to 1, 3 to 1, and 5 to 1 respectively, the flat downstream slope in the last case having the same effect as a large top width. For weirs with flat tops ·66 foot wide, back slopes varying from 2 to 1 to 5 to 1, H'_2 is nearly $\frac{H_1}{2}$, but when the top was 1·32 feet wide H'_2 was $\frac{2H_1}{3}$.

The first two entries in the table on p. 105 show that with a flat-topped weir c rapidly increased as H_2 increased—Q being constant—and became far higher than with a free weir. See table xix.; C in a high weir differs little from c. When H_1 and H_2 are both great, as with a river in flood, much of the stream is not subject to contraction, v_a approaches V, and C must be high, especially if the front and back slopes are somewhat gradual, as is usual in such weirs. Values of ·80 to ·97 have been found, Q being, however, merely calculated from the river section and slope, a difficulty which may occur in such cases.

Reference Number.	Dimensions of the Weirs.				Downstream Head, H_2, on the Weirs.	Discharges per foot run of Standard Weir in Thin Wall: cubic metres per second.					Remarks.
						·061	·110	·169	·310	·480	
	Down-stream Slope.	Top width in metres.	Up-stream Slope.	Height in metres.		Heads, H, on Standard Weir, metres.					
						·10	·15	·20	·30	·40	
						Corresponding Heads, H_1, on the other Weirs, metres.					
					Weirs with sloping face or back.						
1	1 to 1	0·0	Vertical.	·75	E*		·14	·18	·27	·36	$H_1 < H$ for the greater discharges and when H_2 is small.
					–·06		·14	·19	·27	·36	
					·06		·16	·20	·29	·38	
					·12		·18	·22	·31	·40	
					·24		…	…	·35	·43	
2	1 to 1	·2	½ to 1	·75	E*		·16	·21	·29	·37	
					·12		·17	·21	·29	…	
					·24		…	·27	·32	·39	
3	5 to 1	0·0	Vertical.	·75	E*		·16	·21	·31	·42	$H_1 > H$
					·12		·17	·21	…	…	
					·24		…	·27	·33	…	
					·36		…	…	·40	·45	
4	5 to 1	·2	½ to 1	·75	E*		·17	·22	·31	·41	
					·12		·17	·22	…	…	
					·24		…	…	·33	·41	
					·36		…	…	…	·42	
					Weirs with flat top and vertical face and back.						
5		2·0		·75	·12	·14	·18	…	…	…	
					·24	…	…	·27	·35	…	
					·36	…	…	…	·40	…	
6		·2		·75	E*	·12	·17	·21	·29	·38	For small discharges $H_1 > H$. For greater discharges $H_1 < H$ when H_2 is small, and $H_1 > H$ when H_2 is larger.
					·12	·14	·18	·22	·31	·39	
					·24	…	…	·28	·34	·41	
					·36	…	…	…	·41	…	
7		·2		·35	E*	·12	·17	·21	·29	·37	
					·12	·13	·17	·22	·30	·39	
					·24	…	…	…	…	…	
					·36	…	…	…	·41	…	
8		·1		·75	E*	·11	·15	·19	·27	·35	
					·12	·14	·18	·22	·31	·40	
					·24	…	…	…	·35	·43	
9		·1		·35	E*	·11	·15	·19	·27	·37	
					·12	·14	·17	·21	·28	…	
					·24	…	…	·33	·36	·40	
					·36	…	…	…	…	·44	

* Tail water below crest and wave at a distance.

Hughes, adopting equation 51 with $n=1$, has worked out[1] the values of c for weirs Nos. 5 and 6 on the above list, and the results condensed are as follows :—

Discharge in cubic metres per second.	Weir No. 5.			Weir No. 6.		
	H_1 metres.	H_2 metres.	c	H_1 metres.	H_2 metres.	c
·061	·122	·031	·50	·119	·000	·50
	·122	·091	·70	·123	·090	·67
	·161	·150	·87	·135	·120	·85
				·163	·150	·74
·169	·236	·151	·61	·216	·000	·56
	·247	·211	·81	·219	·060	·56
	·293	·271	·84	·220	·120	·63
				·233	·180	·74
				·277	·240	·72
·310	·353	·242	·63	·301	·000	·61
	·360	·303	·80	·307	·120	·63
	·396	·361	·88	·319	·210	·71
				·413	·360	·72
·392	·409	·300	·68			
	·418	·360	·83			
	·439	·389	·84			
·480				·382	·000	·65
				·384	·060	·63
				·406	·240	·70
				·442	·300	·68

The effect of a submerged weir varies greatly according to the state of the discharge. With low water it may act as a free weir, and have great effect, for however small the discharge may be, the upstream water-surface must be higher than the top of the weir. With larger discharges the heading-up is less, and with a great depth of water the weir may be almost imperceptible.

15. Contracted Channels and Weir-like Conditions.—Contracted channels are (chap. ii. arts. 6 and 19) analogous to submerged weirs. The co-efficients are generally not very well known. When an open stream issues from a reservoir, or from a larger channel, or passes

[1] Madras Government Paper on Bazin's New Experiments on Flow over Weirs.

between contracted banks, or bridge abutments, or piers, c may have any value from ·60 to ·95, being smallest when the angles of the apertures are sharp and square (especially if there is a decrease in section both vertically and laterally), greater if the angles are chamfered or curved, and greatest when there are bell-mouths. The co-efficients are also greater for large than for small openings. The values for narrow openings are, roughly, for square piers, ·6; obtuse angled, ·7; curved and acute, ·8 to ·9. For wider openings add ·1 or ·2. The co-efficient may thus be 1·0 in a bell-mouthed opening.

When a bridge or other obstruction in a stream has a waterway less than that of the stream the real obstruction is frequently much less than it seems to be. It is to be measured, not by the difference between the waterway at the obstruction and that upstream of it, but by the difference in the upstream and downstream water-levels. This is very often inconsiderable. A fall of 1 foot gives a theoretical velocity of 8 feet per second, and ·25 foot gives 4 feet per second. Bridges have sometimes been altered or rebuilt owing to 'obstruction' which was nearly harmless. Heading-up is most likely to be considerable with high discharges, because the mean width of the channel is then increased, while perhaps that of the contracted place is not. Thus the effect varies in just the opposite manner to that of a submerged weir.

The real objection to a contraction is very often the expansion which succeeds it and the eddies and scour which occur (chap. ii. arts. 17 and 23, and chap. vii. art. 2).

Submerged weirs and lateral contractions are really varieties of the same type of case, and some aspects of both of them will now be considered.

A typical case of contraction is that caused by bridge piers (shown in plan and elevation in Fig. 80A). As in other cases, the 'drop down' begins at AB where the reduction in the cross section of the forward-moving water begins. It ends where the section attains its minimum value. This is often about D, but it may be at L if the surface slope DL is greater than the bed slope. Below L the section again enlarges, and there may be a rise in the surface or, if V is very high, a standing wave (chap. vii. art. 11). It is pointed out by Houk[1] in discussing floods in the Miami Valley,

[1] *Calculation of Flow in Open Channels.* State of Ohio. The Miami Conservancy District. Technical Reports, Part iv. Dayton, Ohio, 1918.

that Q is simply the discharge of an orifice of area DE under the head AB—B being at the same level as D—with due addition for velocity of approach, that the discharge of AB is not to be calculated by the weir formula and added, and that such addition is based on an erroneous principle, the error being due (a) to the absence of a crest and (b) to the fact that the water treated as flowing over a weir passes—downstream of the drop-down—through the area treated as an orifice. But (a) does not seem to be a cause and (b) is the same if there is a weir mn instead of piers. In any channel which is locally contracted the formula—equation 16, art. 10,

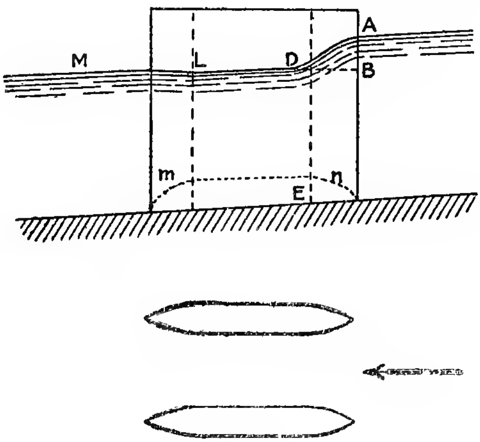


Fig. 80A.

chap. ii.—for variable flow applies. The reasons for adopting the weir formula are given in art. 2. The weir formula is fairly correct for the case of a free weir in a thin wall (Fig. 65, art. 1), or a notch (Figs. 82 and 83, art. 17). But directly there is any sort of drowning (Figs. 66 and 68, art. 1, and Fig. 80, art. 13) complications arise and a variety of co-efficients have to be used. Only two levels are however required in most cases, namely those of crest and upstream water.

The equation quoted expresses the principle that the fall in the water surface, from A to L, less the loss of head from resistances, is equal to the increase in the velocity head. This applies when the weir or narrowing is bell-mouthed. It can also be applied to a sharp lateral contraction [1] if c_c is estimated—it generally differs little from c—and the contracted area thus determined. See chap. iii. art. 5. It is at the contracted area that the velocity head must be taken. Something must be allowed for resistance due to the eddies. Ordinarily the length BD is small and nothing is allowed for the friction in that length so that the first term on the right of the equation disappears. It is partly owing to this and partly to the difficulty in estimating the sectional area at L (Fig. 80) that the equation is not usually applied to submerged weirs. Equations 49 and 50 are used together. It is known that part of the discharge comes from the section AB and part from BK, but the theory is of course imperfect. It is known, however, that both parts of the stream are contracted and pass at an increased velocity through the reduced section at L. The water

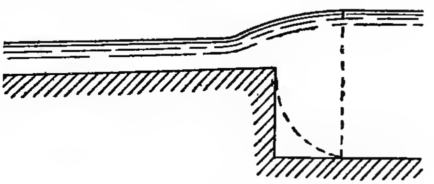


FIG. 80B.

level at M is determined by the discharge of the downstream channel. When a structure is being designed the water level at L is not known, the probable level at M is considered and the weir formulæ are used with such co-efficients as are found in practice to be fairly accurate. When the amount of drowning is very great only one equation is used.

In the cases dealt with in the report under reference the distance BD was generally great—in many cases hundreds of feet,—the stream contracting gradually. The loss of head from friction in that length was separately computed and allowed for.

The formula was then

$$V_2 = \sqrt{2g\left(H + \frac{V_1^2}{2g}\right)} \quad . \quad . \quad . \quad (53\text{A})$$

where V_2 is the velocity at DE and V_1 that at AB. The value of c was 1·0 unless there were angles such as to cause contraction.

[1] In this case the drop-down begins at the commencement of the eddy which replaces the pointed portion of the pier.

The total drop was generally from 1·5 feet to 4·8 feet, the velocity 6·7 to 23·1 feet per second, and the head lost in friction less than 25 per cent. of the total fall. The widths of most of the openings are not stated, but in one case the width was 280 feet and the sectional area at DE 7960 square feet. The depths were generally great. The minimum sectional area was found by soundings and measurements taken after the flood. If there were sharp edges or square corners at the entrance a co-efficient of contraction was applied. This was ·7, ·8, or ·9.

Since the friction is approximately as V^2, Houk considers that, in allowing for friction in a considerable length in which the velocity changes greatly, it is best to calculate the mean velocity V not as $\frac{V_1+V_2}{2}$ but as $\sqrt{\frac{V_1^2+V_2^2}{2}}$. Also—see Notes at end of chapter vii.—a percentage should be added to the loss of head

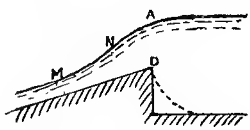


Fig. 80c.

because the square of the mean of the velocities in a cross-section is less than the mean of the squares. In equation 16 and the equations in chap. vii. art. 10, V is taken as $\frac{V_1+V_2}{2}$ and no percentage is added, but in these cases V_1 and V_2 differ only slightly and no appreciable error results.

A weir-like condition, with water surface convex upwards, exists wherever momentum is being imparted to the water, as when a stream issues from a reservoir or from a larger stream (Fig. 80b), or below a closed lock-gate when the water enters the lock from the sides and flows along the lock. The case of a right-angled elbow is similar. In all these cases the water has no previous momentum in the new direction and the fall is approximately $\frac{V^2}{2g}$. V is usually moderate and it is not necessary to calculate the fall in the water surface, but it can often be seen. The case of a rapid with a steep slope (Fig. 80c) is mentioned below.

The statement (chap. ii. art. 11) that downstream of any abrupt change in a uniform channel the flow is uniform is subject to the above qualifications.

In the case of a thin-wall weir the air has access to the lower side of the sheet and C, when $H=1{\cdot}4$ feet and $G=2{\cdot}46$ feet, is about ·66. In the case of a rapid (art. 11 and Fig. 80c) the air is excluded, and when

$S=$	1 to 1	3 to 1	5 to 1	10 to 1
C is about	·75	·65	·60	·56

S is the downstream slope. See Tables xviii. and xxii. With the slope of 1 to 1 the drowning is slight. At about 3 to 1 its effect —for the particular value of H quoted—counteracts that of the exclusion of the air.

The value of the head, h, on the actual crest of a weir is of interest in some cases, as will be seen. Bazin in his experiments observed the head h (at left side of crest, Fig. 76, art. 10) with the following results, W being the crest width and H the head, measured as usual to the right of the weir :—

$\frac{H}{W}=$	·4	·6	·8	1·0	1·4	2
$\frac{h}{H}$ (When $W=$ ·66 ft.) $=$	·94	·92	·91	·89	·87	·85
$\frac{h}{H}$ (When $W=$ ·33 ft.) $=$	·97	·95	·93	·91	·88	·85
$\frac{h}{H}$ (When $W=$ ·164 ft.) $=$	...	...	·95	·93	·89	·86

In the case of a rapid (Fig. 80c) Bazin found the following, S being the slope of the rapid and h the head at the crest :—

$H=$	·33	·66	1	1·3 feet.
$\frac{h}{H}$ (When $S=1$ in 5)	·845	·863	·859	·852
$\frac{h}{H}$ (When $S=1$ in 10)	·851	·869	·876	·873

When S was 1 in 5 and the crest width was ·66 feet instead of zero,

$$\frac{h}{H}=\cdot 876 \quad \cdot 891 \quad \cdot 877 \quad \cdot 871$$

For the greater heads the mean velocity of the stream, where the head is h, is nearly the same as if AD was an orifice under a head $\frac{h}{2}$ and $c_v = 1$. If at any point on the rapid the velocity exceeds the above, or the depth falls short of h, a standing wave (chap. vii. art. 11) can occur. The velocity down the slope of a smooth rapid may be very high. From N to M the surface may be concave upward, as shown in Fig. 80c. Below this there is uniformity of flow. In large rapids in India and Burma, in connection with irrigation works, the slope is about 1 in 10 or 1 in 15, the surface rough—boulder pitching,—and H much greater than in the above experiments—say 3 to 11 feet. It is not known how the ratio $\frac{h}{H}$ is affected, but it is possible that it is not very different from the above. Observations are needed to decide the point. It will then be possible to work out the depths further down the rapid and to attend, in designing, to the question of the standing wave.

Section V.—Special Cases

16. Weirs with Sloping or Stepped Side-walls.—For a weir of triangular section the formula is obtained by putting $H_t = 0$ and $l_t = l$ in equation 36 (p. 71). Thus—

$$Q = \tfrac{4}{15} c \sqrt{2gl} H^{\frac{3}{2}} \quad . \quad . \quad (54).$$

Since l increases as H, in any triangular weir in which c does not vary greatly, Q is nearly as $H^{\frac{5}{2}}$, that is, it varies much more rapidly than with an ordinary weir. If two weirs, one triangular and one rectangular, are so designed (Fig. 81) as to hold up the water of a stream to a given level with ordinary supplies, the triangular weir will allow floods to pass with a smaller head. This applies to any weir with sloping sides. The triangular form is suitable for small drains. By making the sides of a weir at any given level DE (Fig. 81) horizontal, and extending them outwards, the rise of the water above DE can be limited.

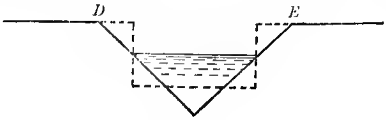


FIG. 81.

The formula for the discharge of a trapezoidal weir (Fig. 82) is

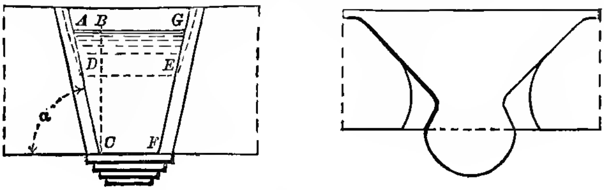


FIG. 82.

obtained by putting $H_t=0$ in equation 38 (p. 71). Thus—

$$Q=\tfrac{2}{3}c\sqrt{2g}H^{\frac{3}{2}}\{l_b+\tfrac{2}{5}(l_t-l_b)\} \quad . \quad . \quad . \quad (55).$$

The quantity in the outer brackets is the crest length of the equivalent ordinary weir. This length is less[1] than $\frac{l_t+l_b}{2}$ because the velocity of the water at the bottom of the section is greater than at the top. If there is velocity of approach $(H+nh)$ must be put for H in equation 55, or else C put for c. If r is the ratio of the side slopes, that is, the ratio of AB to BC, then $\frac{AB}{BC}=r=\cot a$, $AB=rH=H\cot a$, and $l_t-l_b=2rH=2H\cot a$.
Thus equation 55 may be written—

$$Q=\tfrac{2}{3}C\sqrt{2g}H^{\frac{3}{2}}\{l_b+\cdot 8rH\} \quad . \quad . \quad . \quad (56).$$

17. Canal Notches.—A common problem on irrigation canals is to design a weir so that the water-levels, CD, EF, etc. (Fig. 83), upstream of it, corresponding to different discharges in the channel of approach, shall be the same as they would have been if the weir had not existed and the channel had continued uniform and uninterrupted. If the cross-section of the channel of approach is trapezoidal, the form of the aperture will be approximately

[1] In a triangular weir it is $\frac{2}{5}\,l$.

trapezoidal, and its crest will be at the bed-level of the canal. Such a weir is termed a notch. It is usually, for convenience in construction, built exactly trapezoidal and of the form shown in Fig. 82, the lip being added to cause the falling water to spread out and exert less effect on the downstream floor. In a large channel two or more notches are built side by side instead of one very large notch. The co-efficients, so far as known, are given below.

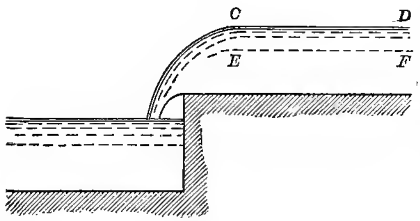


Fig. 83.

If C is the same for all heads the true theoretical form of the notch is curved, the angles at C, F (Fig. 82) being rounded. The slope of the sides is great for small depths because the co-efficient for flow in channels increases rapidly for small depths; but if C increases fast with the head at small depths, as is highly probable, judging from other weir co-efficients, the form is more nearly a trapezoid. To design the notch, find Q and q, the discharges (or the fractions of the discharges if there are to be several openings) of the channel for two depths D and d. Then from equation 56

$$l_b + \cdot 8rd = \frac{q}{\frac{2}{3}C_1\sqrt{2g}d^{\frac{3}{2}}} \quad . \; . \; . \quad (57).$$

$$l_b + \cdot 8rD = \frac{Q}{\frac{2}{3}C_2\sqrt{2g}D^{\frac{3}{2}}} \quad . \quad (58).$$

Therefore $$\cdot 8r(D-d) = \frac{Q}{\frac{2}{3}C_2\sqrt{2g}D^{\frac{3}{2}}} - \frac{q}{\frac{2}{3}C_1\sqrt{2g}d^{\frac{3}{2}}}.$$

Or $$r = \frac{\frac{2}{3}\sqrt{2g}(C_1Qd^{\frac{3}{2}} - C_2qD^{\frac{3}{2}}}{\cdot 8 \times \frac{4}{9} \times 2g(D-d)C_1C_2d^{\frac{3}{2}}D^{\frac{3}{2}}}.$$

$$= \frac{C_1Qd^{\frac{3}{2}} - C_2qD^{\frac{3}{2}}}{4\cdot 28(D-d)C_1C_2d^{\frac{3}{2}}D^{\frac{3}{2}}} \quad . \; . \; . \quad (59).$$

The depths d and D can be so selected as to make the notch specially accurate for any given range of depth. In irrigation canals (and still more in their distributaries) there is a certain minimum depth, d_1, below which the channel is not run. In such a case it does not matter if the notch is inaccurate for depths less than d_1. To make its accuracy a maximum for depths between d_1 and any greater depth, D_1, the range of depth should be divided into four parts and the depths d and D taken at the quarter points. Thus if

$$D_1 - d_1 = k$$
$$d = d_1 + \frac{k}{4}$$
$$D = d_1 + \frac{3k}{4}.$$

If general accuracy is required over a range of depth from zero to D_1, then $d = \frac{D_1}{4}$ and $D = \frac{3D_1}{4}$. The formulæ are, however, most simple when $D = 2d$. In this case equation 59 becomes—

$$r = \frac{C_1 Q d^{\frac{3}{2}} - 2{\cdot}828 C_2 q d^{\frac{3}{2}}}{4{\cdot}28 d C_1 C_2 d^{\frac{3}{2}} \times 2{\cdot}828 d^{\frac{3}{2}}}$$
$$= \frac{C_1 Q - 2{\cdot}828 C_2 q}{12{\cdot}10 C_1 C_2 d^{\frac{5}{2}}} \quad . \quad . \ (60).$$

Substituting this value of r in 57

$$l_b = \frac{q}{\frac{2}{3} C_1 \sqrt{2g} d^{\frac{3}{2}}} - \frac{{\cdot}8(C_1 Q - 2{\cdot}828 C_2 q)}{12{\cdot}10 C_1 C_2 d^{\frac{3}{2}}}$$
$$= \frac{2{\cdot}262 C_2 q - {\cdot}8 C_1 Q + 2{\cdot}262 C_2 q}{12{\cdot}10 C_1 C_2 d^{\frac{3}{2}}}$$
$$= \frac{2{\cdot}262 C_2 q - {\cdot}4 C_1 Q}{6{\cdot}05 C_1 C_2 d^{\frac{3}{2}}} \quad . \ . \ . \ (61).$$

If C_1 and C_2 are each assumed to be equal to C,

$$r = \frac{Q - 2{\cdot}828 q}{12{\cdot}10 C d^{\frac{5}{2}}} \quad . \ . \ (62).$$

And

$$l_b = \frac{2{\cdot}262 q - {\cdot}4 Q}{6{\cdot}05 C d^{\frac{3}{2}}} \quad . \ . \ . \ (63).$$

If it is desired to build a notch to the true form, that is not strictly trapezoidal, the lower part corresponding to a small depth in the channel may first be designed trapezoidal and the upper parts designed in instalments, working upwards.

In deciding in which direction a notch is to deviate from the true form, and for what water-levels accuracy is to be aimed at, regard must be had to the special circumstances of the case. If scour of the canal bed is feared or if there is difficulty, with low supplies, in getting enough water into the distributaries, the notch can be designed narrow.

If a notch is drowned its true form is modified. In Fig. 82 let

DE be the upstream water-level when the tail water is just level with the crest CF. The portion $CDEF$ of the notch obviously need not be altered. As the tail water rises above CF the discharge through the notch becomes gradually less than it would be for a free notch with the same upstream water-level, and the upper part of the notch must be widened as shown by the dotted lines. In this case also a trapezoid can be drawn so as to closely agree with the true form. As before, the trapezoid can be designed so as to give nearly exact discharges for any particular range of depths, or the notch can be designed to the true form as above explained. The formulæ for a drowned notch are as follows: For an upstream depth d let q_1 be the discharge through $ADEG$ and q'_1 through $DCFE$.

$$\begin{aligned} q &= q_1 + q'_1 \\ &= \tfrac{2}{3}C_1\sqrt{2g}(d-h)^{\frac{3}{2}}[l_b + 2rh + \cdot 8r(d-h)] \\ &\quad + C'_1\sqrt{2g(d-h)}(l_b + rh)h \quad . \; . \; . \; (64). \end{aligned}$$

For a greater discharge let D and H be the heights of AG and DE above CF. Then

$$\begin{aligned} Q &= \tfrac{2}{3}C_2\sqrt{2g}(D-H)^{\frac{3}{2}}[l_b + 2rH + \cdot 8r(D-H)] \\ &\quad + C'_2\sqrt{2g(D-H)}(l_b + rH)H \quad . \; . \; . \; (65). \end{aligned}$$

If the upstream and downstream channels are similar in all respects $d-h=D-H$ and $D-d=H-h$. Let $D=2d$. Then $d=D-d=H-h$ and $H=d+h$. Therefore

$$\begin{aligned} Q &= \tfrac{2}{3}C_2\sqrt{2g}(d-h)^{\frac{3}{2}}[l_b + 2rH + \cdot 8r(d-h)] \\ &\quad + C'_2\sqrt{2g(d-h)}(l_b + rH)H \quad . \; . \; (66). \end{aligned}$$

Subtracting 64 from 66 and putting $C_1=C_2=C$ and $C'_1=C'_2=C'$,

$$\begin{aligned} Q-q &= \tfrac{2}{3}C\sqrt{2g}(d-h)^{\frac{3}{2}}[2rd] \\ &\quad + C'\sqrt{2g(d-h)}[l_b(H-h) + r(H^2-h^2)] \quad . \; . \; . \; (67). \end{aligned}$$

from which r can be found, and l_b can then be found from 65, Q and Q_1 being selected at such depths as to make the trapezoid most accurate at the points desired. If D is not taken as $2d$, or if C_1 and C_2 differ, the equation will be complicated, and it may be easiest to adopt the instalment process and design the notch to the true curve, afterwards straightening it if necessary.

For notches having crest lengths of 2 to 6 feet c has been considered in India to be ·65 to ·73 and C ·70 to ·78, the figures being greater the larger the notch. Recent figures given by Harvey[1] are as follows:—

$H=$	3	5	7	8	9 feet.
$C=$	·848*	·945*	·95*	·87†	·9†

It appears that the notches had been built too wide—perhaps because C was taken too low—and have since been narrowed.

18. Oblique and Special Weirs.—If a weir is built obliquely across a stream the discharge is that due to the full length of the weir, provided the section of the stream passing over the weir is small compared to that of the stream at the approach section. In this case the water approaches the weir nearly at right angles. Thus at low water the full length of the weir is utilised. A weir AC (Fig. 83A) must be higher than BD in order to hold up low

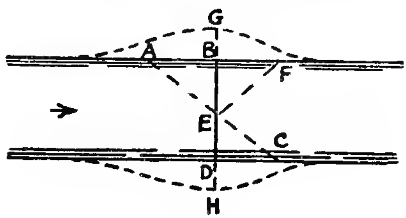


FIG. 83A.

water to the same level. But in floods the water passing over the weir travels nearly parallel to the axis of the stream. AC probably obstructs floods as much as BD does. If the low water discharge is very small, the heights of AC and BD may be almost equal and the oblique weir may give a slight advantage in a flood. The heavier the flood the less the advantage. The above remarks also hold good if the oblique weir is V-shaped (CEF) and in whichever direction the stream is flowing. If the channel is widened as per dotted lines the full length GH is utilised even in floods, but if v_a is high the gain as regards flood level—compared with the weir BD—is not so great as when v_a is low. The problem of constructing a weir so that it will hold up low supplies and yet not form a serious obstruction to floods can best be solved by means of gates or shutters. See also chap. vii. art. 8.

Circular weirs have been used where there was not room for straight weirs (Gourley, *Min. Proc. Inst. C.E.*, vol. clxxxiv.), the

[1] *Proceedings Punjab Engineering Congress*, 1919.
* AG (Fig. 82) = 6 ft, CF = 2·63 ft., BC = 7·5 ft.
† AG (Fig. 82) = 5·4 ft. CF = 1·6 ft., BC = 9 ft.

spigot ends of pipes—6-inch to 24-inch—having been turned true inside and outside and bevelled on the inside and the pipes placed vertically with the spigot ends upwards and submerged, the water thus flowing over the edges and into the pipes. The width of the square edge above the bevel was $\frac{1}{32}$ in. for the 6-inch and 9-inch pipes and $\frac{1}{8}$ in. for the others. A formula involving $H^{1\cdot42}$ was arrived at and applies to heads up to $\cdot2D$. Calculated for the usual weir formula the co-efficients are:—

Outside diameter (D) of pipe (inches)	6·9	10·08	13·7	19·4	25·9
c (when $H = \cdot5$ ft.)	·58	·58	·585	·59	·60
c (when $H = \cdot25$ ft.)	·61	·61	·615	·62	·63

Each pipe stood in a square chamber whose diameter should be $3D$, the width of the channel of approach being $2D$, baffle plates being used to still the water and an air tube opening under the lip of the weir.

Water has been made to flow up a pipe (Stewart and Longwell, *Trans. Am. Soc. C.E.*, vol. lxvii.)—of diameter (D) 2 to 12 inches—and out at the top, which was turned true and bevelled on the outside and had a sharp edge. Let H be the height of the water above the edge. If $H > \cdot1D$ there is a "jet condition" and $Q = 5\cdot84\, D^{2\cdot05}\, H^{\cdot053}$. If $H < \cdot1D$ there is a "weir condition" and $Q = 8\cdot8\, D^{1\cdot29}\, H^{1\cdot29}$. Let $D = 1$ and $H = \cdot1$, then c in the usual weir formula comes out 1·02, the sheet probably clinging to the crest. For smaller heads c is greater. If $D = \cdot5$ then c is some 20 per cent. less for the same values of H.

When the plane of a weir in a thin wall, instead of being vertical, is inclined, the co-efficients can be obtained by multiplying that for a vertical weir by a co-efficient of correction c_k, whose value was found by Bazin to be as follows:—

Inclination of plane of weir—

	Upstream.				Downstream.				
Inclination	1 to 1,	$\frac{2}{3}$ to 1,	$\frac{1}{3}$ to 1;	vertical,	$\frac{1}{3}$ to 1,	$\frac{2}{3}$ to 1,	1 to 1,	2 to 1,	4 to 1.
Average value of c_k—	·93	·94	·96	1·0	1·04	1·07	1·10	1·12	1·09.

The heights of the weirs when vertical were 3·72 feet, 1·64 feet, and 1·15 feet. The co-efficient is a maximum when the weir is inclined downstream at 2 to 1, that is, when the height of the crest above the bed is half the distance of the crest downstream from the base of the weir. The weirs were without end contractions, and the head ranged in each case from about ·33 feet to 1·48 feet.

EXAMPLES

Example 1.—A weir in a thin wall is 25 feet long and 3 feet high, and H is 1 foot. The channel of approach is 30 feet wide. Find Q'.

The crest contraction is complete, and the end contraction so nearly complete that no allowance need be made for it. From table xiv. c is probably ·612. From table xii. $K=3·275$. Then $Q'=25\times3·275=81·88$ cubic feet per second.

To allow for v by the usual method, $A=30\times4=120$ square feet. Let Q be assumed to be, say, 84 cubic feet per second. Then $v=\frac{84}{120}=·70$. From table i. $h=·0076$. Let $n=1·\dot{3}$. Then $nh=·0101$, $H+nh=1·010$. The corresponding correction in $(H+nh)^{\frac{3}{2}}$ and in Q' is 1·5 per cent., and Q is thus 83·11 cubic feet per second.

To allow for v by table xiii. $\dfrac{A}{a}=\dfrac{30\times4}{25\times1}=4·8$. When c is ·60 c_a is about 1·015. When c is ·61 c_a is about 1·016. This makes $Q=83·19$ cubic feet per second.

Example 2.—A river 50 feet wide has a maximum discharge of 600 cubic feet per second, the depth being then 3 feet. A weir with a rounded crest ($c=·80$) is to be built in the river so as to raise the flood level by 1 foot. What must be the height of the crest above the bed?

The discharge, q, per foot run of weir is 12 cubic feet per second, and table xii. for $c=·80$ gives $K=4·28$. Therefore $(H+nh)^{\frac{3}{2}}=\dfrac{12}{4·28}=2·80$. From table xi. $H+nh=1·99$ feet. But $v=3·0$, and h (table i.) $=·14$ foot. Therefore, n being 1·0, H is 1·85 feet, and the crest must be 2·15 feet above the bed. The result is quite accurate, supposing that the channel downstream of the weir is altered for a long distance so as to give a free fall over the weir. Otherwise the weir will be drowned, H_2 being ·85 foot, but judging from Bazin's results (art. 14) with weirs having a moderate top width and sloping back and face, the discharge will hardly be affected, H_2 being only $·46H_1$. Actually H would perhaps be 1·9 or 1·95 feet.

Example 3.—A river whose mean width is 50 feet, depth 10 feet, and mean velocity 3 feet per second, has a bridge built across it. The piers and abutments are square, and the total width of the water-way in the bridge is 30 feet. Find the heading-up caused by the bridge.

Let c be ·60. Since Q is 1500 cubic feet per second, and a is 300, therefore $V=\frac{1500}{300\times\cdot 60}=8\cdot 33$ feet per second. From table i. $H=1\cdot 08$ feet nearly, but as there is high velocity of approach H will be less, say 1·0 foot. Therefore $A=50\times 11\cdot 0=550$ square feet, and $v=\frac{1500}{550}=2\cdot 73$ feet per second. From table i. $h=\cdot 116$. Let $n=1\cdot 0$. Then $H+nh=1\cdot 116$. From table i. $V=8\cdot 47$ feet per second, which is too great by nearly 2 per cent., and H is therefore less than 1 foot by 4 per cent., that is, it is ·96 foot.

Example 4.—The depth of full supply in a canal is 5 feet. The discharges with depths of 4 feet and 2 feet are 153 cubic feet and 46 cubic feet per second respectively. Design a trapezoidal notch for a free fall in the canal. The co-efficient is ·66.

From equation 62, page 109,

$$r=\frac{153-2\cdot 828\times 46}{12\cdot 10\times\cdot 66\times 2^{\frac{5}{2}}}=\cdot 51.$$

From equation 63, page 109,

$$l_b=\frac{2\cdot 262\times 4\cdot 6-\cdot 4\times 153}{6\cdot 05\times\cdot 66\times 2^{\frac{3}{2}}}=3\cdot 78 \text{ feet.}$$

Example 5.—A weir in a thin wall is 4 feet high and H is 1 foot. The bed of the stream becomes filled up, so that the depth above the weir becomes 2·5 feet instead of 5 feet, but Q is unaltered. How is H affected?

The ratios $\frac{A}{a}$ are 5 and 2·5 nearly. From table xiii., c being ·60 and n being $1\cdot 3\dot{3}$, the values of c_a are 1·013 and 1·057, so that Q is increased about 4·4 per cent. if H is the same. H will therefore be less than before by $\frac{2}{3}\times 4\cdot 4$ per cent., that is, it will be ·97 feet

Table XI.

Values of H and $H^{\frac{3}{2}}$. (Art. 1.)

H	$H^{\frac{3}{2}}$	Diff. ·01 H	H	$H^{\frac{3}{2}}$	Diff. ·01 H	H	$H^{\frac{3}{2}}$	Diff. ·01 H
·04	·0080	·0032	·60	·4648	·0119	1·8	2·415	·0202
·05	·0112	·0035	·62	·4882	·0121	1·85	2·516	·0205
·06	·0147	·0038	·64	·5120	·0123	1·90	2·619	·0208
·07	·0185	·0041	·66	·5362	·0125	1·95	2·723	·0210
·08	·0226	·0044	·68	·5607	·0127	2·	2·828	·0214
·09	·0270	·0047	·70	·5857	·0129	2·05	2·935	·0216
·10	·0316	·0049	·72	·6109	·0130	2·1	3·043	·0218
·11	·0365	·0051	·74	·6366	·0131	2·15	3·152	·0221
·12	·0416	·0053	·76	·6626	·0132	2·2	3·263	·0224
·13	·0469	·0055	·78	·6889	·0133	2·25	3·375	·0226
·14	·0524	·0057	·80	·7155	·0135	2·3	3·488	·0228
·15	·0581	·0059	·82	·7426	·0137	2·35	3·602	·0231
·16	·0640	·0061	·84	·7699	·0138	2·4	3·718	·0234
·17	·0701	·0063	·86	·7975	·0140	2·45	3·834	·0237
·18	·0764	·0064	·88	·8255	·0142	2·5	3·953	·0238
·19	·0828	·0066	·90	·8538	·0143	2·55	4·072	·0240
·20	·0894	·0068	·92	·8824	·0145	2·6	4·192	·0242
·22	·1032	·0072	·94	·9114	·0146	2·65	4·314	·0244
·24	·1176	·0075	·96	·9406	·0148	2·7	4·437	·0246
·26	·1326	·0078	·98	·9702	·0149	2·75	4·560	·0250
·28	·1482	·0081	1·0	1·000	·0152	2·8	4·685	·0252
·30	·1643	·0084	1·05	1·076	·0156	2·85	4·811	·0254
·32	·1810	·0087	1·10	1·154	·0158	2·90	4·939	·0255
·34	·1983	·0089	1·15	1·233	·0163	2·95	5·066	·0260
·36	·2160	·0091	1·2	1·315	·0166	3·0	5·196	·0262
·38	·2342	·0094	1·25	1·398	·0168	3·1	5·458	·0266
·40	·2530	·0096	1·3	1·482	·0172	3·2	5·724	·0271
·42	·2722	·0099	1·35	1·568	·0176	3·3	5·995	·0275
·44	·2919	·0101	1·4	1·657	·0178	3·4	6·269	·0279
·46	·3120	·0103	1·45	1·746	·0182	3·5	6·548	·0283
·48	·3326	·0106	1·5	1·837	·0186	3·6	6·831	·0287
·50	·3536	·0109	1·55	1·930	·0188	3·7	7·117	·0291
·52	·3750	·0112	1·6	2·024	·0190	3·8	7·408	·0294
·54	·3968	·0113	1·65	2.119	·0194	3·9	7·702	·0298
·56	·4191	·0116	1·7	2·217	·0197	4·0	8·000	·0302
·58	·4417	·0117	1·75	2·315	·0200			

Table XII.—Values of K or $\frac{2}{3}c\sqrt{2g}$ or $5·35c$. (Art. 1.)

c	K	c	K	c	K
·001	·00535	·61	3·264	·81	4·334
·002	·0107	·62	3·317	·82	4·387
·003	·01605	·63	3·371	·83	4·441
·004	·0214	·64	3·424	·84	4·494
·005	·0268	·65	3·478	·85	4·548
·006	·0321	·66	3·531	·86	4·601
·007	·0375	·67	3·581	·87	4·655
·008	·0428	·68	3·638	·88	4·708
·009	·0482	·69	3·692	·89	4·762
·5	2·675	·7	3·745	·9	4·815
·51	2·729	·71	3·799	·91	4·869
·52	2·782	·72	3·852	·92	4·922
·53	2·836	·73	3·906	·93	4·976
·54	2·889	·74	3·959	·94	5·029
·55	2·943	·75	4·013	·95	5·083
·56	2·996	·76	4·066	·96	5·136
·57	3·050	·77	4·120	·97	5·190
·58	3·103	·78	4·173	·98	5·243
·59	3·157	·79	4·227	·99	5·297
·6	3·21	·8	4·28	1	5·35

Table XIII.—Co-efficients of Correction, c_a, for Velocity of Approach. (Art. 5.)

$\frac{A}{a}$	$c=·60$						$c=·80$		
	Values of n.						Values of n.		
	1·4	$1·3\dot{3}$	1				1·4	$1·3\dot{3}$	1
2	1·098	1·093	1·067				1·198	1·189	1·129
2·2	1·079	1·075	1·055				1·156	1·149	1·105
2·5	1·060	1·057	1·042				1·115	1·110	1·079
3	1·041	1·039	1·028				1·074	1·071	1·050
4	1·022	1·021	1·015				1·041	1·039	1·028
5	1·014	1·013	1·009				1·025	1·024	1·017
7	1·007	1·007	1·005				1·012	1·011	1·008
10	1·003	1·003	1·001				1·006	1·006	1·004

Tables XIV. and XV.—Co-efficients of Discharge, c, for Weirs in Thin Walls with Complete Contraction. (Art. 6.)

XIV.—Ordinary Weirs.

Head in Feet.	Length of Weir in Feet.						
	·5	1	2	3	5	10	19
·1	...	...	...	·652	·653	·655	·656
·15	·598	·605	·630	·638	·640	·641	·642
·2	·593	·600	·623	·630	·631	·633	·634
·25	·583	·595	·617	·624	·626	·628	·629
·3	·578	·593	·612	·619	·621	·624	·625
·4	·578	·591	·607	·613	·615	·618	·620
·5	·582	·589	·602	·608	·611	·615	·617
·6	·584	·587	·598	·605	·608	·613	·615
·7	·585	·585	·594	·603	·606	·612	·614
·8	·588	·584	·590	·600	·604	·611	·613
·9	·590	·584	·587	·598	·603	·609	·612
1	·592	·583	·585	·595	·601	·608	·611
1·2	...	...	...	·591	·597	·605	·610
1·4	...	...	·573	·587	·594	·602	·609
1·6	...	...	·571	·582	·591	·600	·607
1·7	...	...	...	...	...	·599	·607
2	...	...	...	·576	...	...	...

XV.—Short Weirs.

Head in Feet.	Length of Weir in Feet.						
	·033	·066	·099	·164	·246	·329	·654
·03	...	...	...	...	·634	...	...
·05	...	...	·620	...	·618	...	...
·10	...	...	...	·605	·608	·618	·624
·13	...	...	...	·613	·605	·605	·618
·16	...	...	·629	·614	·604	·598	·611
·25	...	·653	·628	·612	·602	...	·594
·33	...	·648	·627	·612	...	...	·591
·39	·679	·645	·627	·612	...	·589	·590
·66	·668	·640	·628	·614	...	·593	·591
.80	·666	·642	·628	·615	...	·594	...

TABLE XVI.—CO-EFFICIENTS OF DISCHARGE, c, FOR WEIRS IN THIN WALLS WITHOUT END CONTRACTIONS, BUT WITH FULL CREST CONTRACTION. (Art. 6.)

Head in Feet.	Length of Weir in Feet.								
	1·6 to 6·6	2 (?)	3 (?)	4	5	7	10	15	19
	Bazin's Co-efficients.	Smith's Co-efficients.							
·1	...	...	...	...	·659	·658	·658	·657	·657
·15	...	·652	·649	·647	·645	·645	·644	·644	·643
·2	·662	·645	·642	·641	·638	·637	·637	·636	·635
·25	·655	·641	·638	·636	·634	·633	·632	·631	·630
·3	·652	·639	·636	·633	·631	·629	·628	·627	·626
·4	·646	·636	·633	·630	·628	·625	·623	·622	·621
·5	·640	·637	·633	·630	·627	·624	·621	·620	·619
·6	·637	·638	·634	·630	·627	·623	·620	·619	·618
·7	·635	·640	·635	·631	·628	·624	·620	·619	·618
·8	·633	·643	·637	·633	·629	·625	·621	·620	·618
·9	·633	·645	·639	·635	·631	·627	·622	·620	·619
1	·632	·648	·641	·637	·633	·628	·624	·621	·619
1·2	·631	...	·646	·641	·636	·632	·626	·623	·620
1·4	·630	...	...	·644	·640	·634	·629	·625	·622
1·6	·627	...	...	·647	·642	·637	·631	·626	·623
1·7	·626	...	...	...	...	·638	·632	·626	·623
1·8	·625	...	...	...	...	...	...	...	...

TABLE XVII.—CORRECTIONS FOR WIDE CRESTS. (Art. 10.)

(The correction is always minus except when marked plus.)

Head in Feet.	Width of Crest in Inches.								
	1	2	3	4	6	8	10	12	24
·10	·007	·016	·018	·018	·017	·017	·017	·017	·017
·15	+ ·002	·017	·023	·024	·025	·025	·025	·025	·026
·20		·012	·024	·029	·031	·032	·033	·033	·034
·30		+ ·005	·017	·030	·041	·045	·047	·048	·050
·40			·010	·022	·045	·055	·060	·062	·066
·45			+ ·009	...	...	...	...	...	...
·50				·006	·041	·060	·069	·074	·082
·60					·031	·059	·075	·083	·097
·70					·017	·052	·075	·089	·112
·80					·000	·040	·071	·091	·125
·90					+ ·019	·027	·062	·089	·137
1·0						·056	·050	·082	·149
1·2						+ ·025	·021	·061	·168
1·4							+ ·013	·032	·180
1·5								·015	...

Tables XVIII. to XXII.—Inclusive Co-efficients, C, for Weirs 6·56 Feet Long without End Contractions.

XVIII.—Weirs in Thin Walls. (Art. 6.)

Height of Weir in Feet.	Head in Feet.												
	·164	·23	·33	·53	·66	·82	·98	1·15	1·31	1·48	1·64	1·80	1·97
·66	·687	·683	·689	·703	·720	·735	·750	...	...	...	...	...	...
·98	·680	·672	·671	·677	·689	·700	·713	·723	·734	·743	...	...	...
1·31	·677	·668	·663	·665	·671	·680	·690	·699	·708	·716	·723	·729	·735
1·64	·675	·665	·659	·657	·660	·668	·675	·683	·689	·696	·712	·709	·714
1·97	·674	·663	·656	·653	·654	·659	·665	·671	·677	·683	·689	·693	·699
2·62	·674	·662	·653	·647	·647	·648	·651	·657	·660	·664	·668	·671	·677
3·28	·674	·660	·651	·644	·642	·643	·645	·648	·650	·653	·656	·658	·662
4·92	·672	·660	·650	·641	·638	·636	·636	·636	·636	·638	·639	·640	·641
6·56	·672	·659	·650	·641	·635	·633	·632	632	·632	·632	·632	·632	·632

XIX.—Weirs with Flat Tops and Vertical Face and Back. (Art. 10.)

Dimensions of Weirs.		Heads in Feet.												
Width in Feet.	Height in Feet.	·3	·7	1·0	1·2	1·4	1·6	2·0	2·3	3·0	3·8	4·3	5·0	6·0
16·30	11·25	...	...	...	...	...	...	...	...	...	...	·50	...	...
12·24	11·25	...	...	...	...	...	...	...	·49	...	...	·50	...	...
6·56	4·57	...	...	...	...	...	...	...	·45	...	...	...	·48	·51 (?)
6·56[1]	2·46	·45	·48	·48	...	·50	...	...	...	...	...	...	...	...
5·88	11·25	...	...	...	·50	...	...	...	...	...	...	...	...	...
3·17	11·25	...	...	...	...	...	...	·51	...	·54	...	...	...	...
2·62	4·57	...	...	...	...	...	...	...	·52	...	...	...	·64	·65 (?)
2·62[1]	2·46	·48	·49	·50	...	·52	...	...	...	...	...	...	...	...
1·65	11·25	...	...	...	...	...	·55	...	...	...	·62			
1·31	2·46	·50	·51	·54	...	·59	...	...						
·66	2·46	·52	·58	·65	...	·70	...	...						
·66	1·15	·53	·60	·67	...	...	...	...						
·48	11·25	...	...	...	...	...	·62	·62						
·33	2·46	·57	·65	·80*	...	·71*								
·33	1·15	·57	·72	·77*	...	...								
·16	2·46	·63	·80*	·77*	...	...								
·16	1·15	·69	·75*	·78*	...	...								

[1] When the width was increased by ·33 feet and the upstream edge rounded to a radius of ·33 feet C was increased 10 to 14 per cent.

* These are for sheets drowned underneath. All other figures are for free sheets, and the corresponding figures for Bazin's weirs for sheets depressed or drowned underneath are the same to within, generally, 4 per cent.

XX.—Weirs with Rounded Tops. (Art. 12.)

Sections of Weirs.	Dimensions of Weirs. Radius of Crest.	Height in Feet.	Head in feet. ·3	·7	1·4	·9	2·5	4·7
Fig. 69, p. 82,	·34 ft. upstream, ·40 ft. downstream	1·64	·67	·79	·86	...	...	...
Fig. 78, p. 99, .	·26 ft.	1·64	·72	·84	·84	...	...	...
Fig. 78A, p. 99, .	3·37 ft.	5·3		·57(?)	·62(?)	·61	·64	·675
Fig. 78B, p. 99,	...	1·64	·57	·59	·65	...	...	
	When the height was 8 feet and radius of curvature 10 feet, c was ·60 when $H=2$ feet. When weir raised by laying a $1' \times 1'$ timber along the crest, c was ·65 when $H=2$ feet or 2·7 feet (Horton, *op. cit.*).							

XXI.—Weirs with Steep Backslopes. (Art. 11.)

Top Width of Weir in Feet.	Height of Weir in Feet.	Slope of Face of Weir.	Back Vertical. Head in Feet. ·3	·7	1·4	1·2	5·0	Back ⅓ to 1. Head in Feet. ·3	·7	1·4	Back ⅔ to 1. Head in Feet. ·3	·7	1·4
0·00	1·64	Vertical						·65	·76	·71	·75	·78	·71
	1·64	⅓ to 1	·68	·78	·74								
	1·64	2 to 1	·75	·79	·77								
·33	1·64	Vertical						·56	·73	·72	·56	·73	·71
	1·64	1 to 1	·59	·72	·80			·57	·73	·78	·60	·73	·80
	1·64	2 to 1	·61	·71	·77								
	4·7	2 to 1				·70	·67						
·66	1·64	2 to 1	·58	·65	·73								
	4·9	2 to 1					·695						
	4·9	3 to 1				·67	·67						
	4·9	5 to 1				·63	·63						

XXII.—Weirs with Flat Back-slopes. (Art. 11.)

Slope of Back of Weir.	Top Width of Weir in Feet.	Height of Weir in Feet.	Face Vertical.			Face 1 to 1.			Face 2 to 1.					
			Head in Feet.			Head in Feet.			Head in Feet.					
			·3	·7	1·4	·3	·7	1·4	·3	·7	1·4	2·3	4	6
1 to 1	·00	1·64				·81	·80	·76	·79	·78	·77			
		2·46	·72	·73	·75									
		4·9								·79 (?)	·77 (?)	·73 (?)	·70 (?)	·68 (?)
	·33	1·64	·57	·71	·78	·60	·72	82	·61	·71	·79			
	·66	1·64							·58	·65	74			
2 to 1	·00	1·64	·65	·69	·72	·72	·75	·79	·73	·74	78			
		2·46	65	·66	·69									
	·33	1·64	·56	·66	·72				·61	·69	·78			
	·66	1·64				·58	·63	·74	·58	·64	·73			
		2·46	48	·56	·68	·51	·61	·71	·55	·62	·70			
		4·9								·63 (?)	·66 (?)	·68 (?)	·69 (?)	·69 (?)
	1·31	2·46	·49	·51	·58									
3 to 1	·00	1·64	·55	·64	·68							Heads.		
												1·7	3·3	4·8
5 to 1	·00	1·64				·66	·66	·69	·68	·69	·71			
		2·46	·58	·58	·60									
	·33	1·64	·54	·58	63	·56	·64	·70	·58	·65	·69			
		4·9							·5	·66 (?)	68 (?)	·67	·64	·67
	·66	1·64							·58	·62	·68			
10 to 1	·00	2·46	·52	·54	·56									

For a weir 8 feet high, with upstream slope 5 to 4 and downstream slope 1 in 6, c was ·69 when $H = 1{\cdot}9$ feet. When the weir was raised by laying a $1' \times 1'$ timber along the crest, c was ·68 when $H = 1{\cdot}2$ feet (Horton, *op. cit.*).

For rapids, C has been found to be ·65 to ·67, the face having a slope of 1 to 1, H being 2·5 to 4·2 feet and v_a being 2 to 3 feet per second (*The Control of Water*, Parker).

CHAPTER V

PIPES

[For preliminary information see chapter ii. articles 8 to 21]

Section I.—Uniform Flow

1. General Information.—In a uniform pipe, AB (Fig. 84), let the length AC, amounting to two or three times the diameter, be termed

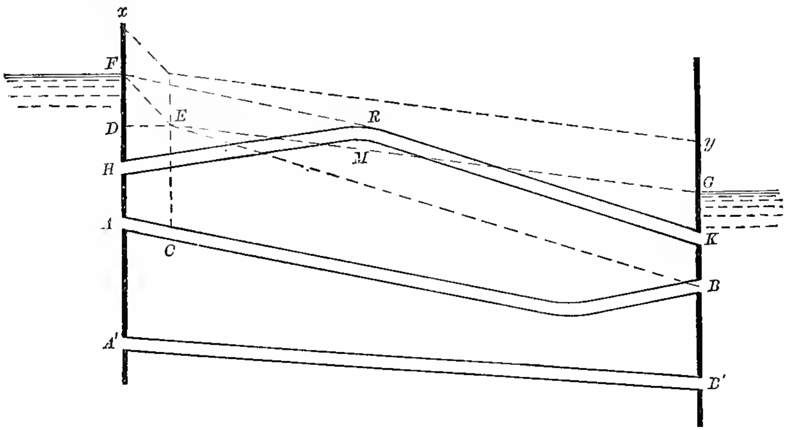


Fig. 84.

the mouthpiece of the pipe. At the entrance of the pipe a head $\frac{V^2}{2g}$ must be spent in imparting momentum to the water. This causes a loss of pressure head only, and not of total head. In exchange for the loss of pressure the water obtains a velocity head $\frac{V^2}{2g}$, but this is finally lost in the receiving reservoir, where the energy possessed by the water is wasted in eddies. There is also a loss in the mouthpiece depending on the co-efficient of resistance (chap. iii. art. 6), and varying from about $\cdot 06\frac{V^2}{2g}$ in a bell-mouthed, to about $\cdot 50\frac{V^2}{2g}$ in a cylindrical mouthpiece. This last occurs if the pipe simply stops short flush with the side of the reservoir without being splayed out. If the pipe projects into the reservoir, and ends without a flange, the loss of head is about $\cdot 93\frac{V^2}{2g}$. The total loss of pressure head at the entrance of a pipe is thus $(1+z_a)\frac{V^2}{2g}$ where z_a varies from ·06 to ·93. This loss of head is the height *FD*. The line of hydraulic gradient is *FEG*.

In equal lengths L_1, L_2, etc., the falls in the line of gradient or losses of head by friction are equal. If the inclination of the pipe is uniform, as in *A'B'*, the line of virtual slope is straight, but not otherwise. Generally, however, the variations in the inclination of the pipe in lengths L_1, L_2, etc., are not enough to cause great differences in the lengths of their horizontal projections, and the line of virtual slope is practically straight. Generally the length of a pipe is so great that the loss of head at the entrance may be neglected in estimating *H*, and the length of the mouthpiece in estimating *L*. *S* is then found more easily. The actual position of the pipe is of no consequence. The virtual slopes and discharges of the pipes *AB*, *A'B'*, etc., are all equal, provided the roughnesses, diameters, and lengths are equal. If the pipe discharges freely into air, the virtual slope is *FB*. Pipes are always assumed to be circular in section unless the contrary is stated.

If at any point *R* the line of the pipe rises above the line of the hydraulic gradient, the pressure is less than the atmospheric pressure. At such a point air may be disengaged from the water and the flow impeded, the line of gradient being shifted to *FR* (loss of head at entrance not considered) and the pipe *RK* running only partly full. If the height *MR* is more than 34 feet and *R* is lower than *X*, flow is still possible.[1] The above refers to cases in which the water is subjected throughout to ordinary atmospheric pressure. If the pressures on the two reservoirs are unequal the heads must be calculated (chap. ii. art. 1) and the gradient *xy* drawn accordingly. Arrangements must be made for periodically drawing off the air which accumulates at 'summits' such as *R* lying above the gradient line.

With small pipes a great increase in the temperature of the water increases the discharge. The following results have been found :—

[1] See Notes at end of chapter.

Diameter of Pipe.	Increase in Temperature of Water.		Increase of Discharge.
	From	To	
Inches.			
1	60°	212°	25 per cent.
1·5	57°	120°	8 per cent. (V about 8·5). 10 per cent. (V about 5·7).
2	52°	59°	Discharge was perceptibly increased.

The pressure in a pipe, after allowing for difference in head, decreases somewhat in going from the circumference to the centre.

Let D be the diameter of a pipe. Then R is $\frac{D}{4}$ or half the actual radius. Since the sectional area is as D^2, $\sqrt{R}$ as $\sqrt{D}$, and since C also increases with D, the discharge increases more rapidly than $D^{\frac{5}{2}}$. If two pipes are nearly equal in diameter, their discharges will be nearly as $D^{\frac{5}{2}}$. Allowing for increase of C, a pipe of 2 feet diameter will discharge nearly as much as six pipes of 1 foot diameter. To double the discharge of a pipe it is only necessary to increase the diameter by about 30 per cent. Since V increases as $\sqrt{S}$, and C also increases slightly with S, the discharge increases rather more rapidly than $\sqrt{S}$. In order to double the discharge S must be more than trebled. Doubling the slope increases the discharge by perhaps 50 per cent. For a given head H the slope is inversely as L, and Q therefore increases more rapidly than $\frac{1}{\sqrt{L}}$. It is clear that slight errors in measuring the diameter of a pipe, or an insufficient number of measurements when the diameter varies—as it nearly always does—may cause considerable errors in discharges or co-efficients.

All the ordinary problems connected with flow in uniform pipes can be solved by means of equations 14 and 15 (p. 21), some directly and some by the tentative process. The problems referred to are those in which one of the quantities Q, S and D has to be found, the others being given. V can, of course, always be found from D and Q without difficulty, or either of those quantities from V and the other. Pipes are generally manufactured of certain fixed sizes, and when the theoretical diameter has been calculated the most suitable of these sizes can be adopted, unless a special size is to be made. To facilitate calculations various tables have been prepared. The method of using them and of dealing with the above problems will be clear from the examples given and the remarks which precede them.

2. **Short Pipes.**—When the length of a pipe is not very great the velocity may be high, the co-efficient C may be outside the range of experimental data, and its value then can only be estimated. For cases in which L is not more than $100D$ the pipe may be treated as a short tube, and equation 7 (p. 13) used. The following values of c have been found:—

MATERIALS AND DIAMETERS IN INCHES.

Ratio of L to D.	Iron.[1]	Cast Iron.[2]				Earthenware.[3]	Cement or Stoneware.[5]						Brickwork.[4]
	1	4	6	8	10	6	4	6	9	12	18	24	·5′ × ·4′.
6		..	..		..		..		..	..	..	·77	
8		..	..	.	..	..	..		..	..	·74		
12		..	..		..	..	..		..	·71	..	.	
15	..	..		..	..	..	..		·68	..	..	..	
21·6	..	..	..	..	·84	..	..	..	..	..	..	..	
24	·73	..		..		·80	..	·64		..	..	..	..
27	..		..	·76		..	..	..	..	..	..	..	·66
36	·68	..	·71	.	..	·69	·60	..		..	..	..	·55
43	..	..	..	..	·74	..	..	..		..	..	..	·53
48	·63	..	..	..		..	..	..	..	..		..	·48
54	..	·63	..	·66									
60	·60	..	..										
72	..	..	·62	..									
100	·55	..	..	..									
108	..	·50	..										

Full details of the experiments are not known. When V is not too high it is best to adopt the usual formula for pipes. See example 4 at end of chapter.

For the brickwork pipe the ratio of L to D has reference to the ·5′ dimension.

[1] Fanning ($H = 2·36$ ft.).

[2] Egyptian Irrigation Experiments ($H = ·8_{-}$ ft.).

[3] Punjab do. ($H = ·3$ to 3 ft.).

[4] do. do. ($H = ·3$ to 3 ft.).

[5] Madras Kistna Division Experiments ($H = ·25$ to $3·2$ ft.).

Punjab Engineering Congress, 1916. (P.B. Varma.) All the pipes were submerged.

All the experiments were made with small heads. The shorter the pipe the greater the proportionate loss of head at the entrance and the less the variation of c for a proportionate increase in L. Thus when L increases from $25L$ to $50L$ c does not decrease so much as when L increases from $50L$ to $100L$.

3. **Combinations of Pipes.**—If a pipe does not simply connect two reservoirs, but is, say, a branch supplied from a larger pipe and itself bifurcating, its discharge can only be ascertained by tapping it and attaching pressure columns.

When a water-main gives off branches it may undergo reductions in diameter. Suppose that the conditions in such a main are to be determined when no water is being drawn off by the branches. If the discharge of the main is known the loss of head and gradient in each length can be found. Suppose, however, that only the total loss of head H is known. Obviously the gradient in any length will be flatter as D is greater, and $\sqrt{S}$ will be roughly as $\frac{1}{D^{\frac{5}{2}}}$ or $\frac{H}{L}$ as $\frac{1}{D^5}$ or H as $\frac{L}{D^5}$. Thus if the total loss of head is known the loss in each length can be roughly found, the gradient being sketched and the discharge computed. When greater accuracy is required let D' be an approximation to the average diameter of the whole main. With this diameter and gradient $\frac{H}{L}$ find an approximate discharge Q', and thence the velocities V_1, V_2, etc. Then for any length L_1, $C_1\sqrt{R_1}=\frac{V_1}{\sqrt{S}}$. The slopes S_1, S_2, etc., can then be found, and the losses of head are $L_1 S_1$, $L_2 S_2$, etc. If these when added together are not equal to H the discharge Q' must be corrected. When Q has been found accurately the diameter D of the equivalent uniform main is known. It is such as gives the discharge Q with the gradient $\frac{H}{L}$. If the above problem again occurs with the same pipe, but a different value of H, there will be no difficulty, for D will be practically unaltered.

Let Fig. 85 represent a main of uniform diameter, and let its discharge be drawn off gradually by branches. If the discharges at M and N are Q and zero respectively, and if the discharge is supposed to decrease uniformly along the whole length of the pipe, then the line of gradient will be a curve. If x and y are the ordinates of any point in the curve, and A and B are constants, $Q=Ax$.

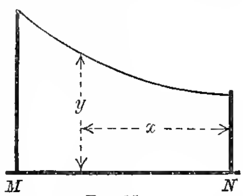


FIG. 85.

But if C is supposed constant, $Q=B\sqrt{S}=B\left(\frac{dy}{dx}\right)^{\frac{1}{2}}$. Therefore $\frac{dy}{dx}=\frac{A^2}{B^2}x^2$. Integrating, $y=\frac{A^2}{3B^2}x^3$.

When $x=L$, $y_1=\frac{A^2}{3B^2}$. L^3, and the mean gradient $\frac{y_1}{L}=\frac{A^2}{3B^2}.L^2$. But when $x=L$, $\frac{dy}{dx}$ is $\frac{A^2}{B^2}.L^2$, or the mean gradient is one-third of the gradient at M. The total loss of head is one-third of what it would have been if the whole discharge Q had been delivered at N. As C increases with S the fraction is really greater than one-third.

If in a branched pipe (Fig. 86) the pressures at A, B, C are known, the discharges can be found by assuming a pressure head, H, at D, and calculating the discharges Q_1, Q_2, Q_3. If Q_1 does not

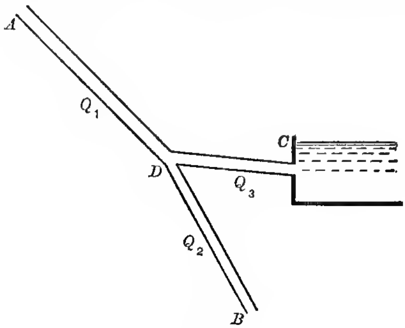


FIG. 86.

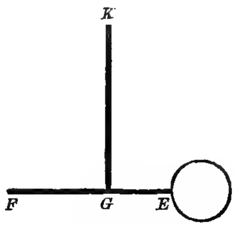


FIG. 87.

equal Q_2+Q_3, then H must be altered and a fresh trial made. Q_3 may be plus, zero, or minus according to the direction in which the water flows.

Let E (Fig. 87) be a water-main, EF a branch, and GK a pressure column, and let there be a three-way cock at G. If no water is being drawn off at F the water rises to a height K, determined by the pressure in the main, whether GK or GF is open; but if water is being drawn off at F the height GK will be less when GF is open. If EF is a house service-pipe and GK a pipe rising to the ground-level outside the house, then by means of a pressure-gauge at K an inspector can tell, without entering the house, whether water is being used in it or not.

In a system of bifurcating pipes (Fig. 88) such as that used for the water-supply of a town, the pressure heads sufficient to force the water to the required levels at various points, L, K, F, having been determined, the gradients corresponding to imaginary pressure columns at these points can be drawn, and the required discharges q_1, q_2, etc., being known, the diameters of the various pipes can be calculated. Suppose the system to be at work, then if the consumption in a branch FG is increased, the pressure head at F will be lowered and the branch FH will not be able to obtain its

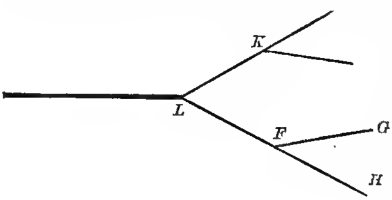


FIG. 88.

estimated supply, unless its conditions are similar to those of FG. The lowering of the pressure at F causes an increased discharge in LF, and a lowering at L, and thus more water is drawn in from the reservoir, but not to the same amount as the increase taken by FG. Thus any excessive consumption tends to partially remedy itself, firstly by preventing water being forced to high levels in its neighbourhood, and secondly, by drawing more water into the main. (Cf. chap. vii. art. 6.)

4. Bends.—The loss of head, H_B, "due to a bend" in a pipe, is the loss over and above the loss, H, from friction in the same length of straight pipe. It is usually put in the form $Z_b \dfrac{V^2}{2g}$. With a view to ascertaining the values of Z_b for bends of 90° in pipes of diameters ranging from about 2 inches to 2·5 feet, experiments have been made by Weisbach,[1] Williams, Hubbell and Fenkell,[2] Schoder,[3] Davis,[3] and Brightmore.[4] The bends experimented on had radii (R) of $2{\cdot}5D$ to $24D$, D being the diameter of the pipe. A detailed review of all the experiments is given in *The Engineer*, 26th May 1911. The general result is roughly that Z_b in a 90° bend may be about ·10 to ·40, and that the loss of head H_B is generally only a fraction of H.

The experiments show that great care is needed to ascertain H_B. The difficulty is to determine what the loss would have been in a straight pipe. A small error in ascertaining this upsets the calculations completely. It is essential that the diameter and condition of the bend should be the same as in the tangents, and the same as in the straight length with which the bend is to be compared, and that the pressure columns should be so placed that they are not affected by disturbances due to the bend itself, or to any other bend or any other cause operating upstream. A length of 100 pipe diameters is perhaps necessary to let a disturbance die away. These conditions have not been completely fulfilled in any of the experiments. Owing to the smallness of H_B its actual value has been obscured by the errors, and the results of the experiments are generally considered to be unreliable. Details of them are, however, given below.

When R is great the resistance per foot run of pipe is small, but the length is great, and this may cause a fairly high value of H_B. As bearing on this point it may be observed that views are discrepant as to the effect of a very slight change in direction.

[1] *Mechanics of Engineering.*
[2] *Trans. Am. Soc. C.E.*, vol. xlvii.
[3] *Trans. Am. Soc. C.E.*, vol. lxii.
[4] *Min. Proc. Inst. C.E.*, vol. clxix.

Williams, Hubbell and Fenkell state that a divergence from the straight of 2° had considerable effect. Schoder found that C, for a pipe laid not strictly straight, *i.e.* with a slight zig-zag appearance, was the same as when it was quite straight, and he quotes the case of a pipe in which gentle bends of several degrees had no effect.

The fact that H_B is caused largely in the downstream tangent (chap. ii. art. 13) was recognised in all the experiments, and it was included in the observations, the normal loss of head due to the tangent length being afterwards deducted. Brightmore found that the loss of head in the bend itself was little, if at all, greater than in an equal length of straight pipe, but the circumstances seem to have been peculiar, as noted below.

In a cross-section a few feet downstream of the termination of a 90-degree curve of 40-feet radius in a 30-inch pipe the maximum velocity was found with low velocities to be in the centre of the pipe, but it moved, when the maximum velocity was 3·5 feet per second, to a distance from the edge of the pipe equal to about ·20 of the diameter. A further increase of 30 per cent. in the velocity failed to shift it further. With curves of 15 feet and 40 feet radius its position was about the same. In Brightmore's experiments on 3-inch and 4-inch pipes the flow in a bend approximated to that in a free vortex, *i.e.* the velocity in going across the pipe, at the lower end of the bend, from the outside to the inside of the bend, was nearly inversely as the radius struck from the centre of the bend. He also found, with the 3-inch pipe, with R equal to $12D$, that when V exceeded 3 feet per second the condition was unstable, H_B being sometimes about a mean between the values for $R=10D$ and $R=14D$, but being sometimes much less.

Weisbach, as well as most of the other experimenters, make H_B equal to $Z_b \frac{V^2}{2g}$. The following are the approximate values found for Z_b for bends of 90°:—

Experimenter.	Diameter of pipe (D).	Radius of Bend (R).								
		Zero (elbow).	2·5D.	8·5D.	5D.	7D.	10D.	14D.	15D.	20D.
	Inches.									
Weisbach .	...	·98	·14	·135	·13	...	...	...	...	...
Davis . .	$2\frac{1}{16}$	...	...	·33	..	·49	...	...	...	...
Brightmore	3	1·17	...	·29	...	·39	...	·15	...	...
Do.	4									
Schoder .	6	...	·12	·11	...	·14	·08	·025	·015	·14
	Feet.									
Williams Hubbell and Fenkell	1	...	...	·35	...	...	...	...	...	...
	2·5	...	...	·40	...	...	...	...	...	...

Whatever is known regarding the relative losses of head in bends subtending different angles is given in chap. ii. art. 13.

5. Relative Velocities in Cross-Section.—The velocities at different points in the cross-section of a pipe have been observed chiefly by means of the Pitot tube (chap viii. art. 14). Bazin found that the velocity curve was convex downstream, and that $r = \cdot 74R$, r being the distance from the axis to the point where the velocity is equal to V—the mean velocity in the pipe—and R being the radius of the pipe. In a 30-inch pipe the form of the velocity curve was found by Williams, Hubbell, and Fenkell to be very nearly a semi-ellipse. The velocity ratios tended to become irregular with low velocities. It is useless to discuss the precise nature of the curve until the ratio of V to the central velocity is better determined.

Regarding this ratio various old experiments show somewhat conflicting results. The ratio increases with V and also with the diameter of the pipe. The following table must be taken as showing probable and approximate values only :—

Kind of Pipe.	Diameter of Pipe in inches.	Mean Velocities in Feet per Second.							
		·78	1·5	2·5	3·5	5	8	14	62·5
Brass, . .	$1\frac{1}{8}$	...	...	...	...	...	...	...	·84
Brass seamless, .	2	·70	·73	·77	·79	·80	...	...	...
Cast-iron, . .	7·5	...	...	·80	·81	·82	·83	·84	...
Cast-iron, . .	9·5	...	·80	·81	·82	·83	·84	·85	...
Cast-iron with deposit, .	9·5	...	·81	·81	·82	·82	·83	·83	...
New iron coated with coal-tar, .	12	...	·83	·83	·84	·85	·85	·85	...
	16	...	·82	·83	·84	·85	...	...	...
	30	·75	·83	·84	·85				
Cement, . .	31·5	...	...	...	·85	·86	...	...	...
New iron coated with coal-tar, .	42	...	...	...	·86	...	...	...	...

Bilton's figures for small pipes, mostly cast-iron (*Proc. Victorian Inst. of Engineers*, 1909, and *Min. Proc. Inst. C.E.*, vol. clxxx.), are as follows :—

Central velocity, ft. per second,	2.	4.	6.	8 and over.
$\frac{1}{2}$-inch pipe	·750	·764	·788	·804
$\frac{3}{4}$-inch pipe	·780	·793	·817	·830
1-inch pipe	·793	·810	·835	·848
$1\frac{1}{2}$-inch pipe	·807	·830	·855	·868
2-inch pipe	·812	·839	·865	·878
3-inch-pipe	...	·843	·872	·888
4-inch pipe and larger	...	...	·873	·890

Bilton considers that the ratio diminishes slightly as the roughness increases. In large pipes it was found that in two cases the ratio was as much as 0·914 and 0·994. Bilton explains this by suggesting that in large pipes the maximum velocity may not always be at the centre of the pipe, but that, owing to obstructions, oscillation may take place, and it may follow a wave-like course; in large pipes at low velocities the ratio is not definitely ascertainable.

SECTION II.—VARIABLE FLOW

6. Abrupt Changes.—The losses of head occurring at abrupt changes in small pipes have been found experimentally by Weisbach, and are as below.

Abrupt Enlargement (Fig. 4, p. 5).—The loss of head is $\frac{(V_1 - V_2)^2}{2g}$ or the head due to the relative velocity, but see remarks in chap. ii. art. 18.[1]

Abrupt Contraction (Fig. 3, p. 5).—The loss of head (and also for a diaphragm (Fig. 90) or for a contraction with a diaphragm) is chiefly caused by the enlargement from EF to MN, and is to be found as above. To find the velocity at EF divide the velocity

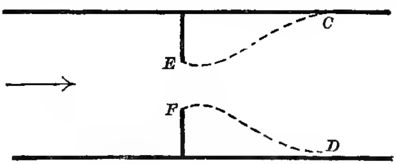


FIG. 90.

at MN by c_c. For a diaphragm[2] (Fig. 90) the values of c_c were found to be as follows:—

$\frac{\text{Area } EF}{\text{Area } CD}$ =	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
c_c =	·624	·632	·643	·659	·681	·712	·755	·813	·892	1·00

These may be accepted for the other cases.

Elbow (Fig. 91).—The loss of head is

$$z_e \frac{V^2}{2g} \text{ where } z_e = \cdot 946 \sin^2 \frac{\theta}{2} + 2\cdot 05 \sin \frac{\theta}{2}$$

[1] See also Notes at end of chapter.
[2] See also chap. viii. art. 17.

The values of z_e are as follows :—

$\theta=$ 20°	40°	60°	80°	90°	100°	110°	120°	130°	140°
$z_e=$·046	·139	·364	·740	·984	1·260	1·556	1·861	2·158	2·431.

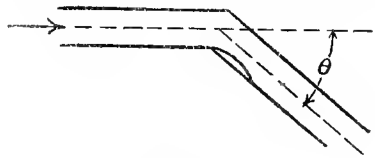


FIG. 91.

Thus at a right-angled elbow nearly the whole head due to the velocity is lost. When two right-angled elbows closely succeed each other the loss of head is double that in one elbow if the two bends are in opposite directions, but is no greater than that in a single elbow if the bends are both in one direction.

Gate-Valve (Fig. 92).—

$\frac{h}{D}=1·0$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{a}{A}=1·0$	·948	·856	·740	·609	·466	·315	·159
$z_e=$ ·0	·07	·26	·81	2·06	5·52	17·0	97·8.

Where A is the sectional area of the pipe and a that of the opening.

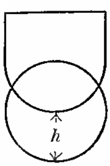


FIG. 92.

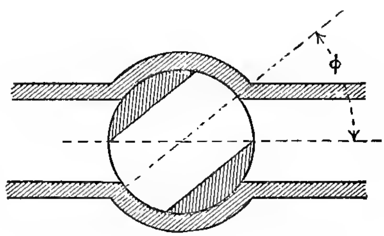


FIG. 93.

Cock (Fig. 93).—

$\phi=$	5°	10°	15°	20°	25°	30°	35°	40°	45°
$\frac{a}{A}=$	·926	·850	·772	·692	·613	·525	·458	·385	·315
$z_e=$	·05	·29	·75	1·56	3·10	5·47	9·68	17·3	31·2
$\phi=$	50°	55°	60°	65°	82°				
$\frac{a}{A}=$	·250	·190	·137	·091	0·0				
$z_e=$	52·6	106	206	486	∞				

Throttle Valve (Fig. 94).—

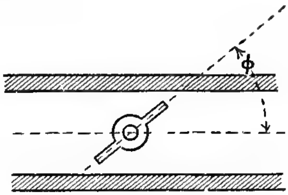


Fig. 94.

$\phi = 5°$	$10°$	$15°$	$20°$	$25°$	$30°$	$35°$	$40°$	$45°$	$50°$
$z_t = {\cdot}24$	$\cdot 52$	$\cdot 90$	$1{\cdot}54$	$2{\cdot}51$	$3{\cdot}91$	$6{\cdot}22$	$10{\cdot}8$	$18{\cdot}7$	$32{\cdot}6$
$\phi = 55°$	$60°$	$65°$	$70°$						
$z_t = 58{\cdot}8$	118	256	$751.$						

In the last three cases z_v, z_c, and z_t are multiplied by $\frac{V^2}{2g}$ to give the loss of head.

It is not at all certain that the above figures apply correctly to large pipes, and in fact it has been proved that some of them do not apply correctly. For a gate in a 2-foot pipe z_v has been found to be as below.

$\frac{h}{D}$	z_v as observed.	z_v by Weisbach's rule given above.
$\frac{13}{72}$	41·2	43
$\frac{15}{72}$	31·35	28
$\frac{1}{4}$	22·7	17
$\frac{1}{3}$	11·9	7·92
$\frac{3}{8}$	8·63	5·52
5/12	6·33	3·77
11/24	4·58	2·87
1/2	3·27	2·06
7/12	1·55	1·11
2/3	·77	·57
1·0	·00	·00

When loss of head due to any of the above causes occurs, the line of hydraulic gradient shows a sudden drop as at GH, Fig. 95, its inclination is reduced, and with it the velocity and discharge of the pipe. If the local loss of head did not exist the slope would be KL. The velocity to be used in calculating the loss of head is that due to KG and not KL. If a second cause operates at M the gradient becomes KG', $H'M$, NL, and the loss of head $G'H'$ is now less than before because the velocity is less. Thus the loss of head does not increase in proportion to the number of causes operating. But where economy of head is desired, it is necessary to avoid abrupt changes of all kinds, using tapering 'reducers' where the diameter changes, and curves of fair radius at all bifurcations or changes in direction.

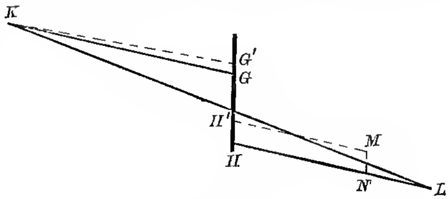


Fig. 95.

It appears that the disturbance of the velocity ratios due to abrupt changes may extend downstream for long distances. Bazin found that the disturbance from the entrance contraction of a 32-inch pipe disappeared at 25 to 50 diameters downstream, but disturbance due to curves has been found to extend to 100 diameters. In the disturbed region the pressures, as indicated by pressure columns, appear to be below normal, or at least to be unreliable. In some important experiments on a 6-foot pipe[1] some of the results are doubtful and probably erroneous, owing to a piezometer being placed just downstream of an abrupt change.

7. Gradual Changes.—When a gradual change occurs in the sectional area of a pipe equation 16, page 22, must be used. At a point where the diameter of a pipe changes a tapering piece is usually put in. If the taper is gradual the loss of head in it from resistances is about the same as in a uniform pipe with the same mean velocity.

The following are examples of accidental changes in the diameters of pipes:—

(1)	(2)	(3—4)		(5—6)		(7)	(8)	(9)	(10)	(11)	(12)
Length of Pipe.	Nominal Diameter.	Actual Diameters.		Velocities.		$\frac{V_1^2 - V_2^2}{2g}$ or h_v	$V = \frac{\sqrt{V_1^2+V_2^2}}{\sqrt{2}}$	C	Loss of Head from Resistances or h' or $\frac{v^2L}{c^2R}$.	Actual Fall in Gradient or h.	Percentage of figure in column 7 to figure in column 10.
		d_1	d_2	V_1	V_2						
Feet.	Ins.	Ins.	Ins.	Feet.	Feet.	Feet.	Feet.		Feet.	Feet.	
100	12	12·5	11·5	4·0	4·73	– ·099	4·38	113	·609	·708	16 2
25	30	$29\frac{7}{8}$	$30\frac{1}{8}$	4·0	3·93	+ ·0082	3·97	128	·0384	·0302	21·4
25	30	$29\frac{7}{8}$	$30\frac{1}{8}$	1·0	·983	+ ·00051	·993	113	·00312	·00261	16·3

[1] *Transactions of the American Society of Civil Engineers*, vol. xxvi.

The figures in column 11 are obtained from those in columns 7 and 10. If the flow were uniform the figures in columns 10 and 11 would be the same, and the ratio of these figures to one another shows the error caused by assuming the pipe to be uniform. If the fall h is observed, and V found from h and C, the value of V found will be erroneous in the ratio (neglecting the small variation in C) of $\sqrt{h}$ to $\sqrt{h'}$, that is, in the first of the cases shown, by about 8 per cent. of the smaller figure. If h and V are observed (V being found, say, by measuring Q in a tank) and C is deduced, the error in C will be similar to the above. If h is not observed, but deduced from known values of V and C, then the percentage error is as shown in column 12. The second and third cases show the same pipe with very different velocities, and it will be noticed that the percentage of error does not vary very greatly. In the first case quoted the variation of the diameter from the nominal diameter is perhaps excessive and hardly likely to occur in practice. With longer lengths of pipe the percentage of error will, of course, be small, but sometimes observations are made on short lengths, and it is clear that in such cases great error may arise, if the diameter is assumed to be uniform.

When the diameter of a pipe is reduced (Fig. 96) the velocity head in the narrow part is increased and the pressure head

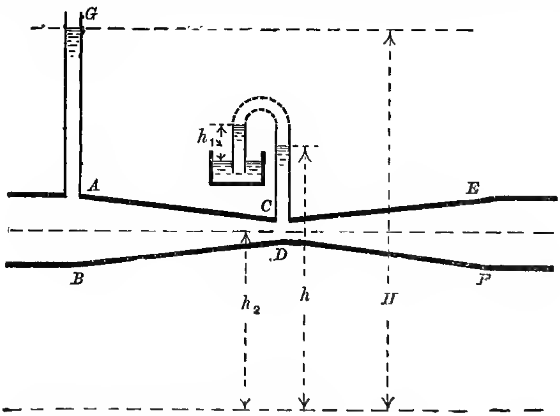


FIG. 96.

reduced. The insertion of a portion like ACE in a pipe causes very little loss of head if the tapers are gradual. The case is similar to that of a compound tube (chap. iii. art. 17). If CD is small enough, the pressure there will fall below the atmospheric pressure P_a, and if holes are bored in the pipe at this section no water will flow out, but air will enter. The pressures on the conical surfaces $ACDB$ and $CDFE$ balance one another, and the water has no more tendency to push the pipe forward than it has in a uniform pipe.

With the arrangement shown in Fig. 97, the orifices being made to correspond as exactly as possible, the water flows with very little waste into the second reservoir, and the head GH is slightly less than KL. The pressure in the jet KG is P_a, and it makes no practical difference whether this portion is enclosed by a pipe or not, so long as the head KL is kept the same.

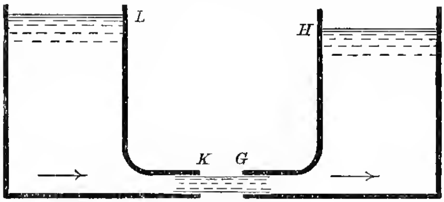


FIG. 97.

If at CD (Fig. 96) another pipe is introduced, pumping can be effected through it, as with the case of a cylindrical or compound tube.

When the hydraulic gradient of a pipe is so flat that the fall between two pressure columns would be too small to be properly observed, the 'Venturi Meter' (Fig. 96) is adopted. It consists of two tapering lengths of pipe with two pressure columns. If the diameters, velocities, and sectional areas at AB and CD are D, v, A and d, V, a, then (chap. ii.)

$$\frac{V^2}{2g}+h=\frac{v^2}{2g}+H.$$

Also $$\frac{V^2}{2g}=\frac{A^2}{a^2}\cdot\frac{v^2}{2g}.$$

Therefore $$\frac{v^2}{2g}\left(\frac{A^2}{a^2}-1\right)=H-h.$$

$$\therefore v^2=\frac{2ga^2}{A^2-a^2}(H-h).$$

$$v=\frac{a}{\sqrt{A^2-a^2}}\sqrt{2g(H-h)}.$$

To allow for loss of head in the tube a co-efficient c must be used, and

$$Q=c\frac{Aa}{\sqrt{A^2-a^2}}\sqrt{2g(H-h)} \quad . \quad . \quad . \quad (70).$$

If the pressure at CD is less than P_a, the height h_1 measures the difference (the pressure tube being bent as shown by the dotted lines), and h_1 must be deducted from h_2 to give h.

The length AC is actually made less than CE. For other details concerning Venturi meters see chap viii. art. 16.

Section III.—Co-efficients and Formulæ

8. General Information.—Pipes of importance are generally of iron. Of these the vast majority are of cast iron. In America some pipes—generally large—are of riveted steel or wrought iron, and some are wood-stave pipes. Pipes are also made of concrete or are lined with cement. An iron or steel pipe if not protected by an inside coating of asphalt—this term also includes coal tar and other compositions—generally becomes affected in time by 'incrustation.' Even if so protected it often becomes affected by incrustation or sometimes by vegetable growths. A 'clean' pipe is one—whether coated or uncoated—not affected in any way or which, if affected, has been cleaned. It is only for clean pipes that definite co-efficients can be given. Others will be referred to below (art. 10).

The sizes of pipes constantly tend to increase. There are cast-iron pipes 5 feet in diameter. A concrete pipe 14·5 feet in diameter is in use, also an 11-foot riveted steel pipe, lined with concrete.

For each class of pipe there is a separate set of co-efficients. C increases with R and also to some extent with S, that is with V. In tables it is usual to show C for different values of V, not of S. There are few observations for high velocities. Ordinary velocities range from 1 or 2 to 5 feet per second. Velocities of more than 10 feet per second are rare. Experiments on pipes have included many sizes and many velocities. Very frequently there are several values of S and V for one pipe. To obtain complete and accurate sets of co-efficients reliable experiments should be made with a large range of velocity on each one of a considerable number of sizes of pipes. It cannot be said that this has been done. To a great extent inference has to be adopted. Knowledge has, however, been improved of late.

It has been shown above that a slight difference in D, or irregularity in D, has a great effect. It must be added that—at least as regards some of the older experiments—the diameter may have been inaccurately stated, the manufacturer's size having been accepted. There may be considerable difficulty in obtaining V or Q with accuracy (chap. viii. section i.). Errors in the measurement of Q, D, and S may be in either direction, those in S and D being relatively greatest with low values of these quantities. But error may arise from unsuspected or unreported incrustation, air lock,[1] losses of head from bends or obstruction by objects which have accidentally got into the pipe or—in small pipes which cannot be got at from inside—by projecting pieces of lead used for the joints. All these tend to give low values of C. Hence, generally, C as reported is likely to be too low rather than too high, and to be worst determined when S or D is small. Small channels are no doubt more sensitive than large channels to variations in the roughness.

From the point of view of economy it is important to obtain reliable co-efficients for pipes. It is sometimes said that certain co-efficients are 'on the safe side,' and sometimes a distinction is drawn between 'laboratory' and 'field' experiments, the former being those in which sources of error are carefully removed. The value of C which is sought is the value for a clean pipe free from sources of error. The engineer can make allowances, and can be on the safe side as much as he thinks necessary.[2] It is not right to compel him to be so by supplying him with low figures. Neither should he be supplied with too high figures. There will always be a small margin within which co-efficients will vary. The value sought is not the one at the highest edge of the margin. It is one which will be obtained under proper conditions, and may possibly be exceeded.

9. **Co-efficients for Ordinary Clean Pipes.**—For cast-iron pipes Darcy obtained a set of co-efficients which vary from 93 to 113 as R varies from ·042 foot to 1 foot. Smith and Fanning framed much more extensive sets, making C increase with both R and V. Their co-efficients apply to clean cast-iron, steel, or wrought-iron pipes (not riveted), coated or uncoated, and with joints smooth and curves of fair radius. Lawford framed a similar set of co-efficients. Kutter's co-efficients ($N = {\cdot}011$) are also much used. A brief abstract of most of the above—for a velocity of 3 feet per second,

[1] See notes at end of chapter.

[2] In America a factor of safety—having reference to discharge and not to strength—is in many cases adopted. Its value can be fixed with reference to the injury likely to result from overestimation of the discharge.

which is about the most useful value—and of seven other sets of co-efficients is given in the following table :—

PIPE CO-EFFICIENTS ($V=3$ ft. per second).

Diameter in feet.	Kutter.	Smith.	Lawford.	Flamant.	Unwin.	Williams.	Saph and Schoder.	Williams and Hazen.	Barnes.	Mallett.
·25	40(?)	99	70	102	99	93	83	101	74	74
1	106	109	106	121	109	110	99	113	102	104
4	139	134	138	144	119	131	118	126	139	129
10	159	153	...	161	126	146	132	137	171	143

Fanning's co-efficients are nearly the same as Smith's. A set by Tutton is nearly the same as Williams'. Smith's figures, obtained by drawing a curve, included diameters up to 8 feet, but the curve has been extended. In each of the seven sets mentioned, and in Tutton's, C is obtained from a formula.[1] It is not known that in every case the author of the formula intended it to be applied to the larger diameters included in the table. It is not known that experiments have been made on any iron pipe, unless riveted, of diameter greater than 6 feet, though experiments on larger circular channels lined with mortar have been made, and some of the co-efficients were meant to include such channels.

It will be seen that in some cases C is persistently high or low, in others high or low for certain diameters. Some of the co-efficients were clearly intended to be on the safe side or to allow for badly laid or otherwise defective pipes. Unwin for large pipes relied partly on an experiment which has been rejected by others.[2] The small pipes on which Lawford experimented had been a year in use. His co-efficients for such pipes—not for others—were rejected by the author in 1911.[2] They were, however, used by others, and largely account for the low values of C, for small pipes, in the table. Barnes gives new experiments by himself on 40-inch and 44-inch pipes. His low figures for small pipes cause his curve to run up steeply, and give very high co-efficients for a 10-foot pipe. Fig. 97A shows C for a few selected sets. The relative differences in most cases are not very great, the zero being far below the diagram.

Kutter's co-efficients[3] (C_K) were derived from observations on open

[1] The formula gives V, but C can be calculated from it. For details and references see art. 11.

[2] *Engineering*, 2nd June 1911.

[3] For details as to Kutter's, Bazin's and Manning's co-efficients see chap vi. arts. 11 to 13.

channels of many sizes and degrees of roughness. It has long been known to engineers that C_K, supposing it to be correct for any large smooth channel, is for the same kind of channel too low when R is

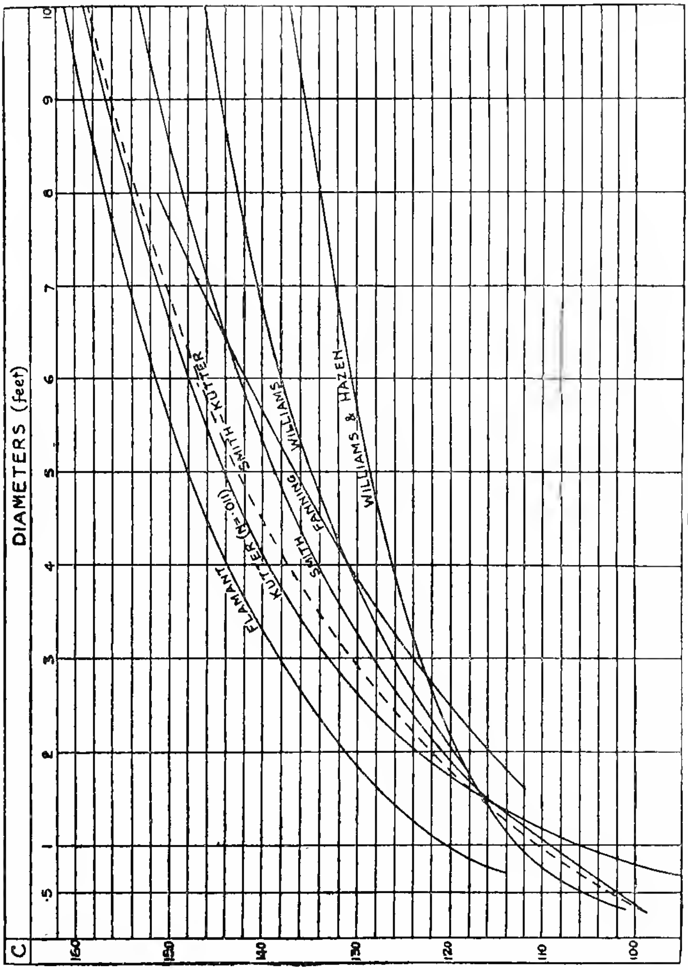


FIG. 97A.

about ·25 foot or less. The left-hand part of the curve of C_K should descend less abruptly. There is every probability of the true curve being higher than most of the others. The curve now suggested for acceptance is shown by a dotted line.[1] For small diameters it is near Smith's curve. For larger diameters it runs below the Kutter

[1] Marked Smith-Kutter.

line—but, as will be seen, C_K is in these cases somewhat high for the particular velocity under consideration—and joins it when D is 13 feet.

Regarding the values of C for velocities other than 3 feet per second, selected sets of co-efficients for various velocities are shown

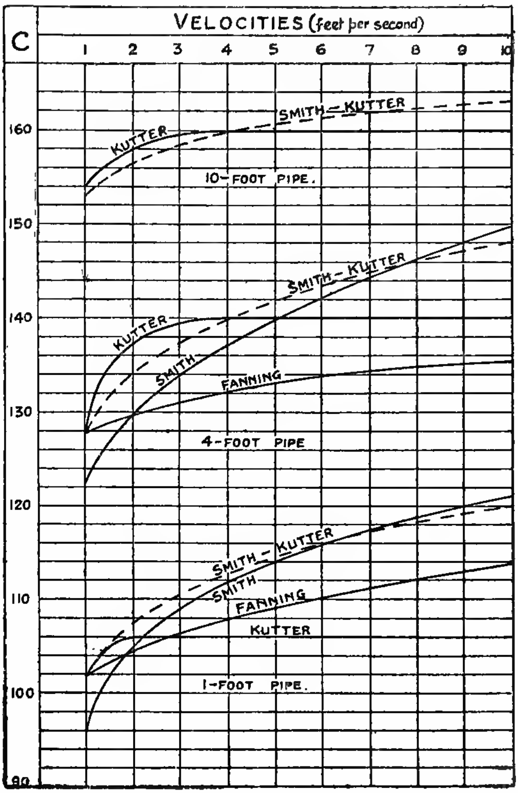


FIG. 97B.

in Fig. 97B for three sizes of pipe. The ordinates for velocities of 3 feet per second agree with those of Fig. 97A. Mallett's formula does not provide for any alteration of C as V changes. It will be mentioned again (art. 11). In the other seven formulæ C increases on the average by 17 per cent. as V varies from 1 to 10 feet per second. The increase is independent of D. Smith and Fanning have about the same average rate of increase, but it is less as D is greater. This is doubtless correct in principle, because in an open channel

C ceases to increase when R is great. Kutter makes it cease to increase when R is 3·28 feet, *i.e.* when D is, say, 13 feet. Accepting this and considering all the figures, the co-efficients of table XXVA. are arrived at. The figures for very high and very low velocities are of course not so well determined as the others.

The law of variation of C_K is peculiar and can hardly be correct. It changes rapidly when V is low, and ceases to change when V is higher. When D is 1 foot the Kutter curve is too low, as explained above. For larger diameters up to 8 feet the agreement is very much as when D is 4 feet. Owing to the bulge in the curve, C_K is relatively high when V is 3 feet per second. For high velocities C_K is too low except when D approaches 13 feet.

Manning's adaptation of C_K does not vary with V. When V is 3 feet per second it agrees closely with Smith's co-efficients.

With regard to small pipes, Schoder and Gehring,[1] with pipes—mostly rusty—of diameters of 3 to 8 inches, found Fanning's figures to be generally some 3 per cent. too low. They have been slightly raised, except for the smallest sizes—the increase, when $V=3$, is 5 per cent. for the 1-foot pipe, and 3 per cent. for the 6-inch—and this brings them into accord with those of the Smith-Kutter curve for larger pipes. They are included in table XXVA. Kutter's co-efficients—corresponding to values of V, not of S—are given in table XXVB. For Fanning's and Smith's original co-efficients see tables XXIV. and XXV.

All the co-efficients apply to cast-iron, wrought-iron, or steel pipes (not riveted), coated or uncoated, well laid, and with joints smooth and curves of fair radius. They apply to pipes of other materials if N is ·011. Kutter's co-efficients apply to all such channels, with the reservations already made.

As regards any possible difference between a coated and an uncoated pipe Smith, with a 1·05-inch pipe, found that coating it made no difference. This was confirmed by the experiments of Schoder and Gehring above referred to—some of the pipes were coated and some uncoated—and it is confirmed by general experience. Most of the largest pipes are coated, and experiments on such pipes when uncoated are wanting.

Kutter's and the other co-efficients dealt with in chap. vi. were meant to apply to open channels. Knowledge regarding small open channels is derived chiefly from Bazin's experiments. In these there are only a few cases in which, in the same channel, V changes while R does not change. In only some of such cases is

[1] *Engineering Record*, 29th August 1908.

there indication of increase of C with V. Kutter considered all Bazin's experiments and others, and concluded that C increases with V until S is 1 in 1000. He clearly did not discover the exact law. Experiments on pipes have been far more numerous, and there are frequently, as has been seen, several values of S and V for the same pipe, and thus clear evidence is obtained of the increase of C with V, co-efficients such as those above discussed can be obtained and the—not very great—inaccuracies of Kutter's co-efficients corrected.

Let V be the velocity in a circular channel running half full. It is improbable that V will be appreciably different—S being the same—when the channel is full. The distribution of the velocities (chap. vi. section iii.) is not the same, but this can hardly affect appreciably the general forward movement. The co-efficients of table XXVA. are probably better suited than any others to open channels of small or moderate size when N is ·011.

For pipes of cement, mortar, concrete, or brickwork there are

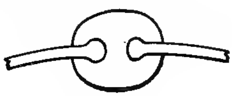


FIG. 97C.

few experiments from which tables such as XXVA. could be framed, and Kutter's co-efficients should be used. They are given in table XXVB. High velocities in such channels are unusual. For a given material, *e.g.* brickwork, the degree of roughness is not exactly the same in all cases. For the selection of the proper value of N for any pipe or channel see chap. vi. art. 12.

10. Co-efficients for Other Pipes.—Riveted pipes are made up of iron or steel sheets. The pipes are generally of large size, say, 2 to 10 feet in diameter. The sheets have lap joints longitudinally. In the 'taper' pattern each length of pipe tapers slightly, the smaller end fitting into the larger end of the next length down-stream—as in a stove pipe—and being riveted to it. There is thus a succession of abrupt but slight enlargements. In the 'cylindrical' pattern each alternate length is made of larger diameter so that the ends of both adjoining lengths fit into it, and are riveted to it. There is thus a succession of enlargements and contractions. In some pipes, however, there are butt joints. There is also a 'locking bar' type of pipe (Fig. 97C) in which the sheets, instead of being riveted longitudinally, are held in the grooves of a longitudinal bar. Usually there are double rows of rivets, both longitudinally and at the joints. The larger the pipe, the thicker generally the plates and the larger the rivet heads. Thus the larger the pipe, the greater its general roughness is likely to be.

The values of C as ascertained for riveted pipes of diameters from 2·75 feet to 8·615 feet are erratic.[1] Generally the change in C with change of R is comparatively small. Sometimes the larger diameter has the smaller value of C. All this is probably due to the larger pipe being the rougher, and to the different patterns. Whether the taper or the cylinder pattern gives the higher co-efficient is not known. By taking values of C for all the diameters

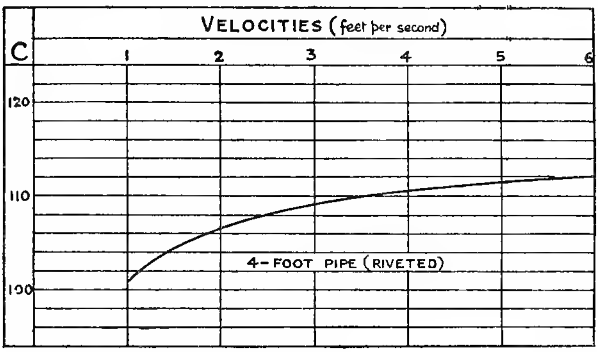


FIG. 97D.

within the range mentioned above—the mean diameter is 4 feet—and striking a general mean, a curve (Fig. 97D) has been arrived at. The curve is flatter than the corresponding curve in Fig. 97B, *i.e.* C is less affected by changes in V. When V is 3 feet per second the discharge of the riveted pipe is 20 per cent. less than that of the cast-iron pipe. N is between ·013 and ·014. The co-efficient for a riveted pipe of any of the sizes above considered, for any given value of V, will probably differ by not more than 5 to 7 per cent. from the corresponding figure on the diagram, but it may be either more or less. Of the largest sizes one is more and one less. And similarly with the smallest sizes. In designing a riveted pipe, figures should be obtained for actual pipes of similar pattern and size. Otherwise—and to some extent in any case—the factor of safety should be higher than for a cast-iron pipe. The

[1] *115 Experiments on Riveted Steel Pipes. Hydraulic Flow Reviewed. Trans. Am. Soc. C.E.*, vols. xl., xliv., and others.

following table, obtained by calculation from Garrett's *Hydraulic Tables and Diagrams*, is, however, given :—

Diameter of Pipe.	Velocity in Feet per Second.				
	1.	3.	5.	8.	10.
Feet.					
1	...	102	104	...	...
$1\frac{1}{2}$	94	105	107	...	...
2	96	106	108	112	...
$2\frac{1}{2}$	97	107	111	114	116
3	99	108	112	116	119
4	103	110	114	116	...
5	104	112	114	...	...
6	108	115	116	...	...

For smaller pipes sheet iron is used, and there may be single rows of rivets. For such pipes, asphalted and with diameters of $10\frac{7}{8}$ inches to $25\frac{7}{8}$ inches, and with V ranging up to 10, 12, and 20 feet per second, Smith found C to be very much the same as for ordinary cast-iron pipes (table XXVA.). The thickness of the sheets was usually only ·0054 foot to ·0091 foot.

If a large riveted pipe is lined with cement so as to be made uniform and smooth, the value of C will be increased accordingly. The discharge—allowance being made for the thickness of the lining—is likely to be increased by some 20 per cent.

For small spiral riveted pipes C has been found by Schoder and

TABLE XXIIA.

Description of Pipe.	Joints.	Diameter of Pipe.	Velocities in Feet per Second.				Remarks.
			1.	3.	5.	10.	
		Inches.					
Spiral riveted, asphalted.	Riveted flange.	6	+3	+ 7	+ 9	+11	Steel 0·05 in. thick.
Spiral riveted, galvanised.	Do.	6	−5	− 6	− 6	9	Steel 0·078 in. thick.
Spiral riveted, asphalted (flow with the laps).	Do.	4	+0	+ 3	+ 4	+ 5	Steel 0·0375 in. thick.
Spiral riveted, asphalted (flow against the laps).	Do.	4	−4	− 1	+ 0	+ 2	Steel 0·0375 in. thick.
Seamless drawn brass.	Special flange.	5	+6	+13	+18	+23	Flange arranged so as to give a continuous smooth pipe.

Gehring to be as given above[1] in table XXIIA. The figures show the differences between the experimental co-efficients and those of

[1] *Engineering Record*, 29th August 1908.

Fanning. In the case of the 6-inch pipes C was the same, whether the flow was with or against the lap. The rivets had very flat heads. 'The asphalt coating tends to fill up and smooth the lap, but the galvanising leaves the edge of the lap sharp.'

For wood-stave pipes the results of a great number of experiments are given by Scobey.[1] The diameters ranged from 4 inches to 13·5 feet. The values of C are in many cases extremely erratic. Some of the observations were carried out under great climatic and other difficulties. Sometimes the increase of C with V is very rapid, but sometimes it is nil, and on the average it is about the same as with cast-iron pipes, and the value of C for wood-stave pipes should be taken as being 9 or 9·5 per cent. less than the value shown in table XXVA. In the *Bulletin* it is suggested that the percentage averages about 4·5 when $V=3$, about 1 when $V=7$, and about 7 when $V=1$, but this refers to discharges of cast-iron pipes calculated by the Williams-Hazen formula. It will be seen (Fig. 97A) that this agrees—owing to the shape of the Williams-Hazen curve—with the figures now proposed. N for wood-stave pipes is about ·012. The discharging capacity of a wood-stave pipe does not usually either increase or decrease with use. The uncertainty as to C makes a comparatively high factor of safety desirable in designing.

The deposits and growths in pipes, already referred to (art. 8), are of various kinds and depend on the character of the water. The reduction of discharge which they are likely to cause is a matter of experience and judgment. Frequently there is a slimy deposit. This may form on the inside of the coating of a pipe or on iron, cement, or masonry. In time it may seriously reduce the discharge. With some waters the slime is succeeded by nodules. In some climates and with some waters vegetable growths occur inside the pipe. They can be prevented by sterilising the water. Incrustation of iron pipes is worst with soft moorland waters. If there is no coating, or at small holes or cracks in the coating, tubercles or nodules are formed. The nodules may be preceded by slime. Limestone water is far less harmful and no coating may be needed. In course of time the discharge of a tuberculated iron pipe may be reduced by 30 per cent. or even, especially with small pipes, by 50 per cent.

In an iron pipe slimy deposit may reduce N to about ·013—that is, by some 16 per cent.—in a few years. On masonry and cement it has less effect, perhaps because the channels are larger.

[1] U.S. Department of Agriculture, *Bulletin No. 376.*

Brickwork may deteriorate with age independently of deposits (chap. vi. art. 11). Barnes has found[1] that with the soft water from Thirlmere, in 40-inch and 44-inch asphalted mains, the discharge was reduced by 13 per cent. in one year and by a smaller percentage year by year, the total reduction in ten years being 31 per cent.

In America it is sometimes estimated that the discharge of a cast-iron pipe is reduced by 15 per cent. in ten years and by 30 per cent. in twenty years, and that of a riveted steel pipe by 9 per cent. in ten years.[2]

For a 2-inch seamless brass pipe Saph and Schoder found C to exceed Fanning's figures, the excess being 18 per cent. when $V=5{\cdot}77$ feet per second. See also table xxiiA.

Schoder and Gehring found that a 6-inch wrought-iron pipe in long service in a steam-heating main had a sort of glaze inside it, and C was some 16 per cent. higher than Fanning's figures.

For small tin, lead, zinc, or glass pipes Fanning's co-efficients are fairly correct. For 2·5-inch hose they are nearly correct when the hose is of rubber or lined with rubber, but they should be reduced by about 16 per cent. when the hose is of linen and unlined.

11. **Formulæ.**—The ordinary formula for flow in pipes is sometimes put in the form $\frac{H}{L}=\frac{V^2}{C^2R}$. This gives the loss of head, H, in a given length when C and R are known. If the diameters of two pipes are equal, the loss of head is as $\frac{1}{C^2}$. A moderate difference in estimating C—as when there is a choice of formulæ—makes a large difference in H.

The formulæ referred to in art. 9 are mostly of the form $S=K\frac{V^n}{D^m}$, where m and n are quantities such as 1·25 and 1·85, and K is constant. They are sometimes called exponential formulæ. There are formulæ of this type for open channels and weirs. It is unlikely that they are the true theoretical formulæ. The main idea is to avoid variable co-efficients. From the practical point of view there are serious objections to the use of such formulæ. Instead of referring to tables of co-efficients it is necessary to use a table of logarithms. The practical engineer has constantly to make rough and rapid calculations in connection, say, with changes which are contemplated or which have come about of themselves,

[1] *Min. Proc. Inst. C.E.*, vol. ccviii.
[2] U.S. Department of Agriculture, *Bulletin No. 376.*

e.g. a change in the width of a channel or of the depth of water in it. Within the range of depth, etc., with which he is concerned the co-efficient may be nearly constant, or, if not, he knows in what direction it changes. The simplicity of the formula is of the first importance. Even the detailed calculations made at the desk are done more quickly with the simple formulæ than with logarithms.

Again there is the question of comparisons. With the existing formulæ it is easy to make a comparison between the discharges, say, of two pipes, one of cast iron and one of riveted steel. With the exponential formulæ no comparisons can be made without working out the discharges. The values of the indices of R and S for two formulæ or two classes of pipes are different. The comparisons made above (art. 9) were not possible until C in the ordinary formula had been calculated. With the present formulæ the engineer can choose any value of c or C in which he believes.[1] Lastly, there are great numbers of persons—for instance, the irrigation subordinates in eastern countries—who can understand $H^{\frac{3}{2}}$ in the weir formula but not $H^{1\cdot47}$, nor could they use logarithms.

The formulæ referred to in art. 9 are as follows:—

Author—	Flamant.[2]	Unwin.[3]	Williams.[4]	Saph and Schoder.[5]	Williams and Hazen.[6]	Barnes.[7]
$K=$	·00036	·0004	·00038	·000469	·000368	·000436
$n=$	1·75	1·85	1·87	1·87	1·852	1·891
$m=$	1·25	1·127	1·25	1·25	1·167	1-454

Tutton's formula[8] is $V=140\,R^{\cdot66}\,S^{\cdot51}$. Mallett's formula[9] is of the Bazin type, and $C=\dfrac{\alpha}{\beta+\delta\dfrac{N}{\sqrt{R}}}$ (where N is Kutter's N), and does not vary with V. For cast-iron, concrete, and locking-bar pipes in best order, $\alpha=172$, $\beta=1$, $\delta=30$, and $N=\cdot011$. For slightly incrusted pipes, $\alpha=162$, $\beta=1$, $\delta=30$, and $N=\cdot013$. For pipes in worse condition there are other figures. The formula is of a general and inclusive type and is meant to give fair approximations under very diverse conditions.

1 When utilising the tables in the present work any value of c or C can be used.

2 *Annales des Ponts et Chaussées*, 1892. *Water*, Dec. 1913.

3 *Industries*, 1886.

4 *Trans. Am. Soc. C.E.*, vol. li.

5 *Trans. Am. Soc. C.E.*, vol. li. For 'commercial pipes' n varies from 1·74 to 2·0. The figures in art. 9 were obtained by taking n as 1·87.

6 U.S. Department of Agriculture, *Bulletin No. 376*.

7 *Hydraulic Flow Reviewed*. *Water*, 15th June 1916.

8 *Journal of Association of Engineering Societies*, vol. xxiii.

9 *Min. Proc. Inst. C.E.*, vol. ccviii.

Notes to Chapter V.

Air in Pipes (art. 1, p. 122).—The quantity of air which water can hold in solution is greater, the greater the pressure and the lower the temperature. At points of low pressure there is a tendency for air to be disengaged from the water. Air, however introduced, impedes the flow of water and reduces the discharge, the condition being known as 'air lock,' and most likely to occur with low pressures at 'summits' such as G, and with low velocities because the air is not then so quickly carried along or absorbed. At summits on important mains there may be automatic air valves. These allow any accumulated air to escape, and they allow air to enter the main when it is emptied for repairs and to escape when it is refilled. When the line of gradient is not far above the pipe, simple stand-pipes (Fig. 5, p. 9) may be used.

Pipes above Line of Hydraulic Gradient (art. 1, p. 122).—The pipe NRM (Fig. 97E) lies above the line of hydraulic gradient. Such cases are not common. The heights of any pressure columns in NRM are less than 34 feet, and the pressures less than atmospheric. Air may thus be disengaged from the water. At any defective joints water will not escape but air will enter. If there is a summit such as S above the gradient line, arrangements must be made for periodically drawing off the air accumulated there. For this purpose an air vessel is attached at the summit. At its lower side is a cock, A, opening to the pipe, at its upper side a cock, B, opening to the outer air. One or other of these must be closed. Suppose B to be closed and the vessel full of air at the same pressure as that in the pipe. To get rid of air A is closed and B opened. Through B water is introduced and the vessel filled, and the air in it expelled. B is now closed and A opened. The water finds its way into the pipe, and if there is air in the pipe it is displaced and enters the air vessel. By repeating the above operation the pipe can be kept free of air and air lock prevented. Another method is to attach an air exhausting pump and remove air from the air vessel until water is drawn at the pump. A pipe with a summit above the gradient line is often called a syphon. Pumping or suction are necessary in order to first start the flow in it.

Let a summit L be above the line xy. It is sometimes incorrectly said that flow is impossible, the idea being that water flows along the pipe HNL with such velocity as to consume in resistance all the head available. If air is regularly drawn off at L flow will take place, the gradient being xL. Flow is impossible only when L is higher than x.

Abrupt Enlargement (art. 6).—Observations by Archer[1] show that the loss of head in pipes 1 to 3 inches in diameter was actually $B\frac{(V_1 - V_2)}{2g}$, where B was as follows:—

Ratio of A_1 to A_2.	Values of V_1, feet per second.				
	2.	6.	12.	30.	80.
1 : $1\frac{1}{4}$	1·225	1·123	1·060	·981	·903
1 : 4	1·055	·965	·911	·846	·780
1 : ∞	1·022	·937	·884	·820	·759

Fluid Friction (chap. ii. art 9).—The friction of water on a plane surface is seldom exactly as V^2, but is as V^n where n varies from about 1·7 to 2·16. See chap. x. art. 5, final paragraph.

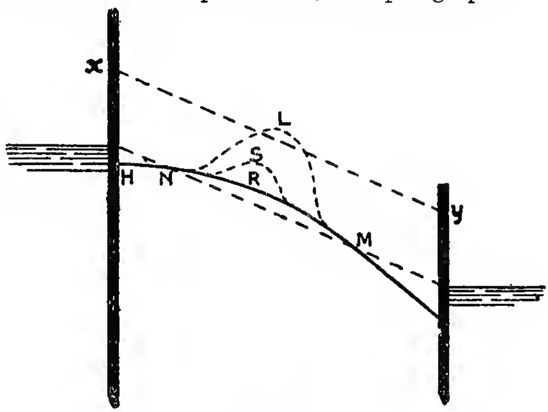


Fig. 97e.

Examples

Explanation.—The problem to be solved may be either to find the discharge in a pipe for which all the data are known, or when the discharge and one of the quantities D or S are known, to find the other. In the first case the solution is direct, in the others (since R and C vary with D and S) indirect. The methods to be adopted will be clear from the following examples.

In the examples Smith's and Fanning's co-efficients happen to have been used, but of course the new Smith-Kutter co-efficients —or any others—can be used in exactly the same manner.

[1] *Proc. Am. Soc. C.E.*, vol. xxxix.

One advantage of the system of tables here adopted, as compared to some others, is that V always enters as a factor. It is a distinct advantage, in designing, that the value of V, and not only of Q, should constantly come to notice.

Example 1.—Using Smith's co-efficients, find the discharge of a C.-I. pipe whose diameter is 3 feet and slope 1 in 1000.

From table xxiv., C is about 123·5 and V about 3·4. Smith's co-efficient for this value of V is 130, so that V will be about 3·6 and C about 130. From table xxiii. $\sqrt{R}=\cdot 866$. From table xxvi. $C\sqrt{R}=112\cdot 5$. From table xxviii. $V=3\cdot 56$, which agrees nearly with the value assumed, and confirms the co-efficient 130. From table xxiii. $A=7\cdot 07$. Then $Q=7\cdot 07\times 3\cdot 56=25\cdot 17$ c. ft. per second.

Example 2.—Using Smith's co-efficients, design a pipe to carry 20 c. ft. per second, the fall being 10 ft. in 5000.

Assume $D=2$ ft. From table xxiii. $A=3\cdot 142$ sq. ft. and $\sqrt{R}=\cdot 707$. Also $V=\dfrac{20}{3\cdot 14}=6\cdot 37$ ft. per second. From table xxv. $C=129$. From table xxvi. $C\sqrt{R}=91\cdot 2$. This value does not appear in table xxviii.; ∴ look out 182·4, which gives (for $S=\frac{1}{500}$) $V=8\cdot 16$; ∴ V is 4·08, which is too low, that is, the assumed diameter was too small.

Let $D=2\cdot 5$ ft. From table xxiii. $A=4\cdot 91$ and $\sqrt{R}=\cdot 791$. Also $V=\dfrac{20}{4\cdot 91}=4\cdot 07$ ft. per second. From table xxv. $C=128$. From table xxvi. $C\sqrt{R}=101$. From table xxviii. $V=4\cdot 52$ ft. per second, which is too high. The diameter 2·5 ft. is thus slightly in excess of what is required. To find the actual discharge, C (for $V=4\cdot 5$) is 129·5, $C\sqrt{R}$ is 102·4, V is 4·58, and Q is $4\cdot 58\times 4\cdot 91=22\cdot 49$ c. ft. per second.

Since $\left(\dfrac{2'\cdot 4''}{2'\cdot 6''}\right)^{\frac{5}{2}}=\left(\dfrac{14}{15}\right)^{\frac{5}{2}}=\dfrac{12\cdot 5}{15}$ nearly, ∴ a 2 ft. 4 in. pipe would be too small.

Example 3.—A $1\frac{1}{2}$-ft. C.-I. pipe has to carry a discharge of 18 c. ft. per second. What will the gradient be? Fanning's co-efficient to be used. From table xxiii. $A=1\cdot 77$. Then $V=\dfrac{18}{1\cdot 77}=10\cdot 2$ ft. per second. From table xxiv. $C=117$ and $S=\cdot 020$ nearly. From table xxvii. $\sqrt{S}=\cdot 1414$. From table xxiii.

$\sqrt{R}=\cdot612$. From table xxvi. $C\sqrt{R}=71\cdot6$ and $71\cdot6\times\cdot1414=10\cdot23$. Therefore $S=\cdot020$ is correct.

Example 4.—A pipe 2 in. in diameter and 20 ft. long connects two reservoirs, the head being 1 ft. and the pipe projecting into the upper reservoir. Find the discharge, using Fanning's co-efficients.

The pipe being short, the loss of head at entrance must be allowed for. This (art. 1) is $z_a=1\cdot93\frac{V^2}{2g}$. Suppose V to be 4 ft. per second. Then from table i. $\frac{V^2}{2g}=\cdot25$ and z_a is $\cdot48$. This loss occurs in the length of, say, $\cdot4$ ft., so that $L=19\cdot6$ ft. and $S=\frac{1\cdot0-\cdot48}{19\cdot6}=\cdot027$. From table xxiv. $S=\cdot040$ is the slope which gives $V=4\cdot0$, so that V has been assumed too high.

Let V be 3·5 ft. per second. Then $\frac{V^2}{2g}=\cdot19$, and z_a is $\cdot37$, and $S=\frac{1\cdot0-\cdot37}{19\cdot6}=\cdot032$. Table xxiv. does not give this slope exactly, but evidently C is about 97. From table xxiii. $\sqrt{R}$ is $\cdot204$. In table xxvi. look out $\cdot408$. Then $C\sqrt{R}$ is $\frac{39\cdot6}{2}=19\cdot8$. The slope $S=\cdot032$ is steeper than those in the tables. Therefore calculate $\sqrt{S}$, which is $\cdot18$, and $C\sqrt{RS}$, which is $19\cdot8\times\cdot18$, or $3\cdot56$ ft. per second, which is near enough.

Example 5.—An open stream discharging 16 c. ft. per second is passed under a road through a syphon or tunnel of smooth plastered brickwork of section 2 ft. $\times$ 2 ft., which first descends 10 ft. vertically, then travels 80 ft. horizontally, and again rises 10 ft. vertically, the bends being right-angled and sharp. What is the loss of head in the tunnel?

Here $V=\frac{16}{4}=4$ ft. per second. There are 4 elbows of 90° each. That at the entrance to the tunnel is opposite in direction to the second. Hence the total loss of head from the elbows is $4\times\cdot984\times\frac{V^2}{2g}=\cdot984$ ft.

To find the approximate loss of head from friction let Fanning's co-efficients be used. Then $R=\cdot5$, $C=117$, $S=\cdot0024$. The fall in 100 ft. is $\cdot24$ ft. The total loss of head is thus $\cdot98+\cdot24=1\cdot22$ ft.

TABLE XXIII.—VALUES OF A AND R FOR CIRCULAR PIPES.

Diameter (D). Feet.	Diameter (D). Inches.	Sectional Area (A). Square Feet.	Hydraulic Radius (R). Feet.	$\sqrt{R}$	Remarks.
	$\frac{1}{2}$	·00136	·0104	·102	
	$\frac{3}{4}$	·00307	·0156	·125	
	1	·00545	·0208	·144	
	$1\frac{1}{4}$	·00852	·0260	·161	
	$1\frac{1}{2}$	·0123	·0312	·177	
	$1\frac{3}{4}$	·0167	·0364	·191	
	2	·0218	·0417	·204	
	$2\frac{1}{2}$	·0341	·0521	·228	
	3	·0491	·0625	·250	
	4	·0873	·0833	·289	
	5	·136	·104	·323	*Diameters not given in Table.* To find A for a larger diameter, look out A for half the diameter and multiply by 4. For a smaller diameter, look out A for double the diameter and divide by 4. To find $\sqrt{R}$ for a larger diameter, look out $\sqrt{R}$ for one-fourth the diameter and multiply by 2. For a smaller diameter, look out $\sqrt{R}$ for 4 times the diameter and divide by 2. *Circular Channels not full.* For a channel of circular section running half full, A is one-half of the value in the table, and $\sqrt{R}$ is the same as in the Table.
	6	·196	·125	·354	
	7	·267	·146	·382	
	8	·349	·166	·408	
	9	·442	·187	·433	
	10	·545	·208	·456	
	11	·660	·229	·479	
1	0	·785	·250	·50	
1	1	·922	·271	·520	
1	2	1·069	·292	·540	
1	3	1·227	·313	·559	
1	4	1·396	·333	·577	
1	5	1·576	·354	·595	
1	6	1·767	·375	·612	
1	7	1·969	·396	·629	
1	8	2·181	·417	·646	
1	9	2·405	·437	·662	
1	10	2·640	·458	·677	
2	0	3·142	·500	·707	
2	2	3·687	·542	·736	
2	4	4·276	·583	·764	
2	6	4·909	·625	·791	
2	8	5·585	·667	·817	
2	10	6·305	·708	·841	
3	0	7·069	·750	·866	
3	3	8·296	·812	·901	
3	6	9·621	·875	·935	
3	9	11·05	·937	·967	
4	0	12·57	1·0	1·0	
4	6	15·90	1·125	1·061	
5	0	19·64	1·25	1·118	
5	6	23·76	1·375	1·173	
6	0	28·27	1·50	1·225	
6	6	33·18	1·625	1·275	
7	0	38·48	1·75	1·323	
7	6	44·18	1·875	1·370	
8	0	50·26	2·0	1·414	
8	6	56·74	2·125	1·458	
9	0	63·62	2·25	1·5	
9	6	70·88	2·375	1·541	
10	0	78·54	2·50	1·581	

Tables XXIV. to XXVb.—Co-Efficients for Pipes corresponding to given Diameters and Velocities. (Art. 9.)

(Also suitable for open channels when R *is the same and* N *the same.)*

Tables xxiv. to xxva. are for ordinary pipes, N being about ·011.

The small figures in table xxiv. show, nearly, the slopes which give the velocities entered in the heading, and they can be used to show the approximate slopes when the co-efficients in table xxv. or xxva. are used.

XXIV.—*Fanning's Co-Efficients.*

Diameter of Pipe.	Velocities in Feet per Second.									
	·1	·5	1	2	3	4	6	10	15	20
Inches. ½	43	51	76 ·0167	87 ·049	93 ·098	94 ·167	96 ·371	100 ·93	102	103
¾	50	75	79 ·0097	88 ·032	93 ·066	96 ·112	98 ·243	101 ·62	103	104
1	73	77	81 ·0083	89 ·024	94 ·048	94 ·083	98 ·180	102 ·47	104	105
1½	77	81	86	90 ·016	94 ·033	96 ·057	100 ·120	102 ·32	104	105
2	85	88	90	94 ·010	96 ·022	98 ·040	101 ·072	104 ·23	106	106
3	89	92	93	96 ·007	98 ·016	100 ·027	102 ·056	105 ·143	106	106
4	93	93	95	97 ·0049	100 ·011	102 ·019	103 ·039	106 ·107	108	108
6	94	95	97	100 ·0032	102 ·0070	103 ·012	106 ·026	108 ·070	109	111
8	96	97	99	102 ·0024	104 ·0050	105 ·0087	107 ·019	110 ·050	112	113
Feet. 1	93	100	102	105 ·0015	106 ·0032	108 ·0054	110 ·012	114 ·032	115	116
1·5	...	104	106	109 ·0092	111 ·0019	113 ·0034	114 ·0074	117 ·020	118	...
2	...	109	111	114 ·00063	116 ·0014	117 ·0024	118 ·0051	121 ·014	122	...
3	...	117	118	121 ·00038	123 ·00082	124 ·0014	127 ·0030	128 ·0082	129	...
4	...	127	128	129 ·00024	131 ·00052	132 ·00094	135 ·0020	135 ·0052	136	...
5	...	134	135	136 ·00017	137 ·00040	137 ·00067	138 ·0014	142 ·0040	142	...
6	...	137	137	137 ·00014	140 ·00032	141 ·00053	143 ·0012	147 ·0031	147	...
7	...	141	143	143 ·00011	146 ·00025	147 ·00045	148 ·00097	151 ·0024	151	...
8	...	149	150	151 ·00009	151 ·00020	152 ·00034	155	158	158	...

XXV.—Smith's Co-efficients.

Diameter of Pipe.	Velocities in Feet per Second.										
	1	2	3	4	5	6	8	10	12	15	20
Feet. ·05	...	78	82	86	88	89	91	91	91	91	...
·1	80	89	94	97	99	101	103	105	105	105	...
1	96	104	109	112	114	116	119	121	123	124	124
1·5	103	111	116	119	121	123	126	129	130	132	133
2	109	116	121	124	127	128	132	135	136	138	...
2·5	113	120	125	128	131	133	136	139	141	143	...
3	117	124	128	132	134	136	140	143	145	147	...
3·5	120	127	131	135	137	139	142	146	149	151	...
4	123	130	134	137	140	142	146	150	152	153	.
5	128	134	139	142	145	147	150	155	...	...	...
6	132	138	142	146	148	151	155	...	...	...	...
7	135	141	145	148	151	...	...	...	...	...	...
8	138	143	148	151	153	...	...	...	...	...	...

Notes on Hydraulic Tables.

The tables in this book, as already noted, admit of the use of any co-efficient which may be selected. The examples given show how they are to be used.

As regards interpolations, these can often be made by mere inspection. When strict accuracy is required the following example (table xxvi.) may be followed. Let C be 109·7 and $\sqrt{R}$ be 1·118. The upper and lower figures of C and $C\sqrt{R}$ are taken from the tables and the differences entered in the last line.

C	Diff.	$C\sqrt{R}$	Diff.
109		121·8	
	·7		·8
109·7		122·6	
	·3		·3
110		122·9	
Total,	1·0		1·1

The 109·7 is interpolated, the differences entered in column 2, the approximately proportionate differences in column 4, and the figure 122·6 arrived at. To interpolate between two values of S or $\sqrt{S}$ (table xxviii.) proceed similarly, but if there is also an interpolation in C it may be best to calculate V for both the values of $\sqrt{S}$ and then interpolate.

XXVA.—Smith-Kutter Co-efficients.

Diameter of Pipe.	Velocities in Feet per Second.							
	1	2	3	5	7	10	15	20
Inches.								
½	77	87	92	96	99	100	101	102
¾	80	88	93	97	100	101	103	104
1	82	90	94	98	101	103	105	106
1½	86	92	95	99	102	105	107	108
2	90	94	97	101	104	106	108	111
3	93	96	99	103	106	108	111	114
4	95	98	101	105	108	110	113	115
6	97	101	103	107	110	113	115	118
8	99	104	106	110	113	115	118	120
Feet.								
1	102	107	111	115	118	120	123	125
1·5	107	113	116	121	124	127	130	133
2	113	119	122	126	129	132	135	...
2·5	118	124	127	131	134	137	140	...
3	122	127	131	135	138	141	144	...
3·5	125	131	134	138	142	144	...	...
4	128	134	137	142	145	148	...	...
5	133	139	143	147	150	153	...	...
6	138	143	147	152	155	158	...	...
7	143	147	151	155	158	161	...	...
8	147	151	154	158	160	...	...	...
9	150	153	156	159	161	...	...	...
10	153	156	158	160	162	...	...	...
11	157	158	160	161	163	...	...	...
12	160	161	162	163	164	...	...	...
13	164	164	164	164	164	...	...	...

*XXV*B.—*Kutter's Co-efficients.*

Diameter of Pipe.	$\sqrt{R}$	Velocities in Feet per Second.											
		1	2	3	4	1	2	3	4	1	2	3	4
		(N = ·009)				(N = ·010)				(N = ·011)			
1·5	·612	138	147	148	148	124	129	131	131	111	116	116	116
2	·707	143	154	154	157	128	136	139	142	117	122	124	124
3	·866	150	163	166	168	135	145	149	149	121	130	132	132
4	1·0	157	169	173	174	141	152	155	155	128	137	139	140
5	1·118	164	173	178	179	147	157	160	160	135	142	144	144
6	1·225	170	177	182	184	153	161	163	164	140	147	148	148
8	1·414	181	186	190	190	162	168	169	171	148	151	154	154
10	1·581	190	192	195	195	170	173	175	176	154	158	159	159
12	1·732	197	198	199	199	177	178	179	179	159	162	163	163
16	2·0	210	209	208	207	191	189	187	186	172	171	170	169
		(N = ·012)				(N = ·013							
1·5	·612	100	104	104	104	93	96	100	100	...	...	...	...
2	·707	106	111	112	112	97	101	106	106	...	...	...	...
3	·866	113	119	121	121	103	109	110	110	...	...	...	...
4	1·0	118	125	127	127	109	114	116	117	...	...	...	...
5	1·118	122	129	132	132	113	118	120	121	...	...	...	...
6	1·225	128	132	135	136	116	121	123	124	...	...	...	...
8	1·414	136	138	140	141	122	128	129	129	...	...	...	...
10	1·581	142	144	145	146	129	132	133	133	...	...	...	...
12	1·732	148	148	149	149	136	137	138	138	...	...	...	...
16	2·0	160	157	156	155	147	145	144	143	...	...	...	...

Note.—When V exceeds 4 feet per second C generally remains the same.

Table XXVI.—Values of $C\sqrt{R}$ for various Values of C and $\sqrt{R}$.

For a value of C lower than 90 look out double the value and halve the result.

For a value of C over 140 look out half the value and double the result.[1]

Values of C.	Values of $\sqrt{R}$.								
	·354[2]	·382	·408	·433	·456	·479	·500	·520	·540
90	31·9	34·4	36·7	39·0	41·0	43·1	45·0	46·8	48·6
91	32·2	34·8	37·1	39·4	41·5	43·6	45·5	47·3	49·1
92	32·6	35·1	37·5	39·8	42·0	44·1	46·0	47·8	49·7
93	32·9	35·5	37·9	40·3	42·4	44·5	46·5	48·4	50·2
94	33·3	35·9	38·4	40·7	42·9	45·0	47·0	48·9	50·8
95	33·6	36·3	38·8	41·1	43·3	45·5	47·5	49·4	51·3
96	34·0	36·7	39·2	41·6	43·8	46·0	48·0	49·9	51·8
97	34·3	37·1	39·6	42·0	44·2	46·5	48·5	50·4	52·4
98	34·7	37·4	40·0	42·4	44·7	46·9	49·0	51·0	52·9
99	35·0	37·8	40·4	42·9	45·1	47·4	49·5	51·5	53·5
100	35·4	38·2	40·8	43·3	45·6	47·9	50·0	52·0	54·0
101	35·8	38·6	41·2	43·7	46·1	48·4	50·5	52·5	54·5
102	36·1	39·0	41·6	44·2	46·5	48·9	51·0	53·0	55·1
103	36·5	39·3	42·0	44·6	47·0	49·3	51·5	53·6	55·6
104	36·8	39·7	42·4	45·0	47·4	49·8	52·0	54·1	56·2
105	37·2	40·1	42·8	45·5	47·9	50·3	52·5	54·6	56·7
106	37·5	40·5	43·2	45·9	48·3	50·8	53·0	55·1	57·2
107	37·9	40·9	43·7	46·3	48·8	51·3	53·5	55·6	57·8
108	38·2	41·3	44·1	46·8	49·2	51·7	54·0	56·2	58·3
109	38·6	41·6	44·5	47·2	49·7	52·2	54·5	56·7	58·9
110	38·9	42·0	44·9	47·6	50·2	52·7	55·0	57·2	59·4
111	39·3	42·4	45·3	48·1	50·6	53·2	55·5	57·7	59·9
112	39·6	42·8	45·7	48·5	51·1	53·6	56·0	58·2	60·5
114	40·4	43·5	46·5	49·4	52·0	54·6	57·0	59·3	61·6
116	41·1	44·3	47·3	50·2	52·9	55·6	58·0	60·3	62·6
118	41·8	45·1	48·1	51·1	53·8	56·5	59·0	61·4	63·7
120	42·5	45·8	49·0	52·0	54·7	57·5	60·0	62·4	64·8
122	43·2	46·6	49·8	52·8	55·6	58·4	61·0	63·4	65·9
124	43·9	47·4	50·6	53·7	56·5	59·4	62·0	64·5	67·0
126	44·6	48·1	51·4	54·6	57·5	60·4	63·0	65·5	68·0
128	45·3	48·9	52·2	55·4	58·4	61·3	64·0	66·6	69·1
130	46·0	49·7	53·0	56·3	59·3	62·3	65·0	67·6	70·2
132	46·7	50·4	53·9	57·2	60·2	63·2	66·0	68·6	71·3
134	47·4	51·2	54·7	58·0	61·1	64·2	67·0	69·7	72·4
136	48·1	52·0	55·5	58·9	62·0	65·1	68·0	70·7	73·4
138	48·9	52·7	56·3	59·8	62·9	66·1	69·0	71·8	74·5
140	49·6	53·5	57·1	60·6	63·8	67·1	70·0	72·8	75·6

[1] Or look out $\frac{2}{3}$ or $\frac{3}{3}$ value and multiply accordingly.

[2] For a lower value, *e.g.* ·204 (see table xxiii.), look out ·408.

TABLE XXVI.—*Continued.*—VALUES OF $C\sqrt{R}$ FOR VARIOUS VALUES OF C AND $\sqrt{R}$.

For a value of C lower than 90 look out double the value and halve the result.

For a value of C over 140 look out half the value and double the result.

Values of C.	Values of $\sqrt{R}$.								
	·559	·577	·595	·612	·629	·646	·662	·677	·707
90	50·3	51·9	53·6	55·1	56·6	58·1	59·6	60·9	63·6
91	50·9	52·5	54·2	55·7	57·2	58·8	60·2	61·6	64·3
92	51·4	53·1	54·7	56·3	57·9	59·4	60·9	62·3	65·0
93	52·0	53·7	55·3	57·9	58·5	60·1	61·6	63·0	65·8
94	52·5	54·2	55·9	57·6	59·1	60·7	62·2	63·6	66·4
95	53·1	54·8	56·5	58·1	59·8	61·4	62·9	64·3	67·2
96	53·7	55·4	57·1	58·8	60·4	62·0	63·6	65·0	67·9
97	54·2	56·0	57·7	59·4	61·0	62·7	64·2	65·7	68·6
98	54·8	56·5	58·3	60·0	61·6	63·3	64·9	66·3	69·3
99	55·3	57·1	58·9	60·6	62·3	64·0	65·5	67·0	70·0
100	55·9	57·7	59·5	61·2	62·9	64·6	66·2	67·7	70·7
101	56·5	58·3	60·1	61·8	63·5	65·3	66·9	68·4	71·4
102	57·0	58·9	60·7	62·4	64·2	65·9	67·5	69·1	72·1
103	57·6	59·5	61·3	63·0	64·8	66·5	68·2	69·7	72·8
104	58·1	60·0	61·9	63·6	65·4	67·2	68·8	70·4	73·5
105	58·7	60·6	62·5	64·3	66·0	67·8	69·5	71·1	74·2
106	59·3	61·2	63·1	64·9	66·7	68·5	70·2	71·8	74·9
107	59·8	61·7	63·7	65·5	67·3	69·1	70·8	72·4	75·7
108	60·4	62·3	64·3	66·1	67·9	69·8	71·5	73·1	76·4
109	60·9	62·9	64·9	66·7	68·6	70·4	72·2	73·8	77·1
110	61·5	63·5	65·5	67·3	69·2	71·1	72·8	74·5	77·8
111	62·1	64·1	66·1	67·9	69·8	71·7	73·5	75·2	78·5
112	62·6	64·6	66·6	68·5	70·4	72·4	74·1	75·8	79·2
114	63·7	65·8	67·8	69·8	71·7	73·6	75·5	77·2	80·6
116	64·8	66·9	69·0	71·0	73·0	74·9	76·8	78·5	82·0
118	66·0	68·1	70·2	72·2	74·2	76·2	78·1	79·9	83·4
120	67·1	69·2	71·4	73·4	75·5	77·5	79·4	81·2	84·8
122	68·2	70·4	72·6	74·7	76·7	78·8	80·8	82·6	86·3
124	69·3	71·5	73·8	75·9	78·0	80·1	82·1	83·9	87·7
126	70·4	72·7	75·0	77·1	79·3	81·4	83·4	85·3	89·1
128	71·6	73·9	76·2	78·3	80·5	82·7	84·7	86·7	90·5
130	72·7	75·0	77·4	79·6	81·8	84·0	86·1	88·0	91·9
132	73·8	76·2	78·5	80·8	83·0	85·3	87·4	89·4	93·3
134	74·9	77·3	79·7	82·0	84·3	86·6	88·7	90·7	94·7
136	76·0	78·5	80·9	83·2	85·5	87·9	90·0	92·1	96·2
138	77·1	79·6	82·1	84·5	86·8	89·1	91·4	93·4	97·6
140	78·3	80·8	83·3	85·7	88·1	90·4	92·7	94·8	99·0

TABLE XXVI.—*Continued.*—VALUES OF $C\sqrt{R}$ FOR VARIOUS VALUES OF C AND $\sqrt{R}$.

For a value of C lower than 100 look out double the value and halve the result.

For a value of C over 160 look out half the value and double the result.

Values of C.	Values of $\sqrt{R}$.						
	·736	·764	·791	·817	·841	·866	·901
100	73·6	76·4	79·1	81·7	84·1	86·6	90·1
101	74·3	77·2	79·9	82·5	84·9	87·5	91·0
102	75·1	77·9	80·7	83·3	85·8	88·3	91·9
103	75·8	78·7	81·5	84·2	86·6	89·2	92·8
104	76·5	79·5	82·3	85·0	87·5	90·1	93·7
105	77·3	80·2	83·1	85·8	88·3	90·9	94·6
106	78·0	81·0	83·8	86·6	89·1	91·8	95·5
107	78·8	81·7	84·6	87·4	90·0	92·7	96·4
108	79·5	82·5	85·4	88·2	90·8	93·5	97·3
109	80·2	83·3	86·2	89·1	91·7	94·4	98·2
110	81·0	84·0	87·0	89·9	92·5	95·3	99·1
111	81·7	84·8	87·8	90·7	93·4	96·1	100·0
112	82·4	85·6	88·6	91·5	94·2	97·0	100·9
113	83·2	86·3	89·4	92·3	95·0	97·9	101·8
114	83·9	87·1	90·2	93·1	95·9	98·7	102·7
115	84·6	87·9	91·0	94·0	96·7	99·6	103·6
116	85·4	88·6	91·8	94·8	97·6	100·4	104·5
118	86·8	90·2	93·3	96·4	99·2	102·1	106·3
120	88·3	91·7	94·9	98·0	100·9	103·9	108·1
122	89·8	93·2	96·5	99·7	102·6	105·6	109·9
124	91·3	94·7	98·1	101·3	104·2	107·3	111·7
126	92·7	96·3	99·6	102·9	105·9	109·0	113·5
128	94·2	97·8	101·2	104·5	107·6	110·8	115·3
130	95·7	99·3	102·8	106·2	109·3	112·5	117·1
132	97·2	100·8	104·4	107·8	111·0	114·2	118·9
134	98·6	102·4	106·0	109·4	112·7	115·9	120·7
136	100·0	103·9	107·5	111·1	114·3	117·7	122·5
138	101·6	105·4	109·1	112·7	116·0	119·4	124·3
140	103·0	106·9	110·7	114·3	117·7	121·2	126·1
142	104·5	108·4	112·3	116·0	119·4	122·9	127·9
144	105·9	110·0	113·9	117·6	121·1	124·7	129·7
146	107·4	111·5	115·5	119·2	122·7	126·4	131·5
148	108·9	113·0	117·0	120·9	124·4	128·1	133·3
150	110·4	114·6	118·6	122·5	126·1	129·8	135·1
152	111·8	116·1	120·2	124·1	127·8	131·6	136·9
154	113·3	117·6	121·8	125·7	129·4	133·3	138·7
156	114·8	119·1	123·3	127·4	131·1	135·0	140·5
158	116·3	120·7	124·9	129·1	132·8	136·7	142·3
160	117·7	122·2	126·5	130·7	134·5	138·5	144·1

TABLE XXVI.—*Continued.*—VALUES OF $C\sqrt{R}$ FOR VARIOUS VALUES OF C AND $\sqrt{R}$.

For a value of C lower than 100 look out double the value and halve the result.

For a value of C over 160 look out half the value and double the result.

Values of C.	Values of $\sqrt{R}$.						
	·935	·967	1·00	1·061	1·118	1·173	1·225[1]
100	93·5	96·7	100·0	106·1	111·8	117·3	122·5
101	94·4	97·7	101·0	107·1	112·9	118·5	123·7
102	95·4	98·6	102·0	108·2	114·0	119·6	124·9
103	96·3	99·6	103·0	109·3	115·1	120·8	126·1
104	97·2	100·6	104·0	110·3	116·2	121·9	127·4
105	98·2	101·6	105·0	111·4	117·3	123·1	128·6
106	99·1	102·6	106·0	112·4	118·5	124·3	129·8
107	100·1	103·5	107·0	113·5	119·6	125·5	131·0
108	100·9	104·4	108·0	114·5	120·7	126·6	132·3
109	101·8	105·4	109·0	115·6	121·8	127·8	133·5
110	102·8	106·3	110·0	116·7	122·9	129·0	134·7
111	103·7	107·3	111·0	117·8	124·0	130·2	135·9
112	104·7	108·3	112·0	118·8	125·1	131·3	137·1
113	105·6	109·2	113·0	119·9	126·2	132·5	138·3
114	106·5	110·2	114·0	120·9	127·3	133·6	139·6
115	107·5	111·2	115·0	122·0	128·4	134·8	140·8
116	108·4	112·1	116·0	123·0	129·6	136·0	142·0
118	110·3	114·0	118·0	125·1	131·8	138·3	144·4
120	112·2	116·0	120·0	127·3	134·1	140·7	147·0
122	114·1	117·9	122·0	129·4	136·3	143·0	149·4
124	115·9	119·8	124·0	131·5	138·5	145·3	151·9
126	117·8	121·7	126·0	133·6	140·7	147·6	154·3
128	119·6	123·7	128·0	135·7	143·0	150·0	156·8
130	121·5	125·6	130·0	137·8	145·2	152·3	159·2
132	123·8	127·6	132·0	140·0	147·5	154·7	161·6
134	125·2	129·5	134·0	142·1	149·7	157·0	164·0
136	127·1	131·5	136·0	144·2	152·0	159·4	166·5
138	129·0	133·4	138·0	146·3	154·2	161·7	168·9
140	130·9	135·3	140·0	148·5	156·5	164·2	171·5
142	132·8	137·2	142·0	150·6	158·7	166·5	173·9
144	134·6	139·1	144·0	152·7	160·9	168·8	176·4
146	136·5	141·0	146·0	154·8	163·1	171·1	178·8
148	138·3	143·0	148·0	156·9	165·4	173·5	181·3
150	140·2	144·9	150·0	159·0	167·6	175·8	183·7
152	142·0	146·9	152·0	161·2	169·9	178·2	186·1
154	143·9	148·8	154·0	163·3	172·1	180·5	188·5
156	145·8	150·8	156·0	165·4	174·4	182·9	191·0
158	147·7	152·7	158·0	167·5	176·6	185·2	193·4
160	149·6	154·7	160·0	169·7	178·8	187·6	196·0

[1] For a higher value, *e.g.* 1·581 (see table xxiii.), look out ·791.

Table XXVII.—Values of S and $\sqrt{S}$.

(For steep slopes not included in Table xxviii.)

To find $\sqrt{S}$ for a steeper slope, look out a slope 4 times as flat[1] and multiply $\sqrt{S}$ by 2. Thus, for 1 in 50, $\sqrt{S}$ is ·07071 × 2 = ·14142.

Slope 1 in	Fall per Foot or S.	$\sqrt{S}$	Slope 1 in	Fall per Foot or S.	$\sqrt{S}$
100	·010	·1	230	·004348	·06594
105	·0095238	·09759	240	·004167	·06455
110	·009091	·095346	250	·004000	·06325
115	·008696	·093250	260	·003847	·06202
120	·008333	·091287	270	·003704	·06086
125	·008	·089442	280	·003571	·05976
130	·007692	·08771	290	·003448	·05872
135	·007407	·08607	300	·003333	·05774
140	·007143	·08452	310	·003226	·05680
145	·006897	·08305	320	·003125	·05590
150	·006667	·08165	330	·003030	·05505
155	·006452	·08032	340	·002941	·05423
160	·00625	·07906	350	·002857	·05345
165	·006061	·07785	360	·06278	·05271
170	·005882	·07670	370	·002703	·05199
175	·005714	·07559	380	·002632	·05130
180	·005556	·07454	390	·002564	·05064
185	·005405	·07352	400	·0025	·05
190	·005263	·07255	420	·002381	·04880
195	·005128	·07161	440	·002273	·04767
200	·005	·07071	460	·002174	·04663
210	·004762	·06901	480	·002083	·04564
220	·004545	·06742	500	·002	·04472

Note to table xxviii.—This table shows values of V for given values of $C\sqrt{R}$ and $\sqrt{S}$.

The first line of the heading shows $\frac{1}{S}$, the third line $\sqrt{S}$. The figures in brackets show the amount by which $\frac{1}{S}$ must be altered to alter $\sqrt{S}$ and V by 1 per cent. Thus for $S=\frac{1}{2200}$ the slopes $\frac{1}{2157}$ and $\frac{1}{2245}$ give V 1 per cent. more or less than in the table. For $C\sqrt{R}=108$, V is 2·32 and 2·28 feet per second.

Slopes not given in table.—To find $\sqrt{S}$ or V see following examples:—

$S=1$ in	10	100	200	2,500	15,000	40,000	50,000
See 1 in	1,000	10,000	800	10,000	3,750	10,000	500
Multiply by	10	10	2	2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{10}$

[1] Also see above note to table xxviii. Thus for 1 in 10, $\sqrt{S}$ is ·3162.

TABLE XXVIII. (See note on preceding page.)

Values of $C\sqrt{R}$	500 (10) ·04472	550 (11) ·04264	600 (12) ·04083	650 (13) ·03922	700 (14) ·03780	750 (15) ·03652	800 (16) ·03536	900 (18) ·03333
100	4·47	4·26	4·08	3·92	3·78	3·65	3·54	3·33
102	4·56	4·35	4·17	4·00	3·86	3·73	3·61	3·40
104	4·65	4·44	4·25	4·08	3·93	3·80	3·68	3·47
106	4·74	4·52	4·33	4·16	4·01	3·87	3·75	3·53
108	4·83	4·61	4·41	4·24	4·08	3·94	3·82	3·60
110	4·92	4·69	4·49	4·31	4·16	4·02	3·89	3·67
112	5·01	4·78	4·57	4·39	4·23	4·09	3·96	3·73
114	5·10	4·86	4·66	4·47	4·31	4·16	4·03	3·80
116	5·19	4·95	4·74	4·55	4·39	4·24	4·10	3·87
118	5·28	5·03	4·82	4·63	4·46	4·31	4·17	3·93
120	5·37	5·12	4·90	4·71	4·54	4·38	4·24	4·00
123	5·50	5·25	5·02	4·82	4·65	4·49	4·35	4·10
126	5·63	5·37	5·15	4·94	4·76	4·60	4·46	4·20
129	5·77	5·50	5·27	5·06	4·88	4·71	4·56	4·30
132	5·90	5·63	5·39	5·18	4·99	4·82	4·67	4·40
135	6·04	5·76	5·51	5·30	5·10	4·93	4·77	4·50
138	6·17	5·88	5·64	5·41	5·22	5·04	4·88	4·60
141	6·31	6·01	5·76	5·53	5·33	5·15	4·99	4·70
144	6·44	6·14	5·88	5·65	5·44	5·26	5·09	4·80
147	6·57	6·26	6·00	5·77	5·56	5·37	5·20	4·90
150	6·71	6·40	6·13	5·88	5·67	5·48	5·30	5·00
153	6.84	6·52	6.25	6·00	5·78	5·59	5·41	5·10
156	6·98	6·65	6·37	6·12	5·90	5·70	5·52	5·20
160	7·16	6·82	6.53	6·28	6·05	5·84	5·66	5·33
164	7·33	6·99	6·70	6·43	6·20	5·99	5·80	5·47
168	7·51	7·16	6·86	6·59	6·35	6·14	5·94	5·60
172	7·69	7·33	7·02	6·75	6·50	6·28	6·08	5·73
176	7·87	7·51	7·19	6·90	6·65	6·43	6·22	5·87
180	8·05	7·68	7·35	7·06	6·80	6·57	6·37	6·00
185	8·27	7·89	7·55	7·26	6·99	6·76	6·54	6·17
190	8·50	8·10	7·76	7·45	7·18	6·94	6·72	6·33
195	8·72	8·32	7·96	7·65	7·37	7·12	6·90	6·50
200	8·94	8·53	8·17	7·84	7·56	7·30	7·07	6·67
205	8·17	8·74	8·37	8·04	7·75	7·49	7·25	6·83
210	9·39	8·95	8·57	8·24	7·94	7·67	7·43	7·00
215	9·62	9·17	8·78	8·43	8·13	7·85	7·60	7·17
220	9·84	9·38	8·98	8·63	8·32	8·03	7·78	7·33
225	10·1	9·59	9·19	8·82	8·51	8·22	7·96	7·50
230	10·3	9·81	9·39	9·02	8·69	8·40	8·13	7·67
235	10·5	10·0	9·60	9·22	8·88	8·58	8·31	7·83
240	10·7	10·2	9·80	9·41	9·07	8·77	8·49	8·00
246	11·0	10·5	10·0	9·65	9·30	8·98	8·70	8·20
252	11·3	10·8	10·3	9·88	9·53	9·20	8·91	8·40
258	11·5	11·0	10·5	10·1	9·75	9·42	9·12	8·60
264	11·8	11·3	10·8	10·4	9·98	9·64	9·34	8·80
270	12·1	11·5	11·0	10·6	10·2	9·86	9·55	9·00
276	12·3	11·8	11·3	10·8	10·4	10·1	9·76	9·20
282	12·6	12·0	11·5	11·1	10·7	10·3	9·97	9·40
288	12·9	12·3	11·8	11·3	10·9	10·5	10·2	9·60
294	13·2	12·5	12·0	11·5	11·1	10·7	10·4	9·80
300	13·4	12·8	12·3	11·8	11·3	11·0	10·6	10·0

TABLE XXVIII.—*Continued.*

Values of $C\sqrt{R}$	1,000 (20) ·03162	1,100 (22) ·03015	1,200 (24) ·02887	1,300 (26) ·02774	1,400 (28) ·02673	1,500 (30) ·02582	1,000 (32) ·02500	1,800 (36) ·02357	2,000 (39-41) ·02236
100	3·16	3·02	2·89	2·77	2·67	2·58	2·50	2·36	2·24
102	3·23	3·08	2·95	2·83	2·73	2·63	2·55	2·40	2.28
104	3·29	3·14	3·00	2·89	2·78	2·68	2·60	2·45	2·33
106	3·35	3·20	3·06	2·94	2·83	2·74	2·65	2·50	2·37
108	3·42	3·26	3·12	3·00	2·89	2·79	2·70	2·55	2·42
110	3·48	3·32	3·18	3·05	2·94	2·84	2·75	2·59	2·46
112	3·54	3·39	3·23	3·11	2·99	2·89	2·80	2·64	2·51
114	3·60	3·44	3·29	3·16	3 05	2·94	2·85	2·69	2·55
116	3·67	3·50	3·35	3·22	3·10	3·00	2·90	2·73	2·59
118	3·73	3·56	3·41	3·27	3·15	3·05	2·95	2·78	2·64
120	3·79	3·62	3·46	3·33	3·21	3·10	3·00	2·83	2·68
123	3·89	3·71	3·55	3·41	3·29	3·18	3·08	2·90	2·75
126	3·98	3·80	3·64	3·50	3·37	3·25	3·15	2·97	2·82
129	4·08	3·89	3·71	3·58	3·45	3·33	3·23	3·04	2·88
132	4·17	3·98	3·81	3·66	3·53	3·41	3·30	3·11	2·95
135	4·27	4·07	3·90	3·74	3·61	3·49	3·38	3·18	3·02
138	4·36	4·16	3·98	3·83	3·69	3·56	3·45	3·25	3·09
141	4·46	4·25	4·07	3·91	4·77	3·64	3·53	3·32	3·15
144	4·55	4·34	4·16	4·00	3·85	3·72	3·60	3·39	3·22
147	4·65	4·43	4·24	4·08	3·93	3·80	3·68	3·47	3·29
150	4·74	4·52	4·33	4·16	4·01	3·87	3·75	3·54	3·35
153	4·84	4·61	4·42	4·24	4·09	3·95	3·83	3·61	3·42
156	4·93	4·70	4·50	4·33	4·17	4·03	3·90	3·68	3·49
160	5·06	4·83	4·62	4·44	4·23	4·13	4·00	3·77	3·58
164	5·19	4·95	4·73	4·55	4·38	4·23	4·10	3·87	3·67
168	5·31	5·07	4·85	4·66	4 49	4·34	4·20	3·96	3·76
172	5·44	5·19	4·97	4·77	4·60	4·44	4·30	4·05	3·85
176	5·56	5·31	5 08	4·88	4·70	4·54	4·40	4·15	3·94
180	5·69	5·43	5·20	4·99	4·81	4·61	4·50	4·24	4·03
185	5·85	5·58	5·34	5·13	4·95	4·74	4·63	4·36	4·14
190	6·01	5·73	5·49	5·27	5·08	4·91	4·75	4·48	4·25
195	6·17	5·88	5·63	5·41	5·21	5·04	4·88	4·60	4·36
200	6·32	6·03	5·77	5·55	5·35	5·16	5·00	4·71	4·47
205	6·48	6.18	5·93	5·69	5·48	5·29	5·13	4·83	4·58
210	6·64	6·33	6·06	5·83	5·61	5·42	5·25	4·95	4·70
215	6·80	6·48	6·21	5·96	5·75	5·55	5·38	5·07	4·81
220	6·96	6·63	6·35	6·10	5·88	5·68	5·50	5·19	4·92
225	7·11	6·78	6·50	6·24	6·02	5·81	5·63	5 30	5·03
230	7·27	6·94	6·64	6·38	6·15	5·94	5·75	5·42	5·14
235	7·43	7·09	6·78	6·52	6·28	6·07	5·88	5·54	5·26
240	7·59	7·24	6·93	6·66	6·42	6·20	6·00	5·66	5·37
246	7·78	7·42	7·10	6·82	6·58	6·35	6·15	5·80	5·50
252	7·97	7·60	7·29	6·99	6·74	6·51	6·30	5·94	5·64
258	8·16	7·78	7·45	7·16	6·90	6·66	6·45	6·08	5·77
264	8·35	7·96	7·62	7·32	7·06	6·82	6·60	6·22	5·90
270	8·54	8·14	7·79	7·49	7·22	6·97	6·75	6·36	6·04
276	8·73	8·32	7·97	7·66	7·39	7·13	6·90	6·51	6·17
282	8·92	8·50	8·14	7·82	7·55	7·28	7·05	6·65	6·31
288	9·11	8·68	8·32	7·99	7·80	7 44	7·20	6·79	6·44
294	9·30	8·86	8·49	8·16	7·96	7·59	7·35	6·93	6·57
300	9·49	9·05	8·66	8·32	8·02	7·75	7·50	7·07	6·71

TABLE XXVIII.—*Continued.*

Values of $C\sqrt{R}$	2,200 (43-45) ·02132	2,400 (47-49) ·02041	2,700 (53-55) ·01925	3,000 (59-61) ·01826	3,300 (65-67) ·01741	3,600 (71-73) ·01667	4,000 (79-81) ·01581	4,500 (89-91) ·01491	5,000 (99-102) ·01414
100	2·13	2·04	1·93	1·83	1·74	1·67	1·58	1·49	1·41
102	2·18	2·08	1·96	1·86	1·78	1·70	1·61	1·52	1·44
104	2·22	2·12	2·00	1·90	1·81	1·73	1·64	1·55	1·47
106	2·26	2·16	2·04	1·94	1·85	1·77	1·68	1·58	1·50
108	2·30	2·20	2·08	1·97	1·88	1·80	1·71	1·61	1·53
110	2·35	2·25	2·12	2·01	1·92	1·83	1·74	1·64	1·56
112	2·39	2.29	2·16	2·05	1·95	1·87	1·77	1·67	1·58
114	2·43	2·33	2·20	2·08	1·99	1·90	1·80	1·70	1·61
116	2·47	2·37	2·23	2·12	2·02	1·93	1·83	1·73	1·64
118	2·52	2·41	2·27	2·16	2·05	1·97	1·87	1·76	1·67
120	2·56	2·45	2·31	2·19	2·09	2·00	1·90	1·79	1·70
123	2·62	2·51	2·37	2·24	2·14	2·05	1·44	1·83	1·74
126	2·69	2·57	2·43	2·30	2·19	2·10	1·99	1·88	1·78
129	2·75	2·63	2·48	2·36	2·25	2·15	2·04	1·92	1·82
132	2·82	2·69	2·54	2·41	2·30	2·20	2·09	1·97	1·87
135	2·88	2·76	2·60	2·47	2·35	2·25	2·13	2·01	1·91
138	2·94	2·82	2·66	2·52	2·40	2·30	2·18	2·06	1·95
141	3·01	2·88	2·72	2·58	2·45	2·35	2·23	2·10	1·99
144	3·07	2·94	2·77	2·63	2·51	2·40	2·28	2·15	2·04
147	3·13	3·00	2·83	2·68	2·56	2·45	2·32	2·19	2·08
150	3·20	3·06	2·89	2·74	2·61	2·50	2·37	2·24	2·12
153	3·26	3·12	2·95	2·79	2·66	2·55	2·42	2·28	2·16
156	3·33	3·18	3·00	2·85	2·72	2·60	2·47	2·33	2·21
160	3·41	3·27	3·08	2·92	2·79	2·67	2·53	2·39	2·26
164	3·50	3·35	3·16	3·00	2·86	2·73	2·59	2·45	2·32
168	3·58	3·43	3·23	3·07	2·93	2·80	2·66	2·51	2·38
172	3·67	3·51	3·31	3·14	3·00	2·87	2·72	2·57	2·43
176	3·75	3·59	3·39	3·21	3·07	2·93	2·78	2·63	2·49
180	3·84	3·67	3·47	3·29	3·13	3·00	2·85	2·68	2·55
185	3·95	3·78	3·56	3·38	3·22	3·08	2·93	2·76	2·62
190	4·05	3·88	3·66	3·47	3·31	3·17	3·00	2·83	2·69
195	4·16	3·98	3·75	3·56	3·50	3·25	3·08	2·91	2·76
200	4·26	4·08	3·85	3·65	3·48	3·33	3·16	2·98	2·83
205	4·37	4·18	3·95	3·74	3·57	3·42	3·24	3·06	2·90
210	4·48	4·29	4·04	3·84	3·66	3·50	3·32	3·13	2·97
215	4·58	4·39	4·14	3·93	3·74	3·58	3·40	3·21	3·04
220	4·69	4·49	4·24	4·02	3·83	3·67	3·48	3·28	3·11
225	4·80	4·59	4·33	4·11	3·92	3·75	3·56	3·36	3·18
230	4·90	4·69	4·43	4·20	4·00	3·83	3·64	3·43	3·25
235	5·01	4·80	4·52	4·29	4·09	3·92	3·72	3·50	3·32
240	5·12	4·90	4·62	4·38	4·19	4·00	3·79	3·58	3·39
246	5·25	5·02	4·74	4·49	4·28	4·10	3·89	3·67	3·48
252	5·37	5·14	4·85	4·60	4·39	4·20	3·99	3·76	3·56
258	5·50	5·27	4·97	4·71	4·49	4·30	4·08	3·85	3·65
264	5·63	5·39	5·08	4·82	4·60	4·40	4·17	3·94	3·73
270	5·76	5·51	5·20	4·93	4·70	4·50	4·27	4·03	3·82
276	5·88	5·63	5·31	5·04	4·81	4·60	4·36	4·12	3·90
282	6·01	5·76	5·43	5·15	4·91	4·70	4·46	4·20	3·99
288	6·14	5·88	5·54	5·26	5·01	4·80	4·55	4·29	4·07
294	6·27	6·00	5·66	5·37	5·12	4·90	4·65	4·38	4·16
300	6·40	6·12	5·78	5·48	5·22	5·00	4·74	4·47	4·24

Table XXVIII.—*Continued.*

Values of $C\sqrt{R}$.	5,500 (108-112) ·01349	6,000 (118-122) ·01291	6,500 (128-132) ·01240	7,000 (138-142) ·01195	7,500 (148-152) ·01155	8,000 (158-162) ·01118	8,500 (167-173) ·01085	9,000 (177-183) ·01054	10,000 (197-203) ·0100
100	1·35	1·29	1·24	1·20	1·16	1·12	1·09	1·05	1·00
102	1·38	1·32	1·47	1·22	1·18	1·14	1·11	1·08	1·02
104	1·40	1·34	1·29	1·24	1·20	1·16	1·13	1·10	1·04
106	1·43	1·37	1·32	1·27	1·22	1·19	1·15	1·12	1·06
108	1·46	1·39	1·34	1·29	1·25	1·21	1·17	1·14	1·08
110	1·48	1·42	1·36	1·32	1·27	1·23	1·19	1·16	1·10
112	1·51	1·45	1·39	1·34	1·29	1·25	1·22	1·18	1·12
114	1·54	1·47	1·41	1·36	1·32	1·27	1·23	1·20	1·14
116	1·57	1·50	1·44	1·39	1·34	1·30	1·26	1·22	1·16
118	1·59	1·52	1·46	1·41	1·36	1·32	1·28	1·24	1·18
120	1·62	1·55	1·49	1·43	1·39	1·34	1·30	1·27	1·20
123	1·66	1·59	1·53	1·47	1·42	1·38	1·34	1·30	1·23
126	1·70	1·63	1·56	1·51	1·46	1·41	1·37	1·33	1·26
129	1·74	1·67	1·60	1·54	1·49	1·44	1·40	1·36	1·29
132	1·78	1·70	1·64	1·58	1·53	1·48	1·43	1·39	1·32
135	1·83	1·74	1·66	1·61	1·56	1·51	1·47	1·42	1·35
138	1·86	1·78	1·71	1·65	1·59	1·54	1·50	1·45	1·38
141	1·90	1·82	1·75	1·69	1·63	1·58	1·53	1·49	1·41
144	1·94	1·86	1·79	1·72	1·66	1·61	1·56	1·52	1·44
147	1·98	1·90	1·82	1·76	1·70	1·64	1·60	1·56	1·47
150	2·02	1·94	1·86	1·79	1·73	1·68	1·63	1·58	1·50
153	2·06	1·98	1·90	1·83	1·77	1·71	1·66	1·61	1·53
156	2·11	2·01	1·93	1·87	1·80	1·74	1·69	1·64	1·56
160	2·16	2·07	1·98	1·91	1·85	1·79	1·74	1·69	1·60
164	2·21	2·12	2·03	1·96	1·89	1·83	1·78	1·73	1·64
168	2·27	2·17	2·08	2·01	1·94	1·88	1·82	1·77	1·68
172	2·32	2·22	2·13	2·06	1·99	1·92	1·87	1·81	1·72
176	2·37	2·27	2·18	2·10	2·03	1·97	1·91	1·86	1·76
180	2·43	2·32	2·23	2·15	2·08	2·01	1·95	1·90	1·80
185	2·50	2·39	2·30	2·21	2·14	2·07	2·01	1·95	1·85
190	2·56	2·45	2·36	2·27	2·20	2·12	2·06	2·00	1·90
195	2·63	2·52	2·42	2·33	2·25	2·18	2·12	2·06	1·95
200	2·70	2·58	2·48	2·39	2·31	2·24	2·17	2·11	2·00
205	2·77	2·65	2·54	2·45	2·37	2·29	2·22	2·16	2·05
210	2·83	2·71	2·60	2·51	2·43	2·35	2·28	2·21	2·10
215	2·90	2·78	2·67	2·57	2·48	2·40	2·33	2·27	2·15
220	2·97	2·84	2·73	2·63	2·54	2·46	2·39	2·32	2·20
225	3·04	2·91	2·79	2·69	2·60	2·52	2·44	2·37	2·25
230	3·10	2·97	2·85	2·75	2·66	2·57	2·50	2·42	2·30
235	3·27	3·03	2·91	2·81	2·72	2·63	2·55	2·48	2·35
240	3·24	3·10	2·98	2·87	2·77	2·68	2·60	2·53	2·40
246	3·32	3·18	3·05	2·94	2·84	2·75	2·67	2·59	2·46
252	3·40	3·25	3·13	3·01	2·91	2·82	2·74	2·66	2·52
258	3·48	3·33	3·20	3·08	2·98	2·88	2·80	2·72	2·58
264	3·56	3·41	3·27	3·16	3·05	2·95	2·86	2·78	2·64
270	3·64	3·48	3·35	3·23	3·12	3·02	2·93	2·85	2·70
276	3·72	3·56	3·42	3·30	3·19	3·09	3·00	2·91	2·76
282	3·80	3·64	3·50	3·37	3·26	3·15	3·06	2·97	2·82
288	3·89	3·72	3·57	3·44	3·33	3·22	3·13	3·04	2·88
294	3·97	3·80	3·65	3·51	3·40	3·29	3·19	3·10	2·94
300	4·05	3·87	3·72	3·59	3·47	3·35	3·26	3·16	3·00

CHAPTER VI

OPEN CHANNELS—UNIFORM FLOW

[For preliminary information see chapter ii. articles 8-16 and 22-24]

Section I.—Open Channels in General

1. **General Remarks.**—Uniform flow can take place only in a uniform channel. Strictly speaking, a uniform channel is one which has a uniform bed-slope, and all its cross-sections equal and similar; but if the cross-sections, though differing somewhat in form, as in Fig. 98, are of equal areas and equal wet borders, the channel is to all intents and purposes uniform, provided the form of the section changes gradually. The term 'uniform channel' will be used in this extended sense.[1] Breaches of uniformity in a channel may be frequent, and the reaches in which the flow is variable may be of great length. The flow in a uniform channel is thus by no means everywhere uniform. Bends are for convenience treated of in chap. vii., but flow round a bend may be uniform. Thus a uniform stream need not be assumed to be straight. It will be seen hereafter (chap. vii. art. 16) that nearly everything which is true for uniform flow is true, with some modifications, for variable flow.

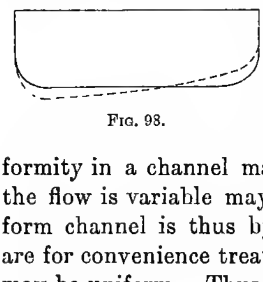


Fig. 98.

The mean depth D (Fig. 99) of a stream is the sectional area A divided by the surface-width W. Since $A=DW=RB$, therefore the hydraulic radius is less than the mean depth in the same ratio as the surface-width is less than the border. This will often assist in forming an idea of the hydraulic radius. The greater the width of a stream in proportion to its depth, and the

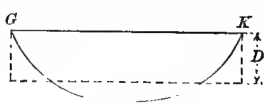


Fig. 99.

[1] If R varies in the opposite manner to S the flow may be uniform in a variable channel, but this is very rare.

fewer the undulations in the border, the more nearly will the surface-width approach to the border and the hydraulic radius to the mean depth. If the depth of water in a channel alters, the hydraulic radius alters in the same manner. When the water-level rises A increases faster than W, and R therefore increases; but $\frac{W}{B}$ decreases (unless the side-slopes are flat), so that R increases less rapidly than D. For small changes of water-level R and D both change at about the same rate.

2. Laws of Variation of Velocity and Discharge.—For orifices, weirs, and pipes it was possible to describe in a few words the general laws according to which the velocities and discharges vary, but for open streams it is not so. One law is simple, and that is, that for any channel whatever V and Q are nearly as $\sqrt{S}$. To double V or Q it is necessary to quadruple S. For other factors it is necessary to consider the shape of the cross-section.

For a stream of 'shallow section,' that is, one in which W greatly exceeds D, a change in W has hardly any effect on R or on V, while Q is directly as W. Also R is very nearly as D. For depths not very small C is approximately as $D^{\frac{1}{6}}$, so that V is as $D^{\frac{2}{3}}$. In this case, if D is doubled, V is increased in the ratio 1·59 to 1. On comparing velocities, taken from tables, for channels from 8 to 300 feet wide with sides vertical, or 1 to 1, and with various velocities, the actual ratio is found to vary from 1·52 to 1·73. If the sides are steep A is nearly as D, and Q therefore as $D^{\frac{5}{3}}$ or thereabouts. For a stream of 'medium section'—that is, one in which W is 2 to 6 times D—with vertical sides A is as D, and for moderate changes of water-level and depths not very small V is nearly as $D^{\frac{1}{2}}$, so that Q is as $D^{\frac{3}{2}}$. Both these kinds of section are extremely common. A flattening of the side-slopes may make Q vary as D^2. If a stream has vertical sides and a depth far exceeding its width—a rare case—the effects of W and D are reversed. For a triangular section—used for small drains—R is as D, A as D^2, C probably as $D^{\frac{1}{6}}$, and Q as $D^{\frac{8}{3}}$.

For other kinds of section no definite laws can be framed, but the effect of D is nearly always greater than that of W, so that D is the most important factor in the discharge, especially if the side-slopes are flat, and S is always the least important factor.

If two streams have equal discharges, and have one factor in the discharge equal, the approximate relation between the other two factors can be found. Let two streams of shallow section have equal slopes, and let one be twice as deep as the other. The latter must be $(2)^{\frac{5}{3}}$ or 3·2 times as wide as the first. This law is nearly the same as for weirs. When two reaches of a canal have different bed-slopes, but equal and similar cross-sections, the depth of water is, of course, less in the reach of steeper slope. If the discharge is approximately as $S^{\frac{1}{2}}D^{2}$, the depths in the two reaches will be inversely as the fourth roots of the slopes. The velocities are inversely as the depths, and are, therefore, as the fourth roots of the slopes. A change of 40 per cent. in the slope will cause a change of only about 10 per cent. in the velocity, and a change of the same proportion, but of opposite kind, in the depth of water. When the changes in the two factors are relatively small they are inversely as the indices in the formulæ. Suppose a stream of medium section with depth D and slope S gives a certain discharge Q. Let D be increased by a small amount $\frac{D}{n}$. Then the compensatory change in S will be $\frac{3S}{n}$. This principle may be applied in designing a channel to carry a given discharge, whenever for any reason it becomes necessary to make a slight change in the value first assumed for any factor.

The discharging power of a stream can be increased by increasing the depth of water, the width or the slope, the last being often effected by cutting off bends. The efficiencies of these processes are in the order named. In any channel having sloping sides both V and Q are more increased by raising the surface-level than by deepening the bed by the same amount. It follows that embanking a river is more effective than deepening it for increasing its discharging power and enabling it to carry off floods. It is in fact the most effective plan that can be devised.

In clearing out the head reaches of Indian inundation canals—so called because they flow only for a few months, when the rivers are swollen—it used to be the custom to place the bed rather high, at the off-take, in order to obtain a good slope. Of late years it has been the custom to lower the bed, giving a flatter slope but a greater depth of water. The velocity is about the same in both cases, the increase in depth making up for the decrease in slope, but the lowered bed of course gives a greatly augmented discharge. On the other hand, the lowered bed must cause the introduction of water more heavily charged with silt. Moreover, the ratio of depth to velocity in the canal is greater than before, and this (chap. ii. art. 23) tends to cause increased deposit. Under the old system of high beds the heads of the canals silted more or less. It has been impossible to find out whether more silt has actually deposited since the introduction of the low-level system, because, owing to changes in the course of the river, the same head channel is seldom cleared for several years in succession, and also because the quantity of silt deposited depends on other factors, such as the position of the head, a canal taken off from the highly silt-laden main stream silting more than one taken from a side channel. Obviously the tendency of the low bed is to silt more than the high one, but the worst that can happen is its silting up till it assumes the level of the high one. This takes time, and while it is going on an increased discharge is obtained.

Section II.—Special Forms of Channel

3. Section of 'Best Form.'—A stream is of the 'best form' when for a given sectional area the border is a minimum, and the hydraulic radius, therefore, a maximum. The velocity and discharge are greater than in any other stream of the same sectional area, slope, and roughness. The form which complies with this condition is a semicircle whose diameter coincides with the line of water surface. This form is used in concrete channels, but not often in others, because of the difficulty of constructing curved surfaces. Of rectilineal figures the best form is half a regular polygon. The greater the number of sides the better, but in practice the form of section is usually restricted to that having a bed level across and two sides vertical or sloping. The best form for vertical sides is the half-square (Fig. 100), and for sloping sides the semi-hexagon (Fig. 101). If the angle of the side-slopes is fixed (as it generally is) at some angle other than 60°,

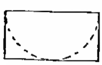


Fig. 100.

Fig. 101.

Fig. 102.

the best form is that in which the bed and sides are all tangents to a semicircle (Fig. 102). The bed-width is $D(\sqrt{n^2+1}-n)$, where n is the ratio of the side-slopes. In every channel of the best form the hydraulic radius is half the depth of water, and if the section is rectilineal, the surface-width is equal to the sum of the two slopes, so that the border is the sum of the surface and bed widths.

The following statement shows the sectional areas of various channels of the best form. All the channels have the same central depth D, the same hydraulic radius $\frac{D}{2}$, and therefore the same velocity.

Description of Cross-section.				Sectional Area.	Ratio of Sectional Area to that of the Inscribed Semicircle.
Semicircle,				$1{\cdot}57\ D^2$	1·00
Half-square,				$2\ D^2$	1·27
Semi-hexagon,				$1{\cdot}732D^2$	1·10
Trapezoid, side-slopes	½ to 1,			$1{\cdot}736D^2$	1·11
,,	,,	1 ,, 1,		$1{\cdot}828D^2$	1·16
,,	,,	1½ ,, 1,		$2{\cdot}106D^2$	1·34
,,	,,	2 ,, 1,		$2{\cdot}472D^2$	1·57
,,	,,	3 ,, 1,		$3{\cdot}325D^2$	2·12

A channel of the best form is not usually the cheapest. If made of iron, wood, or masonry the cost will probably be reduced by somewhat increasing the width and reducing the depth, thereby enabling the sides to be made lighter, though the length of border is slightly increased. In an excavated channel, where the water-surface is to be at the ground-level, the best form will give the minimum quantity of work and will be the cheapest if the material excavated is rock, but if it is earth an increase of width and decrease of depth will reduce the lift of the earth, and therefore the cost. If the water-surface is not to be at the ground-level the cheapest form may differ greatly from the best form.

If it is desired simply to deliver a stream of water of given discharge with as high a velocity as possible, the best form is suitable. If the object is to obtain high silt-supporting power, so that the channel may not silt or may scour and enlarge itself, the question of ratio of depth to velocity must be taken into account; and even when the object is to discourage the growth of weeds the question of depth comes in.

If the depth of water in a channel fluctuates, the section can, of course, be of the best form for only one water-level. Sewers are often made of oval sections in order that the stream may be of the best form, or nearly so, when the water-level is low, the object being to prevent deposits. In Fig. 103 (Metropolitan Ovoid) the radius of the invert is half that of the crown, and in Fig. 104 (Hawkesley's Ovoid) nearly three-fifths. There is also a form known as Jackson's Peg-top Section. In each case the velocity with the sewer one-third full is about three-fourths of the velocity when it is two-thirds full.

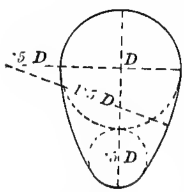


FIG. 103.

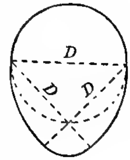


FIG. 104.

4. Irregular Sections.—The cross-section of a stream may be called 'irregular' when the border contains undulations or saliences of such a character as to divide the section into well-marked divisions (Fig. 105). In this case the water in each division has a velocity of its own, and in order to calculate the discharge of the whole stream by the use of the formula $V=C\sqrt{RS}$, it is necessary to consider each division separately, finding its hydraulic radius from its area and border.

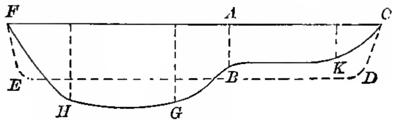


FIG. 105.

The length AB is not included in the border of either division, since if there is any friction along it, it accelerates the motion in one division and retards it in the other. If A_1 A_2 are the sectional areas, and R_1 R_2 the hydraulic radii,

$$Q_1=C_1A_1\sqrt{R_1S}$$
$$Q_2=C_2A_2\sqrt{R_2S}.$$

The discharge of the whole channel, calculated from the equation $Q=CA\sqrt{RS}$, equals Q_1+Q_2 only when $R_1=R_2$, otherwise it is less. The more R_1 and R_2 differ, the more Q differs from Q_1+Q_2, and for given values of R_1 and R_2 the difference is greatest when $A_1=A_2$. If either A_1 or A_2 is relatively very small, the difference between Q and Q_1+Q_2 will be small. It may happen that R_1 and R_2 differ greatly with low supplies, and not much with high supplies. If without altering either the length of the border or the sectional area of the stream the border be changed to $CDEF$, the section is no longer irregular, and the equation $V=C\sqrt{RS}$ is the proper one to use. There are thus two cross-sections with equal values of R and different mean velocities, that is, different values of C. Even in a regular section the same principle holds good. The discharge is the sum of the discharges of a number of parts, and may be affected by a change in the form of the border alone. (See also art. 13.)

An instance of an irregular section occurs when a stream overflows its banks (Fig. 106). As the overflow occurs the border of the whole stream may increase far more rapidly than the sectional area, and Q, if calculated as a whole, would diminish with rise of the water-level. The velocity and discharge of the main

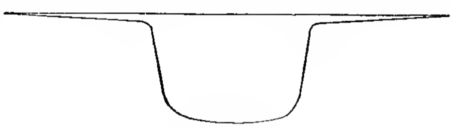


FIG. 106.

body and of the overflow must be considered separately, and both will increase as the water-level rises. Similarly, if there are longitudinal grooves or ruts in the bed of a stream, such, for instance, as those caused by longitudinal battens, the water in the grooves has a separate velocity of its own, and the velocity of the main body cannot be reduced indefinitely by increasing the number and depth of the grooves, although the border can be increased in this manner to any extent. If the river is winding, the spill-water, which flows straight, may have a slope greater than that in the river channel, but its velocity may still be very low, especially if the country is covered with crops or vegetation. Some of the spill-water, however, disappears by absorption, and it is clear that in every case it takes off some of the discharge of the river. Thus the embanking of a river, so as to shut off spills, must necessarily, to start with, raise the flood-level. Whether scour of the channel subsequently reduces the level is another matter.

5. Channels of Constant Velocity or Discharge.—Let A be the area, B the border, and W the surface-width of any stream whose water-level is RS (Fig. 107), and let the water-level rise to TU, the increase in depth being a small quantity d and the increase in the surface-width being $2w$. Then if the slopes RT, SU be made such that

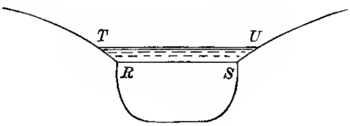


FIG. 107.

$\frac{(W+w)d}{2\sqrt{d^2+w^2}}=\frac{A}{B}$, the border will have increased in the same ratio as the area, and R will be unaltered. By using the new values of A and B, corresponding to the raised surface, the process can be continued, but the slope becomes rapidly flatter. If the surface falls below RS; R is no longer constant, but decreases. It is impossible to design a section such that R will remain constant as the depth decreases to zero. And even within the limits in which R is constant, the mean velocity is not constant. The channel is irregular, and the velocity, both in the main body of water and in the minor ones, increases as the water-level rises. The investigations which have at times been made to find the equation to the curve of the border when R is constant are useful only as mathematical exercises.

The velocity as the water-level rises is nearly constant in a very deep, narrow channel with vertical sides, and it may be kept quite constant by making the sides overhang—as in a sewer running nearly full—but the process is speedily terminated by the meeting of the two sides.

To keep the discharge constant for different water-levels is still more difficult, but would be of great practical use, especially in irrigation distributaries. It could be effected by making the sides overhang, but they would have to project almost horizontally and would very soon meet, thus giving only a small range of depth. Any form of section adopted for giving either constant velocity or constant discharge must be continuous along the channel from its head for a great distance. If of short length the slope or hydraulic gradient in it would be liable to vary greatly, and with it the velocity. (Cf. chap. ii. art. 14.)

6. Circular Sections.—A channel of circular section is an open channel when it is not running full. In such a channel the hydraulic radius, and therefore the velocity, is a maximum when the angle subtended by the dry portion of the border is 103°, or the depth is ·81 of the full depth. If the depth is further increased R decreases, but at first the increase of area more than compensates for this, and the discharge goes on increasing. When the angle above-mentioned is about 52°, or the depth is ·95 of the full depth, the expression $AC\sqrt{R}$ is a maximum, and Q is then about 5 per cent. more than when the channel is flowing full.

Section III.—Relative Velocities in Cross-section

7. General Laws.—Except near abrupt changes the water at every point of a cross-section of a stream has its chief velocity parallel to the axis of the stream and in the direction of flow, and the velocity varies gradually from point to point. Although the velocity at any point in a cross-section is affected to some extent by its distance from every part of the border, it depends chiefly on its distance from that part of the border which is near to it. Those portions of the border which are remote from the point have a small, often an inappreciable effect. In

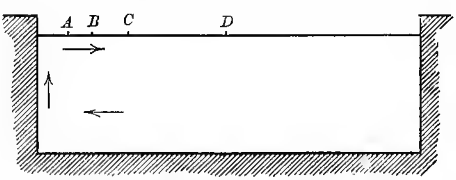


Fig. 108.

Fig. 108 the velocity at A is less than at B because of the effect of the neighbouring side. At all points between C and D the velocities are nearly equal because both sides are remote. Given the cross-section of a stream, the forms of the velocity curves are known in a general way but not with accuracy. In other words, their equations are not known.

The law that the velocity is greatest at points furthest from the border is subject to one important exception. The maximum velocity in any vertical plane parallel to the axis of the stream is generally at a point somewhat below the surface and not at the surface. If D is the depth of water and D_m the depth of the point of maximum velocity, the ratio $\frac{D_m}{D}$ in a stream of shallow section at points not near the sides may have any value from zero to ·30, and if the side-slopes are not steep the same ratio may be maintained right across the channel. When the sides are very steep or vertical the ratio $\frac{D_m}{D}$ close to the side is about ·50 or ·60, and it decreases towards the centre of the stream, attaining its normal value in a shallow section at a distance from the side equal to about $2D$ or $2·5D$, and thereafter remains constant or nearly so.

The depression of the maximum velocity has been sometimes attributed to the resistance of the air, but this theory is now quite discredited. Air resistance could cause only a very minute depression, and it cannot account for the variation of the depression at different parts of a cross-section. It is true that wind acting on waves and ripples may produce some effect. The water-level in the Red Sea at Suez is raised during certain seasons of the year when the wind blows steadily up the Red Sea. On the Mississippi, with depths ranging from 45 to 110 feet, an upstream wind was found to reduce the surface velocity and increase the ratio $\frac{D_m}{D}$. A downstream wind produced opposite effects, but even with a downstream wind the maximum velocity was below the surface, and the same thing has been observed elsewhere. Wind acting on ripples[1] is a different thing from simple air resistance. The depression is attributed by Thomson to the eddies which rise from the bed to the surface. The water of which the eddies are composed is slow-moving, and though the eddies retard the velocity at all points which they traverse, they have most effect at the surface, because they spread out and accumulate there. This explanation seems to be the true one, at least as regards the central portions of a stream. When no

[1] Wind which produces waves can cause currents in large bodies of water.

depression exists there, it is because the eddies are weak relatively to the other factors. The increased depression of the maximum velocity near the sides when these are steep or vertical is clearly connected with certain currents which circulate transversely in a stream. Near the side there is an upward current (Fig. 108), at least in the upper portion of the section, and there is a surface current from the side outwards. It is this current which causes floating matter to accumulate in mid-stream. At a lower level there must be an inward current which brings quick-moving water towards the sides, while the slow-moving water near the surface travels outwards and reduces the surface velocity at all points which it reaches.

As to the cause of the currents, Stearns, who has investigated the subject,[1] considers that they are due to eddies produced at the sides. The eddies from the side tend on the average to move at right angles to it, but they also tend to move chiefly in the direction of the least resistance, that is, towards the surface.

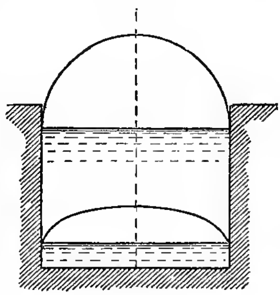


Fig. 109.

8. Horizontal Velocity Curves.—A horizontal 'mean velocity curve' is one whose ordinates are the mean velocities on different verticals extending from surface to bed. The general forms of these curves for a rectangular section are shown in Fig. 109 for two water-levels. When the section is shallow the velocities on different verticals, at a distance from the side exceeding $2D$ or $3D$, become nearly equal. Fig. 110 shows a channel with sloping sides. The length in which the velocity is practically constant is somewhat greater than before, and the curves in this portion are nearly as before, but the part in

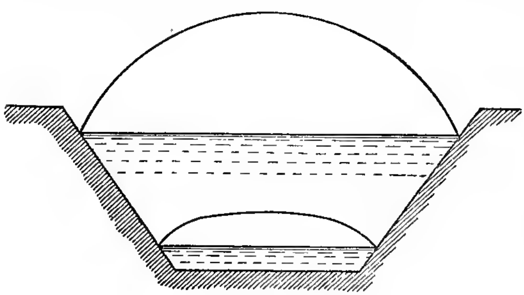


Fig. 110.

[1] *Transactions of the American Society of Civil Engineers*, vol. xii.

which the velocity varies is longer, both actually and relatively to the whole width. If the bed is not level across (Fig. 105, p. 177) the velocity is greater where the depth is greater. If there are, at a distance from the sides, divisions of considerable width and constant depth, as *HG* and *BK*, the velocity in each such division is nearly constant. The rough rule for a channel of shallow section considered as a whole, that V is approximately as $D^{\frac{2}{3}}$ where D is the mean depth, probably applies to any two divisions such as those under consideration and to the same division for different water-levels. But if a division is of small width its velocity is affected by those adjoining it. The velocity at B is affected by the greater velocity between B and G. This, combined with the fact that V is approximately as $D^{\frac{2}{3}}$, causes the velocity curve to be one which tones down the irregularities of the bed. On the South American rivers with depths of 9 to 73 feet, gradually increasing from the bank to the centre of the stream, Révy found the velocity to vary as D^n where n is greater than unity, but this conclusion appears to be unsound.[1] The form of the velocity curves in a channel of irregular sections changes, as it does in regular channels, with the water-level. Irregularities which have a marked effect at low water may have no perceptible effect at high water.

The nature of the horizontal mean velocity curve depends on the shape of the cross-section, and not on its size. From observations made by Bazin on small artificial channels lined with plaster, plank, or gravel, with widths of about 6·5 feet, and depths up to 1·5 feet, and observations made by Cunningham on the Ganges Canal in an earthen channel about 170 feet wide and 5 feet deep, and in a masonry channel 85 feet wide, with depths of 2 feet to 3·5 feet, it is also proved that if the velocity is altered by altering the surface-slope (and in the case of Bazin's channels by altering the roughness), the velocities on different verticals all alter in about the same proportion. It is probable, considering the complications arising from eddies and transverse currents, that the actual size of the channel has some effect, but it is negligible, at least in streams of shallow section, and under the conditions which occur in practice.

Let U be the mean velocity on the central vertical, and V that in the whole cross-section. Let $\frac{V}{U} = \alpha$. The values of the coefficient α are as follows:—

[1] See Notes at end of chapter.

Ratio of mean width to depth,	1	1·5	2	3	4	5	6	7	10	20	30	50	90
Value of a,	·86	·87	·88	·89	·90	·91	·92	·93	·94	·95	·96	·97	·98.

These co-efficients are applicable to rectangular and trapezoidal channels, but may not be very accurate for the latter when the ratio of the mean width to the depth is small, especially if the side-slopes are flat. In other cases they are probably correct to within 1 or 2 per cent. for the deeper sections, and to within ·5 per cent. for shallower sections. The co-efficients have been found chiefly from the observations above mentioned. Bazin did not work out this particular co-efficient, but his figures enable it to be found. In any particular channel the co-efficient increases as the water-level falls.

The co-efficient a was determined in the observations on the Solani aqueduct in the Roorkee experiments. In the aqueduct there is a central wall which divides the canal into two channels, each 85 feet wide. The aqueduct is 932 feet long, and the observations were made in the middle, that is, only 466 feet from the upper end. Upstream of the aqueduct the canal consists of one undivided channel, and the greatest velocities are in the centre. Owing to this fact the maximum velocities at the observation sites in the aqueduct at times of high supply are not in the centres of the channels, but nearer the central wall.[1] The velocities observed to determine a were, however, made in the centres of the channels, and the resulting values of a were therefore too high. The depth varied from 4 to 10 feet, and the ratio of width to depth therefore from 21 to 8·5. The values of a were nearly constant at ·95 or ·96. For the lower depths the co-efficient agrees with that in the above table. For the higher depths it was overestimated for the reason just given. (See chap. ii. art. 21.)

The co-efficients are strictly applicable only when the bed, as seen in cross-section, is a straight and horizontal line, but practically they are applicable whenever the central depth is the mean depth (not counting the sections over the side-slopes), and does not differ much from the others. If there is a shallow in the centre the co-efficient may exceed 1·0, and may increase greatly at low water. For some particular sections somewhat hollow in the centre the co-efficient may not vary as the water-level changes.

The above refers to horizontal mean velocity curves. The properties of horizontal curves at particular levels, for instance at the surface, mid-depth, or bed, are, generally speaking, similar to the above. In the central portions of the stream the curves are

[1] Not at low supply, *cf.* notes on momentum at end of chapter vii.

probably all parallel projections of one another. Near to vertical or very steep sides, owing to the greater depression of the line of maximum velocity, the mid-depth velocity curve, and to some extent the bed-velocity curve, become more protuberant, and the surface curve less so. Fig. 111 shows the distribution of velocities

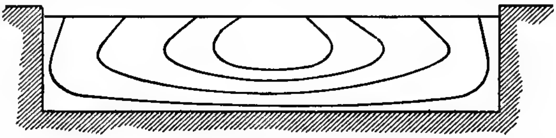


FIG. 111.

found by Bazin in a channel 6 feet wide and 1·5 feet deep, lined with coarse gravel. Each line passes through points where the velocities are equal.

9. **Vertical Velocity Curves.**—The general forms of the curves are shown in Figs. 112 and 113.[1] Many attempts have been made

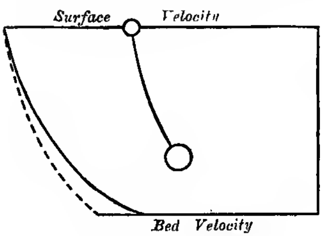


FIG. 112.

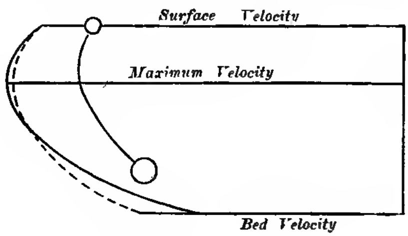


FIG. 113.

to find the equations to the curves, and it is sometimes said that the curve is a parabola with a horizontal axis corresponding to the line of maximum velocity. This is improbable. The transverse curve is certainly not a parabola. The bed of a channel retards the flow in the same manner as the side retards it, and the velocity probably decreases very rapidly close to the bed just as it does close to a vertical or steep bank. Except near the bed, almost any geometric curve can be made to fit the velocity curve. The equation to the curve is not nearly of so much practical importance as the ratios of the different velocities to one another. If these are known, the observation of surface velocities enables the bed-velocities and mean velocities to be ascertained. A slight difference in the ratios may make a great difference in the equation. Even the information regarding the ratios is very imperfect, and

[1] The floats and dotted lines are referred to in chap. viii.

until it is improved it is useless to discuss the equation. When the depths on adjoining verticals are not equal, the curves are probably of a highly complex nature, since each must influence those near it.

Let U_s, U_m, U, and U_b be the surface, maximum, mean and bed velocities on any vertical not near a steep side of a channel, then the ratios which are of most practical importance are those of D_m to D, and of U to each of the other velocities. The results as to these ratios furnished by experiments show great discrepancies. The fact seems to be that the ratios are easily disturbed. A change in depth,[1] roughness, or surface-slope may cause the eddies to rise in greater or less proportion, and so alter the ratios. The quantity of solids moved perhaps affects them, since some of the work of the eddies is expended in lifting or moving the materials. Wind may affect the surface velocities and unsteadiness in the flow may affect the ratios.[2] The depth D_m is seldom accurately observed. This is because the velocities above and below the line of U_m differ very slightly from U_m, and also because the velocities are not generally observed at close intervals. A greater defect is in the observation of bed velocities. They are seldom observed really close to the bed. When so observed a rapid decrease of velocity has been noticed.

Generally the different ratios roughly follow one another. When the eddies reach the surface in greater proportion the ratio $\frac{D_m}{D}$ increases. At the same time U_m is diminished and U_b is increased, because more quickly moving water takes the place of that which rises. Thus the different velocities tend to become equal and the ratios to approach unity. It will be sufficient to consider for the present only the ratios $\frac{D_m}{D}$ and $\frac{U}{U_s}$*. On examining the results of experiments no clear connection between these ratios and the quantities U and D is apparent, but by considering the two separate elements on which, for any given depth, U depends, namely N and S, some more definite, though not very satisfactory results are obtained. The following table contains an abstract of the results of some of the chief observations. Each group consists generally of several series, each series having a separate value of D and U, and sometimes of N or S. The table is a mere abstract, and is intended to show only what experiments have been considered and their general results. On the Mississippi and Irrawaddy and Ganges Canal the observations were made with

[1] Changes in the bed often occur and may not be noticed.

[2] But see chap. ix. art. 5.

* Or $\frac{U}{U_m}$, which is nearly the same.

Abstract of Results of Observations on Verticals not near the Sides of the Channels.

Serial Number of Group.	Channel.	Observer.	Depth, Roughness, and Velocity on Vertical.			Ratios.	
			D	N	U	$\frac{D_m}{D}$	$\frac{U}{U_m}$
			Division I.—Great Rivers.				
1	Mississippi.	Humphreys and Abbott.	76	·027	3·5	·38	·98
2	,,	,,	79	·027	2·1	·13	·94
3	,,	,,	65	·031	5·3	·27	·97
4	,,	,,	27	·025	4·7	·28	·97
5	Irrawaddy.	Gordon.	50	...	5·4	·03	·95
6	,,	,,	29	...	1·8	zero	·93
7	Parana de las Palmas.	Revy.	50	...	2·4	zero	·83
8	La Plata	,,	24	...	1·3	zero	·69
			Division II.—Ordinary Streams.				
9	Saone.	Leveillé.	14	·028	2·2	·15	·90
10	Garonne.	Baumgarten.	11	·0275	5·0	·10	·90
11	Seine.	Emmery.	9	·026	2·5	·05	·89
12	Rhine.	International Commission.	7	·030	7·1	zero	·85
13	Branch of Rhine	Defontaine.	5	·0275	3·5	zero	·87
14	Ganges Canal.	Cunningham.	9	·025	3·5	·12	·88
15	,,	,,	6·5	·013	4·2	·19	·93
			Division III.—Small Streams.				
16	Artificial Channels.	Bazin.	1·3	·020	5·9	·05	·84
17	,,	,,	1·1	·015	6·6	zero	·89
18	,,	,,	1	·012	6·5	zero	·91
19	,,	,,	·9	·010	9·1	zero	·92

the double float, and the ratio $\frac{U}{U_m}$ was thus seriously vitiated (chap. viii. art. 9), the values of U obtained being too high. On the Ganges Canal U was, however, observed separately by means of rod-floats, and by making certain corrections for the length of rod used, corrected values of U have been found and used. In Revy's observations the flow was unsteady.

By considering the figures of each separate series in divisions

ii. and iii. it is quite clear that the ratio $\frac{U}{U_m}$ increases as N decreases. This result had previously been found by Bazin for his small channels. The ratio also increases with the depth. In division i. the figures are unreliable, as above explained, but to some extent they confirm the above laws. From a consideration of the various results the following table has been prepared. The figures are an advance on the former rough rule that the ratio is '·85 to ·90.' The blanks in the table may be filled in according to judgment. In some small and rough channels the ratio has been found to be as low as ·60. The ratio $\frac{U}{U_s}$ may be designated β.

Probable Ratios of Mean to Surface Velocities $\left(\beta \text{ or } \frac{U}{U_s}\right)$ on Verticals not near the Sides of a Channel.

Depth on Vertical.	Values of N.								
	·030	·0275	·0250	·0225	·020	·0175	·015	·013	·010
Feet.									
·90	...	...	...		·83	·86	·88	·89	·91
1·0	·78	...	·82		...	...	...	...	...
1·10	...	...	...		·84	·87	·89	·90	·91
1·25	...	...	...		·85	·87	·89	·91	·91
1·50	...	...	...		·87	·88	·90	·91	·92
2·00	·80	...	·86		...	...	...	...	...
3·00	·83	...	·88		...	...	...	...	...
5·0	·85	·87	·89		...	...	...	·93	...
7·0	...	...	·90		...	...	...	...	...
10·0	·86	·89	·90		...	...	...	·92	...
13·0	...	·91	...		...	...	...	...	...
15·0	·87	...	·91		...	...	...	...	...
18·0	·88	·91	·91		·91	...	·91	·91	...
20·0	·88	...	·92		...	...	...	...	...
23·0	...	·93	...		...	...	...	...	...
28·0	...	·95	...		...	...	...	...	

After the preparation of the above table for depths up to 18 feet the author's attention was drawn to an extensive and careful series of observations made with current-meters by Marr on the Mississippi.[1] The results worked up and abstracted are as follows:—

[1] Report on Current-meter Observations in the Mississippi, near Burlington. The figures for depths of 15 and 20 feet have been obtained from Parker's *Control of Water*. On the Irrawaddy (N not known) the average ratio was found to be ·89, ·90, ·93, and ·97 for average depths of about 53, 32, 64, and 34 feet respectively. Individual observations showed great irregularities, *e.g.* the ·93 ratio varied from 1·01 to ·90 (*Note on the Irrawaddy River*, Samuelson, Government Press, Rangoon).

	Feet.	Feet.	Feet.	Feet.	Feet.
Depth =	11·2	13·2	20·4	21·6	27·6
V =	2·0	2·6	1·9	2·2	2·2
$U \div U_s$ =	·89	·91	·93	·93	·945
$D_m \div D$ =	·09	·09	·26	·21	·09.

The values of N and S are not stated, but N is judged to have been about ·0275, and the above table has been accordingly extended to depths of 28 feet. The velocities were not observed near enough to the bed to enable U_b to be found.

When the maximum velocity is at the surface the ratio $\frac{U}{U_m}$ is the same as $\frac{U}{U_s}$. Otherwise it is 1 to 3 per cent. lower.

No law for the variation of $\frac{D_m}{D}$ can be traced, except that in small streams the ratio is greater the rougher the channel. The ratio never exceeds ·20 except on the Mississippi. On the Irrawaddy, with not dissimilar depths and velocities, it is very small or zero. The difference may possibly be due to differences in N and S. It appears that in very deep rivers all the ratios are more sensitive.

The ratio $\frac{U_b}{U_m}$ or $\frac{U_b}{U_s}$ generally follows the ratio $\frac{U}{U_m}$. In the detailed series of division iii. of the table on page 168, both ratios attain maximum and minimum values together. Values ranging from ·58 to ·63 have been found for the ratio on the Lower Rhine, Meuse, Oder, Worth, and Messel. It is probable that in nearly all experiments the ratios found are too high because the velocities are hardly ever observed close to the bed, and also because of the rapid decrease of velocity near the bed. On the Sâone the current-meter was placed as near to the bed as possible, and the ratio comes out very low. The following table shows such probable values of this ratio as it has been possible to arrive at:—

N	·030 ·0275	·025	·020	·015	·010
Depths, .	Feet. 5 to 18		Feet. 1 to 1·5	Feet. 1 to $1\frac{1}{4}$	Feet. 1
$U_b \div U_m$, .	·50 to ·55		·50 to 55	·60	·65

When the various ratios are known the vertical velocity curve can be drawn. The curves are, of course, sharper the less the depth of water. The depth at which the velocity is equal to the mean velocity on the vertical varies somewhat, being generally deeper as D_m is deeper. It has been found to vary from $\cdot 55D$ to $\cdot 67D$. On the average it is at about $\cdot 60D$ or $\cdot 625D$. The mid-depth velocity is greater than the mean, but generally by only 1 or 2 per cent. On the Mississippi it was found to remain constant while U was constant, even though U_s was increased or decreased by wind, a compensating change occurring near the bed. The mean velocity can be found approximately by an observation at about ·60 of the full depth. It can be found very nearly, as has been shown by Cunningham, by observing the velocities at ·21 and ·79 of the full depth and taking the mean of the two.

10. Central Surface Velocity Co-efficients.—Sometimes the mean velocity V in a cross-section is inferred from an observation in the centre of a stream. If U is the velocity on the central vertical $V=\alpha U$. Sometimes U_s, the central surface velocity, is observed and multiplied by a co-efficient δ. It is clear that δ must be $\alpha\times\beta$. It has been seen that α depends on the shape of the section, and is practically independent of the size, roughness, and slope, while β, at least in streams of shallow section, seems to depend on two of these factors. In a given stream of shallow section and fairly level bed α decreases as D increases, but β increases. Hence δ does not in ordinary cases show any very great fluctuation. On the Ganges Canal, with earthen channels 190 to 60 feet wide, and masonry channels 85 feet wide, and with depths of water from 2 to 11 feet, δ varied from ·84 to ·89. Neither α nor β varied much. With widths of 10 to 20 feet, and depths of 1 to 3 feet, α was somewhat reduced, and δ was also less, its values being ·81 to ·85. At one site, where there was a shallow in the middle, α rose at low water to 1·07 and δ to ·95. Ordinarily δ is seldom below ·80.

Bazin found for small channels the values of a co-efficient Δ, giving the ratio of U_m to V. Its values do not differ very much from those of δ. Bazin, however, assumed that Δ depended only on N and R, and on this assumption he worked out values of the co-efficient for values of R, extending up to 20 feet, or far beyond the limits of his experiments. It has been the custom to use these co-efficients as values of δ, that is, to use them for obtaining V from U_s. This in itself would not cause any very large error, but the values of the co-efficients, when applied to channels of slopes, sizes, and roughnesses, differing greatly from those used by Bazin, are entirely wrong. Neither δ nor Δ can depend only on R and N, but must depend on the values of α and β.

Other general expressions for δ have been proposed by Prony and others, but they, in common with those of Bazin, are almost useless as general formulæ.

Section IV.—Co-efficients

11. **Bazin's and Kutter's Co-efficients.**—Setting aside obsolete and discarded figures, the first important set of co-efficients for open channels is that obtained by Darcy and Bazin from experiments on artificial channels, whose width did not exceed 6·56 feet in masonry and wood and 21 feet in earth. Bazin, from these experiments, framed tables of C (connecting them by an empirical formula, and extending them far outside the range of the experiments) for four classes of channel, namely, earth, rubble masonry, ashlar or brickwork, and smooth cemented surfaces. It has been found that these co-efficients, though correct enough for small channels, often fail for others. More recently two Swiss engineers, Ganguillet and Kutter, went thoroughly into the subject, and after investigating the results of the principal observations, and making some themselves, arrived at various sets of co-efficients for channels of different degrees of roughness, the roughness being defined by a 'rugosity-co-efficient' N. The following statement shows some selected values of Bazin's and Kutter's co-efficients. The last three columns will be referred to below :—

Hydraulic Radius (R).	Bazin's Co-efficients.			Kutter's Co-efficients for Channels having a Slope of 1 in 5000.			Bazin's New Co-efficients.		
	Cement, etc.	Rubble Masonry.	Earth.	Cement, Plaster, etc.	Earthen Channels in Good Order.	Earthen Channels in Bad Order.	Cement, etc.	Regular Channels.	Very Rough Channels.
				N = ·010	N = ·020	N = ·030	γ = ·109	γ = 1·54	γ = 3·17
·5	135	72	36	132	57	35	136	50	29
1·0	141	87	48	152	69	43	142	60	36
2·0	144	98	62	170	82	53	146	75	49
4·0	146	106	76	185	94	63	149	89	61
6·0	147	110	84	193	101	69	151	97	69
10·0	147	112	91	201	108	76	152	106	79

It will be seen that C always increases with R, and that the increase is less rapid as R becomes greater, and that as R increases

C becomes less affected by the degree of roughness. Also that, with change of R, Kutter's co-efficient varies more than Bazin's for smooth channels, and less than Bazin's for rough channels.

Bazin's co-efficients are independent of S, but Kutter's depend to some extent on S, as will appear from the following statement:—

Value of R.	Kutter's Co-efficients for different Slopes.			
	$N=\cdot 010$		$N=\cdot 030$	
	Slope 1 in 10,000	Slope 1 in 1000 and Steeper Slopes.	Slope 1 in 10,000.	Slope 1 in 1000 and Steeper Slopes.
·5	126	138	33	36
1·0	148	156	42	45
2·0	168	172	52	54
4·0	186	185	64	63
6·0	195	191	70	68
10·0	206	197	78	74

When R is about 3·2, C is independent of S. It increases or decreases with S according as R is below or above 3·2, but it varies only slightly for a great change of S, the variation being greatest when S is between 1 in 2500 and 1 in 5000. For slopes steeper than 1 in 1000 the variation is negligible. For all values of N the variation of C with S is very similar in relative amount.

Kutter's co-efficients for flat slopes are based on the Mississippi observations of Humphreys and Abbott. The fall here was small, sometimes only ·02 foot per mile, and doubt has been cast on the reliability of the slope observations. Bazin, who subsequently reviewed the whole question and considered all the best-known experiments, arrived at a new set of co-efficients, some of whose general values are given in the last three columns of the first of the above tables. As before, he makes C independent of S, and his different sets of co-efficients correspond to certain values of γ which is analogous to Kutter's N. The rate at which C varies with change of R conforms more nearly than before to that of Kutter's co-efficients. Bazin in his discussion includes some results which are known to be wrong, such as those obtained on the Irrawaddy (art. 9) and in the Solani aqueduct, Ganges Canal (chap. vii. art. 5), but the rejection of these would not appreciably alter his figures.

The question has recently been discussed by Houk[1] who concludes that the Mississippi observation at Columbus and two of those at Carrollton should be rejected—the fall in these cases having been so small that S may easily have been from 55 to 161 per cent. in error,—but that in the case of the observations at Vicksburg and two others at Carrollton the error would be, say, 7·5 to 27 per cent. He concludes that though it is not proved that Kutter has ascertained the exact law, he is correct in making C increase with decrease of S in deep streams, and he gives details of subsequent observations on the Irrawaddy, Mississippi, Bogue Phalia and Volga—with R averaging 20 to 50 feet—all tending to confirm this law. Considering all the information available, including Bazin's figures and arguments, and the various formulæ which have been propounded, including some recent ones, Houk concludes that the Bazin formula is inferior to Kutter's for all types of open channels, and that although the Kutter formula is not ideal it is the best available. This conclusion is accepted.

Manning adapts Kutter's co-efficients, by putting

$$C = \frac{1 \cdot 486}{N} R^{1/6}.$$

C is independent of S. It varies in the manner described for a stream of shallow section (art. 2). Complete sets of Kutter's, Bazin's, and Manning's co-efficients—C_K, C_B, and C_M—are given in tables xxix. to xlii. A diagram (Fig. 113A) is also given. In the diagram there are shown, for various values of N, curves of C_K, for slopes of 1 in 1000 and 1 in 20,000, by continuous lines, and the curves of C_M by small dashes. The curves of C_B are shown by longer dashes.

In C_B the number of classes is far too small. C_K is far more used than either of the others. It is sometimes said to be complicated, but this chiefly means that for one value of N there are six columns of figures. This causes little trouble when proper tables or diagrams are used.

Small smooth open channels have been dealt with in chapter v. art. 9. The rapid decrease of C_K when R is small is there mentioned and dealt with, and it is stated that small channels are the most sensitive to changes in roughness. This has doubtless been a cause of the error in C_K. The best co-efficients for small open channels—say R less than 2 feet, when N is ·011—are probably those in table XXVA.

[1] *Calculation of Flow in Open Channels.* See chap. iv. art. 15.

For large smooth channels the difference between C_B and the other co-efficients is often great. The number of such channels is limited and the number of observations in them has been small. The question is generally obscured by variations in the roughness. In order to bring out the law of variation of C with R, observations are required on the same channel with different depths of water. These are not often obtained. For the rougher channels—these are generally channels in earth—the differences among the three sets of co-efficients are not excessive.

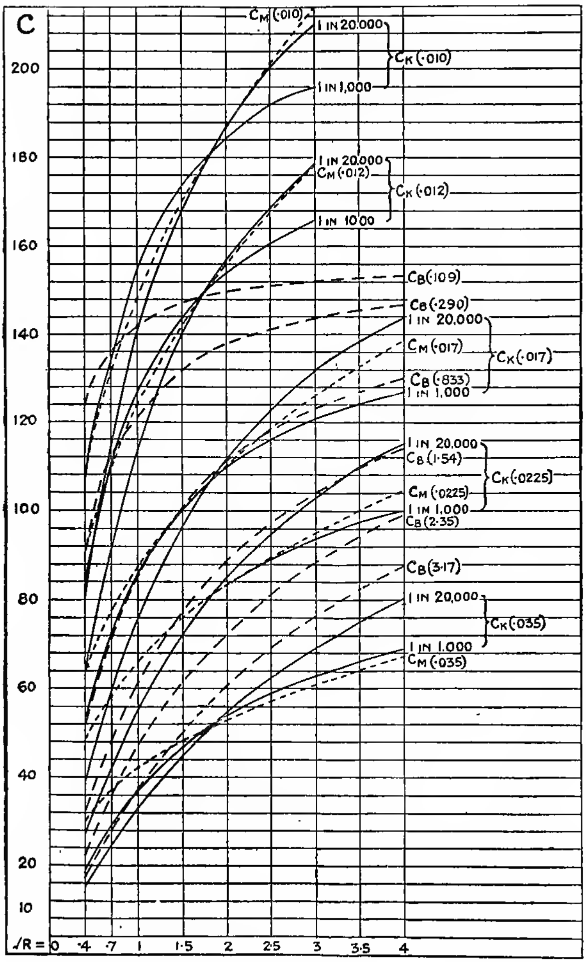


FIG. 113A.

The empirical formulæ connecting the different values of the co-efficients are as follows:—

Bazin's original co-efficients:[1]

$$C = \frac{1}{\sqrt{\alpha\left(1+\frac{\beta}{R}\right)}}.$$

Kutter's co-efficients:

$$C = \frac{41{\cdot}6 + \frac{1{\cdot}811}{N} + \frac{{\cdot}00281}{S}}{\sqrt{R} + N\left(41{\cdot}6 + \frac{{\cdot}00281}{S}\right)}\sqrt{R}.$$

Bazin's new co-efficients:

$$C = \frac{157{\cdot}6}{1+\frac{\gamma}{\sqrt{R}}}.$$

The quantities α, β, N and γ are all constants depending on the nature of the channel.

12. Rugosity Co-efficients.—The kinds of materials for which various values of N have been generally accepted are as follows. Unless otherwise stated all are supposed to be in good order and joints smooth.

·009	Timber planed and perfectly continuous.
·010	Timber planed. Glazed and enamelled materials. Cement and plaster.
·011	Plaster and cement with one-third of sand. Iron, coated or uncoated.
·012	Timber unplaned and perfectly continuous. Concrete. New brickwork (joints in perfect order).
·013	Unglazed stoneware and earthenware. Foul and slightly tuberculated iron. Good brickwork and ashlar.
·015	Wooden frames covered with canvas. Rough-faced brickwork. Well-dressed stonework.
·017	Fine gravel well rammed. Rubble in cement. Tuberculated iron. Brickwork, stonework, and ashlar in inferior condition.
·020	Coarse gravel well rammed. Coarse rubble laid dry. Rubble in inferior condition.

[1] These are not now used All tables and diagrams show Bazin's new co-efficients.

For earthen channels the following are the general values :—

·017	Channels in	very good order.
·020	„	good order.
·0225	„	order above the average.
·025	„	average order.
·0275	„	order below the average.
·030	„	bad order.
·035	„	very bad order.

A channel in very good order is free from irregularities, sharp bends, lumps, hollows, snags, or other obstructions, weeds and overhanging growth. A channel having all the above irregularities (or even a few of them in excess) would be in very bad order. The above descriptions are of course brief and general. The selection of the proper value of N in any particular case requires judgment and experience. There are of course channels requiring values of N intermediate to the above. The larger a stream the less its velocity is affected by changes in roughness. In any description of a channel some idea of its size should be conveyed. Overhanging growth which would have little effect on a large stream may have great effect on a small one. Small streams have—allowing for the difference in size—sharper bends and greater irregularity of cross-section. There is a tendency to underestimate N in such streams.

Regarding rivers and large canals some values of N are given in art. 9, but the figure ·013 refers to a brick channel. In other rivers Kutter found N to vary from ·025 to ·042. In small torrents —discharges of such are often observed to ascertain the run-off of the rainfall—N may be ·05 to ·08 or even more. The roughness of a channel is not necessarily the same at all parts of the bed and sides. Therefore in any channel N may vary as D varies.

In the Punjab canals N is generally taken to be ·0225, but when the channel has been worn very smooth and even, N has sometimes been found to be as low as ·016. In designing the large canals of the Punjab Triple Canal Project, N was taken, by Sir John Benton, to be ·020 for the Upper Jhelum Canal but ·0225 for the Upper Chenab and Lower Bari Doab Canals, where it was expected that more silt would be brought in, channels carrying much silt being considered liable to have rougher beds than others.[1] When mud has deposited in a canal the channel may be very smooth. It may be rough when sand deposits or when scour is going on.

For concrete pipes of 30 inches and 46 inches N has been found to

[1] *Min. Proc. Inst. C.E.*, vol. cci.

be ·012. In the 14·5-foot concrete-lined tunnel recently constructed for the New York water supply N was found to be ·0124. For very smooth concrete N has been found to be ·011. Reinforced concrete is now used for large pipes. The deposits which occur in brick sewers may increase the roughness somewhat, but they may fill up and make smooth any eroded mortar joints. Vitrified stoneware in large sewers gives great smoothness as compared with concrete, but this is in practice no advantage, because the distortion of the pipes in burning causes irregularity at the joints.

The kinds of channels corresponding to Bazin's γ are as follows:—

·109	Cement, planed wood.
·290	Planks, bricks, cut stone.
·833	Rubble masonry.
1·54	Earth if very regular, stone revetments.
2·35	Ordinary earth.
3·17	Exceptionally rough (beds covered with boulders, sides with grass, etc.).

13. **Remarks.**—Besides the causes of discrepancies among the values of C mentioned in chapter ii. (arts. 9 and 11) there are others. On the Mississippi and Irrawaddy V was obtained by the double float which gives erroneous results (chap. viii. art. 9). The results of over a hundred discharges observed near the head of a large canal in India, when arranged into groups according to the depth of silt in the canal, show the average value of N to be ·025 when there is little or no silt, but ·013 when the depth of silt is from ·5 foot upwards. Silt generally deposits in a wedge, the depth being greatest near the head of the canal. It is therefore probable that the want of uniformity of the flow gave a somewhat enhanced value to C, and consequently too low a value to N. This would, however, account only partially for the low value of N, and it is probable that its correct value is not more than ·016 in the silted channel. The above values are the average ones. In individual discharges N varies enormously. For one particular depth of silt it varies from ·009 to ·030. These variations may be accounted for partly by real variations in the roughness of the channel, which often becomes very irregular when scouring is going on actively, partly by errors[1] in the observations of the individual surface-slopes, and partly by variations in the degree of the variability of the flow.

For two channels equal as regards roughness of surface and value of R, N is less when the profile of the section is semicircular or curved than when it is angular. In Bazin's experiments on

[1] These may have been considerable (see chap. viii. art. 2).

small channels C is 5 to 9 per cent. less for a rectangular section, even though the depth was only $\frac{1}{13}$ to $\frac{1}{7}$ of the width, than for a semicircular channel. The difference is probably due to the effect of the eddies produced at the sides (art. 7). The co-efficients in the tables may be taken to be for average sections, the section being neither a segment of a circle nor a rectangle. (See also art. 4.)

In earthen channels N seems to be particularly low when the ratio of width to depth is great. On the river Ravi at Sidhnai the value of N, deduced from a long series of observations, is often ·008 or ·010, and never very much higher. The bed is often silted, but not always. The flow is practically uniform, and the slope observations were checked with a view to discovering any error. The river is straight, very regular, about 800 feet wide, and 6 feet to 10 feet deep. The case was specially investigated, and it seems to be proved that N at this site is not above ·010. It is probable that the low value is due to the small effect of eddies from the sides, as compared with narrower streams and to the regularity of the flow. Generally streams as wide as the Ravi are irregular. The river is straight for five miles upstream of the discharge site and one mile downstream, a reach unique, perhaps, among the rivers of the world, but its great length cannot be the cause of the low value of N. The silt is caused by a dam a mile below the discharge site. In floods the dam is removed, and the silt then scours out. Thus the bed is probably roughest for the greatest depths of water. In spite of this, N is very much the same for all the depths from 6 feet to 10 feet, and C somewhere about 200.

Section V.—Movement of Solids by a Stream

14. Formulæ and their Application.—The observations made by Kennedy, and referred to in chap. ii. (art. 23), were made in India on the Bari Doab Canal and its branches, the widths of the channels varying from 8 feet to 91 feet, and the depths of water from 2·3 feet to 7·3 feet. The beds of these channels have, in the course of years, adjusted themselves by silting or scouring, so that there is a state of permanent *régime*, each stream carrying its full charge of silt. It was found that the relation between D and V in any channel was nearly given by the equation

$$V = \cdot 84 D^{\cdot 64} \quad . \; . \; . \quad (71)$$

Put in a general form, the equation is

$$V = cD^{m} \quad . \; . \; . \; . \quad (72)$$

The theory advanced in the paper quoted is that the silt supported per square foot of bed is P_1D where P_1 is the charge of silt, and the force of the eddies as V^2, so that P_1D is as V^2. If the solids consisted only of silt m would be perhaps $\frac{1}{2}$, but there is also rolled material. The silt discharge is $BDVP_1$, or is as BV^3. The rolled material is supposed to be as BV, and relatively small, and the total solid discharge is thus as a function of V, varying less rapidly than V^3, say as V^n. On the Bari Doab Canal n was 2·56. For, since $D^{\cdot 64}$ is as V, D is as $V^{1\cdot 56}$, and $BDVP$ as $BPV^{2\cdot 56}$.

The equation

$$V = 1{\cdot}05D^{\frac{1}{2}} \quad . \quad . \quad . \quad (73)$$

agrees nearly as closely as equation 71 with the observed results.

The equation

$$V = {\cdot}95D^{\cdot 57} \quad . \quad . \quad . \quad (73\text{A})$$

has also been suggested.[1]

All the above equations are partly empirical, and obviously apply only when the silt and rolled material bear some sort of proportion to each other. In theoretical equations of general application silt and rolled material would have to be considered separately. If there is silt alone, equation 72 may be of the true form for all cases, m being probably $\frac{1}{2}$ or less. If there is rolled material and no silt, as in a clear stream rolling gravel or boulders, the moving force depends on the bed velocity, V_b, and D will be absent from the equation, or will enter into it only in so far as the ratio $\frac{V_b}{V}$ may depend on D.

Regarding equation 71 as a semi-empirical working equation—and no more has been claimed for it—applicable to canal systems and streams carrying silt and fine sand, its practical importance is very great. It is now known that in order to prevent, say, a deposit in any reach or branch, V must not be kept constant, but be altered in the same manner as D. Whether it be altered as $D^{\cdot 64}$ or $D^{\frac{1}{2}}$ does not, for moderate changes, make very much difference. The exact figures will in time be better known. In designing a channel the proper relation of depth to velocity can be arranged for, or, at least, one quantity or the other kept in the ascendant, according as scouring or silting is the evil to be guarded against.

The old idea was that an increase in V, even if accompanied by an increase in D, gave increased silt-transporting power. In a stream of shallow section this is probably correct, for V increases as $D^{\frac{2}{3}}$,

[1] *Proceedings of Punjab Engineering Congress*, 1919.

that is, as fast as required by equation 71, and faster than required by equation 73. In a stream of deep section a decrease in D gives increased silt-transporting power. If the discharge is fixed, a change in D or W must be met by a change of the opposite kind in the other quantity. In this case widening or narrowing the channel may be proper according to circumstances. In a deep section widening will decrease the depth of water, and may also increase the velocity, and it will thus give increased scouring power. In a shallow section narrowing will increase the velocity more than it increases $D^{\frac{1}{4}}$. In a medium section it is a matter of exact calculation to find out whether widening or narrowing will improve matters.

If the water entering a canal has a higher silt-charge than can be carried in the canal some of it must deposit. Suppose an increased discharge to be run, and that this gives a higher silt-carrying power and a smaller rate of deposit per cubic foot of discharge, it does not follow that the deposit will be less because the quantity of silt entering the canal is now greater than before. Owing to want of knowledge regarding the proportion of rolled material, and to want of exactness in the formulæ, reliable calculations regarding proportions deposited cannot be made.

Assuming equation 71 to be correct, Kennedy has determined the following 'critical velocities,' or velocities below which silting will occur in channels supplied with turbid water, such as that of the Indian rivers, and has also published diagrams giving details.

$D=$	1	2	3	4	5	6	7	8	9	10
$V=$	·84	1·30	1·70	2·04	2·35	2·64	2·92	3·18	3·43	3·67.

The preceding figures refer to heavy silt and fine sand, such as enters canals taking off from the upper reaches of the Northern Indian rivers. For reaches of the canals distant from their heads or for canals taking off lower down the rivers, a velocity of $\cdot75\,V$ to $\cdot9\,V$ may be substituted for V. For the fine sand of Sind, ·84 in equation 71 becomes ·63, and for the coarse sand of the Cauvery and Kistna rivers in Southern India 1·01. The proper figure becomes known in each case from experience. Thrupp (*Min. Proc. Inst. C.E.*, vol. clxxi.) gives the following ranges of velocities as those which will enable streams to carry different kinds of silt:—

$D=$	1·0	10·0	
$V=$	1·5 to 2·3	3·5 to 4·5	Coarse sand.
$V=$	·95 to 1·5	2·3 to 3·5	Heavy silt and fine sand.
$V=$	·45 to ·95	1·2 to 2·3	Fine silt.

In the channels on which Kennedy made his observations the charge of silt was supposed to be equal in all cases. But actually some of the coarser solids were gradually deposited, or drawn off by the irrigation distributaries—(small branches)—so that in the lower reaches the silt charge was reduced. In these lower reaches the lower values of D and V occur. If the silt charge had been the same as in the upper reaches V would have been greater in relation to D. Therefore Kennedy's formula tends to show a somewhat too rapid decrease of V as D decreases. Possibly the index of D in equation 73 or 73A is really more correct than in 71 and in the table of Kennedy's critical velocities given above, V, though correct for a depth of about 7 feet, should perhaps be somewhat higher than ·84 for a depth of 1 foot.

The effect of a rising or falling stream on the movement of solids is mentioned in chap. ix. art. 5. On the Irrawaddy it was found that on the day of a high flood a great deepening of the channel occurred at all the observation sites.[1] This may have been due either to the rise or to the greater depth of water after the rise, or to both. When a falling flood is accompanied by silting it may be because water heavily charged with silt has entered the river during the flood.

For special circumstances affecting silting or scour see chap. vii. arts. 1, 2, 3, 7, 8, and 9.

The moving of rolled material must depend on V independently of D. In a reach of the Sirhind Canal in Northern India the rolled material formed, in a period of 20 days, 39 per cent. of the whole.

It is probable that the force exerted by a stream on a solid which it is rolling is more nearly as $V^{1 \cdot 8}$ than V^2. This affects the above mathematical investigation but not the practical results.[2] It has been seen that the exact form of the equation is not of extreme importance.

Observations on circular sewers by Currall[3] tend to show that in order that road detritus and rubbish may be moved by rolling or dragging, D must not be less than 2·5 inches in a 9-inch pipe and 4·5 inches in a 27-inch pipe. For sewers a velocity of 2 to 3 feet per second is generally considered correct. The movement or scour of solids, other than those in suspension, depends greatly

[1] *Note on the Irrawaddy River.* Samuelson (Government Press, Rangoon).

[2] If a number of bodies have similar shapes, and if D is the diameter of one of them and V the velocity of the water relatively to it, the supporting or rolling force is perhaps as $V^{1 \cdot 8} D^2$. and the resisting force or weight as D^3. If these are just balanced D varies as $V^{1 \cdot 8}$, or the diameters of similarly shaped bodies which can just be supported or rolled are as $V^{1 \cdot 8}$ and their weights as V^5 nearly.

[3] *Min. Proc. Inst. C.E.*, vol. cxcii.

on how closely they are packed or stuck together, and the question is outside the domain of Hydraulics.

One theory is that the power of a stream to transport solids depends on the difference between the velocities of two adjacent horizontal layers. Such layers of course do not slide on one another but are eddying and intermixed. When V is below the critical velocity V_c (chap. ii. art. 15) there are no eddies at all and probably no sliding of one layer on another, the greater velocity near the centre of the stream being accompanied by a general deformation of the mass, as it might be in a column of india-rubber. When V rises above V_c there are eddies everywhere and still no sliding. There are general differences of velocity among the horizontal layers. These differences are greater the rougher the bed. So are the eddies caused at the bed. The theory just mentioned does not seem to be practically different from the one already considered.

The action of a stream on a vertical or very steep bank seems to depend chiefly on V alone and not on the relation of V to D. If V is less than 1 foot per second and the water is heavily silted a deposit may occur on the bank, tending to narrow the channel. This is especially likely to occur if there is vegetation on the bank. If V is about 3 feet per second, scour of the bank is, with many soils, likely to occur. This is independent of scour due to bends (chap. vii. art. 1), and again is affected by vegetation.

Let it be required to design a channel to carry a given discharge and to have a given relation of V to D so as to prevent silting or scour. If S is not fixed there is an infinite number of such channels. In deciding which to adopt the question of the actual velocity comes in with reference to possible action on the bank. Owing to these considerations and to general convenience it has been found necessary in the Punjab to fix the approximate ratio of W to D. Some of the figures are as follows:—

$Q=2$	12	80	300	600	1100	2200	3000	c. ft. per second.
$\frac{W}{D}=2$	3	4	5	6	8	12	15	

Let there be two channels, equal as to D and V but one having a rougher bed than the other, and of course a steeper slope. The bed velocity in the rougher channel will be the less but the difference perhaps not very great, and in spite of it the strength of the eddies formed at the bed will probably be greater in the rougher channel. If a short length of channel is roughened the local surface slope is increased, but, owing to the smallness of the length, D and V are not affected. A greater proportion of silt is thrown up to the surface. This in no way affects Kennedy's conclusions but is outside them. His channels did not vary much in roughness.

15. **Remarks.**—The channels in which the observations above referred to were made have all, as stated, assumed nearly rectangular cross-sections, the sides having become vertical (Fig. 114) by the deposit on them of finer silt; but the equations probably apply approximately to any channel if D is the mean depth from side to side, and V the mean velocity in the whole section.

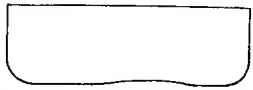


Fig. 114.

If the ratio of V to D^m, say V to $D^{\cdot 64}$, differs in different parts of a cross-section, there is a tendency towards deposit in the parts where the ratio is least, or to scour where it is greatest. There is, of course, a tendency for the silt-charge to adjust itself to the circumstances of each part of the stream, that is, to become less where the above ratio is less, but the irregular movements of the stream cause a transference of water transversely as well as vertically, and this tends to equalise the silt-charge. In a channel with not very steep side-slopes the angles at M, N (Fig. 115) frequently silt up—the velocity there being relatively low—and the sides become steep or vertical. Sometimes, even when the sides are vertical, fine silt adheres to them, and the channel contracts, even though there may be no deposit in the bed. When the bed is level across there frequently occurs a shoaling near the sides, or a scour in the middle, and a marked rounding-off at the lower angles. The section thus tends to assume the form shown in Fig. 116. When the bed is of sand, as in the Bari Doab Canal channels, it remains nearly level, because the sand at the sides rolls towards the centre.

Fig. 115.

Fig 116.

It is clearly impossible to answer, in a general manner, questions such as whether the embanking of a river, or confining it by training-walls, will cause its bed to rise or to scour; whether silt will deposit on flooded land; whether the minor arm of a stream will tend to silt and become obliterated. Everything depends on the charge of silt originally carried, on the hardness of the channels, and on the relations between D and V.

When a channel is sandy the longitudinal section is often a succession of small abrupt falls. After each fall there is a long gentle upward slope till the next fall is reached. The sand is rolled up the long slope and falls over the steep one. It soon becomes buried. The positions of the falls of course keep moving downstream. The height of a fall in a large channel is perhaps 6 inches or 1 foot, and the distance between the falls 20 to 30 feet. A fall does not extend straight across the bed but zigzags, so that the channel as viewed from above presents the appearance of waves.

Some rivers in the northern hemisphere which flow in a southerly direction have a tendency to shift their channels westwards. This is especially noticeable in some of the Indian rivers. The revolution of the earth has been ascribed as a cause. As the water approaches the equator its velocity of rotation about the earth's axis increases. In latitude 30° a stream flowing south at 2 miles an hour has its velocity of rotation increased in one hour from about 1300 feet per second to 1300·37 feet per second, or by ·37 feet per second. This is not a large amount in an hour, and the pressure due to it must be a negligible quantity. Moreover, scour depends on velocity not on pressure (chap. ii. art 23, *cf.* chap. vii. art 1).

An irrigation branch channel, whether large or small, taking off at a right angle from a canal, often receives more than its due share of silt deposit. This is probably owing to the stirring up of rolled material by the eddies formed at the off-take (*cf.* chap. vii. art. 9). The off-take is a masonry 'head.' One of the commonest remedies for silting is to make the floor of the head higher than the bed of the canal—or to make the water pass over a raised 'sill' or gate or both—so as to try to exclude rolled material. But even in such cases the branch may silt. Fig. 116A (section) shows the canal on the left. There are of course at the off-take, transverse to the canal, velocity-of-approach currents somewhat as shown by the arrows. It has been stated in chap. ii. art. 20—also see chap. vii. art. 3—that high velocity in the canal reduces the discharge of a branch taking off from it at right angles, and it has been argued that the branch draws in most of its water from the lower half of the aperture because the water at that level is moving relatively slowly. This consideration, however, has not much force with the usual large apertures and moderate velocities. It has also been argued that the cross currents (art. 7 and fig. 108) cause silt to be carried towards the sides of the canal at a somewhat low level, but it has been seen that the general effect of the currents is to equalise the silt charge. In the absence of currents the water near the sides would be less highly charged than that in the centre.

An arrangement devised by King,[1] to reduce or prevent the deposit of silt in a distributary, consists in the fixing of vanes on the bed of the canal at AB in such a way as to throw off the lower water towards C. A compensating surface flow occurs from D to F. The water in the canal is given a rotatory movement. This has been tried with excellent results, the main channel, however, being a distributary and the branch channel a 'water-course' whose head was only ·5 foot square with no wing walls, and its floor level with the distributary bed. It seems probable that the chief benefit is due simply to the throwing off of the silted water and its replacement by clearer water. Simple roughening of the bed might not be effectual. Another plan is to substitute for the vanes a low masonry spur whose width is gradually reduced, in going upstream, at the rate of 1 in 4 so that the spur throws off the lower water of the canal. This has been tested with complete success, the silting of a distributary—not merely a water-course—having been cured. In all cases the bed of the main channel at the off-take has to be pitched; otherwise severe scour would be caused by the disturbance.

It has been argued that a low velocity of inflow through MN is desirable. Water flowing upwards from the bed of the canal will be able to carry more sand the greater its velocity. Velocity of approach, however, depends on the discharge of the aperture, and this depends not only on the velocity at MN but on the depth MN. A remedy for silting is no doubt the reduction of the depth MN—the length of the aperture being increased to give the proper value of Q—and the increase of the height NP.

The case of the head-works of a large canal is similar. The water flows over a sill which can be further raised by gates. This is independent of the formation of a 'silt trap' in the river by a closure of the gates of the weir which runs across the river, succeeded in due course by their reopening and the simultaneous closure of the canal.

If a distributary has no raised sill or gate the flow of entry is still like that over a submerged weir (chap iv. art. 15), and the lower the velocity of approach the better. Opinion tends to favour wide head openings for distributaries. Not only is velocity

[1] *Proceedings of Punjab Engineering Conference*, 1918.

of approach reduced but eddies are reduced. They would be further reduced by making the opening bell-mouthed (*cf.* chap. ii. art. 20).

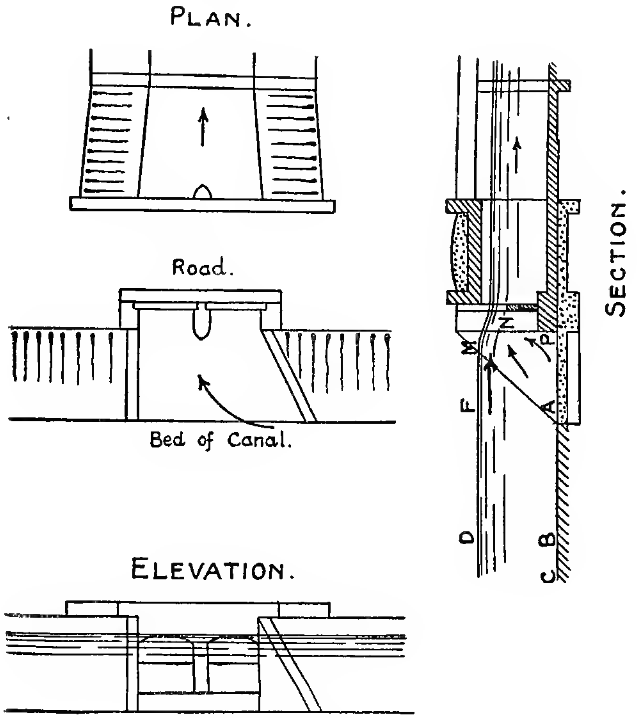


FIG. 116A.

NOTES TO CHAPTER VI.

Dependence of U on D on a Vertical in a Cross-Section (art. 8).—"We have here a most remarkable section of a great river, in which from one bank the bottom slopes in the same direction for a distance of over 3700 feet with the regularity of railway gradients, the depths increasing from nil to 72 feet." The above refers to the Paraná.[1] Révy found that the surface velocity U_s varied as D,

[1] *Hydraulics of Great Rivers.*

and concluded that since $\frac{U}{U_s}$ increases somewhat with D, U must vary as D^n where n is greater than unity. He found a similar result on the Uruguay. But in each case he rejects an observation —at or about the maximum depth—which if accepted would tend to show that U_s did not increase so rapidly as D. The rejections were made on the ground that the depths at the points under consideration were probably local, *i.e.* that they occurred only on the single cross-section taken. There seems to be no proper evidence as to this. The observations at the points considered seem to have about as much weight as any of the others. Moreover, on the Paraná—this was the site of the most important experiments —the observation site was at a bend of about 12 miles radius, the width of the river being nearly a mile. The greatest depths were of course near the concave bank (chap. vii. art. 1) and the velocity would be somewhat greater than if the stream had been straight, even with the same section. The total number of Révy's observations was quite small. The flow appears to have been steady. It seems clear, however, that U varied more nearly as D than as $D^{\frac{2}{3}}$. It cannot be supposed that the law governing two wide portions of a stream is different from that governing two separate wide streams, and these observations of Révy's afford some evidence that C increases, with great depths, more than has been supposed. He himself left the matter to be explained by others.

Average Sections.—An earthen channel is seldom so regular that any two parallel and neighbouring longitudinal or cross-sections are exactly alike. In discussions such as those in the present chapter average sections are always meant. A single section may, as in the above case of the Paraná, contain, for instance, a shallow which is local, that is, does not extend to adjacent sections. At such a shallow V, instead of being less than on neighbouring verticals, is likely to be greater because of the rush of water over it (*cf.* chap. vii. art. 2).

Examples

Explanation.—The explanation given under examples in Chapter v. applies also to open channels. If only one factor, say S, is fixed an infinite number of channels can be designed to carry a given discharge, but usually other factors are determined by practical considerations: the ratio of the side-slopes, say, by the nature of the soil, and the ratio of W to D, say, by the velocity desirable or the solid-moving power required. If V must not fall below a certain minimum this can be arranged by keeping R large enough, or if this cannot be done, by altering S, N, or Q. If V is not to exceed a certain maximum R can be kept down, or S can be reduced to any extent by placing falls in the channel.

Example 1.—Find the discharge of a stream with vertical sides and 15 ft. wide when $D=5{\cdot}0$ ft., $N={\cdot}017$, and $S=1$ in 5225.

From table xliii. $A=75$ and $\sqrt{R}=1{\cdot}74$. From table xxxv. $C\sqrt{R}=183$. From table xxviii. a slope of $\frac{1}{5000}$ gives $V=2{\cdot}59$, and the percentage to be deducted is $\frac{225}{102}=2{\cdot}2$, making $V=2{\cdot}53$. Then $Q=75\times2{\cdot}53=189{\cdot}8$ c. ft. per second.

Example 2.—Design a channel with side-slopes 1 to 1 to discharge 1000 c. ft. per second, S being $\frac{1}{5000}$ and $N={\cdot}0225$. The figures in the annexed statement show the results of successive trials, the bed-width being 40 ft. It is clear that a depth of 7·13 ft. gives the requisite discharge.

	1st trial.	2nd trial.	3rd trial.
Bed-width,	40	40	40
Depth,	7·5	7·25	7·0
A from table xlv.,	356·3	342·6	329
$\sqrt{R}$ from table xlv.,	2·41	2·38	2·34
$C\sqrt{R}$ from table xxxvii.,	216	212	208
V from table xxviii.,	3·05	3·00	2·94
$Q=AV$,	1087	1028	967

Example 3.—In the preceding example let V be limited to 2·5 ft. per second. Find the minimum bed-width.

A must be 400. From table xxviii. $C\sqrt{R}$ is 177, and this in table xxxvii. gives $\sqrt{R}=2{\cdot}08$. From table xlv., a bed-width of 80 ft. and depth 4·75 ft. gives practically the required values of A and $\sqrt{R}$.

Example 4.—A channel 20 ft. wide with side-slopes $\frac{1}{2}$ to 1 and depth 5 ft. has to discharge 240 c. ft. per second, N being ·025. Find S.

From table xliv. $A=112{\cdot}5$ and $\sqrt{R}=1{\cdot}90$. Then $V=\dfrac{240}{112{\cdot}5}=$ 2·13 ft. per second. Assume S to be $\frac{1}{3000}$. Then table xxviii. gives $C\sqrt{R}=151$, which corresponds in table xxxviii. to $\sqrt{R}=2{\cdot}0$. Therefore S has been assumed too low. Assume it to be $\frac{1}{4500}$, then $C\sqrt{R}=142{\cdot}8$ and $\sqrt{R}=1{\cdot}92$. To be exact $\sqrt{S}$ must be increased in the ratio $\frac{1 \cdot 92}{1 \cdot 90}$, or by 1 per cent. nearly, that is, $S=\frac{1}{4411}$.

Example 5.—Keeping Q the same, alter D and S in the last case so as to give the necessary ratio of V to D to prevent silting according to the rules of art. 14.

The statement given below shows that if D is reduced to 3·25 ft. S will be as before (1 in 4410), but W must be increased to 40 ft. If W is left unaltered D can be 4·75, but S must be increased to about 1 in 3572. In a short channel, or one containing falls, it would be easiest to increase S, but otherwise it would be necessary to widen.

Depth of water, ...	5·0	4·5	4·0	3·5	3·0
Velocity according to above rule,	2·35	2·20	2·04	1·87	1·70
Mean width of channel to make $Q=240$ c. ft. per second, .	20·5	24·2	29·4	36·6	47
Bed-width of channel to nearest foot, ...	18	22	27	35	45
$\sqrt{R}$ from table xliv., . . .	1·87	1·85	1·79	1·73	1·64
$C\sqrt{R}$ from table xxxviii., .	137	135	129	123	114
S (from table xxviii.) to give V as above, 1 in . . .	3380	3764	4000	4320	4500

Example 6.—In a channel A is found to be 48 sq. ft., $\sqrt{R}$ is 1·4 ft., Q is known to be 100 c. ft. per second, and S is $\frac{1}{3100}$. Find C and N.

V is $\frac{100}{48}=2 \cdot 08$ ft. per second. From table xxviii., if $S=\frac{1}{3000}$, $C\sqrt{R}=114$. An addition of 61 to 3000 decreases V by 1 per cent., ∴ an addition of 100 decreases V by 1·6 per cent., and $C\sqrt{R}$ must be increased by 1·6 per cent., that is, it is 115·8. Then $C=\frac{115 \cdot 8}{1 \cdot 4}=82 \cdot 7$, which (table xxxvi.) corresponds very nearly to $N=\cdot 020$.

Example 7.—In a channel with vertical sides, 70 ft. wide and 5 ft. deep, the central surface velocity is 3 ft. per second, N is ·025. What is V?

From the table on page 187 β is ·89. From the table on page 183 α is ·945. Then $V=3 \times \cdot 89 \times \cdot 945=2 \cdot 52$ ft. per second.

Tables of Kutter's and Bazin's Co-efficients

These are given to three figures, and the engineer who uses them will be fortunate if the actuals come out so as to agree with the third figure or even come near it. To add a fourth figure is useless, and it would render the tables bulky and less convenient. The values of $C\sqrt{R}$ have been obtained from the four-figure values of C, and the figures in excess of three struck out.

As N increases the difference in C becomes less in proportion to the change in N. Hence it is not necessary to give C for $N=\cdot 0325$.

Table XXIX.—Kutter's Co-efficients ($N=\cdot 009$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	93·4	37·4	98·9	39·5	106	42·2	114	45·6	119	47·7	123	49·1
·45	101	45·6	107	48	113	51·0	122	54·7	127	57	130	58·5
·5	108	54·2	114	56·8	120	60·1	128	64·2	133	66·6	137	68·2
·55	115	63·3	120	66·2	127	69·6	134	73·9	139	76·5	142	78·1
·6	121	72·7	126	75·8	132	79·4	140	84	145	86·7	147	88·4
·65	127	82·5	132	85·8	138	89·5	145	94·2	149	97	152	98·8
·7	133	92·7	137	96·1	143	100	150	105	154	108	156	109
·8	142	114	147	117	152	122	158	126	162	129	164	131
·9	151	136	155	140	160	144	165	149	168	151	170	153
1	159	159	163	163	167	167	171	171	174	174	175	175
1·1	166	183	169	186	173	190	177	194	179	197	180	198
1·2	173	207	175	210	178	214	181	218	183	220	184	221
1·3	178	232	180	235	183	238	185	241	187	243	188	244
1·4	184	257	185	259	187	262	189	265	190	267	191	267
1·5	188	283	190	285	191	287	193	289	193	290	194	291
1·6	193	309	194	310	195	311	196	313	196	314	197	314
1·7	197	335	197	336	198	336	198	337	199	338	199	338
1·8	201	362	201	362	201	362	201	362	201	362	201	362
1·9	204	388	204	388	204	387	203	386	203	386	203	386
2	208	415	207	414	206	413	205	411	205	410	205	409
2·1	211	443	210	440	209	438	207	436	207	434	206	433
2·2	214	470	212	467	211	464	209	460	208	459	208	457
2·3	216	497	215	494	213	490	211	485	210	483	209	481
2·4	219	525	217	520	215	516	213	510	211	507	211	506
2·5	221	553	219	547	217	541	214	535	213	532	212	529
2·6	223	581	221	574	218	568	215	560	214	556	213	554
2·7	226	609	223	601	220	594	217	585	215	581	214	578
2·8	228	637	224	629	221	620	218	610	216	605	215	602
2·9	229	665	226	656	223	646	219	635	217	630	216	626
3	231	694	228	683	224	673	220	660	218	654	217	650

TABLE XXX.—KUTTER'S CO-EFFICIENTS ($N=\cdot01$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	81	32·4	85·7	34·3	91·4	36·6	99	39·6	104	41·5	107	42·7
·45	87·9	39·6	92·6	41·3	98·3	44·3	106	47·6	110	49·7	114	51·1
·5	94·4	47·2	99·1	49·6	105	52·4	112	56	117	58·2	120	59·7
·55	100	55·2	105	57·8	111	60·8	118	64·7	122	67	125	68·6
·6	106	63·6	111	66·3	116	69·6	123	73·7	127	76·1	130	77·7
·65	111	72·3	115	75·3	121	78·7	128	82·9	131	85·4	134	87
·7	116	81·4	121	84·5	126	88	132	92·3	136	94·9	138	96·6
·8	126	100	130	104	134	107	140	112	143	114	145	116
·9	134	120	137	124	141	127	146	132	149	134	151	136
1	141	141	144	144	148	148	152	152	155	155	156	156
1·1	148	162	150	166	154	169	157	173	159	175	161	177
1·2	154	184	156	187	159	190	162	194	164	196	165	198
1·3	159	207	161	209	163	212	166	216	167	217	168	219
1·4	164	230	166	232	167	234	169	237	171	239	171	240
1·5	169	253	170	255	171	257	173	259	174	260	174	261
1·6	173	277	174	278	175	280	176	281	176	282	177	282
1·7	177	301	178	302	178	302	178	303	179	304	179	304
1·8	181	325	181	325	181	325	181	326	181	326	181	326
1·9	184	350	184	349	184	349	183	348	183	348	183	347
2	187	375	187	373	186	372	185	370	185	370	185	369
2·1	190	400	189	398	188	395	187	393	187	392	186	391
2·2	193	425	192	422	191	419	189	416	188	414	188	413
2·3	196	450	194	447	193	443	191	438	190	436	189	435
2·4	198	476	196	471	194	466	192	461	191	458	190	457
2·5	201	501	198	496	196	490	194	484	192	481	192	479
2·6	203	527	200	521	198	514	195	507	194	503	193	501
2·7	205	553	202	546	199	538	196	530	195	526	194	523
2·8	207	579	204	571	201	562	198	553	196	548	195	545
2·9	209	605	206	596	202	586	199	576	197	571	196	567
3	210	631	207	621	204	611	200	599	198	593	196	589

Table XXXI.—Kutter's Co-efficients ($N = \cdot 011$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	71·1	28·5	75·3	39·1	80·3	32·1	87·1	34·8	91·3	36·5	94·1	37·6
·45	77·4	34·9	81·6	36·8	86·6	39	93·1	42	97·5	43·8	100	45·1
·5	83·3	41·6	87·4	43·7	92·5	46·2	99	49·5	103	51·5	106	52·8
·55	88·8	48·8	92·9	51·1	97·9	53·8	104	57·3	108	59·5	111	60·8
·6	94	56·4	98	58·8	103	61·7	109	65·4	113	67·7	115	69·1
·65	98·9	64·2	103	66·8	108	69·8	113	73·7	117	76·1	119	77·6
·7	104	72·4	107	75·1	112	78·2	118	82·3	121	84·7	123	86·2
·8	112	89·5	115	92·3	120	95·6	125	99·8	128	102	130	104
·9	120	108	123	110	127	114	131	118	134	120	136	122
1	126	126	129	129	133	133	137	137	139	139	140	140
1·1	133	146	135	149	138	152	142	156	144	158	145	159
1·2	138	166	141	169	143	172	146	175	148	177	149	178
1·3	144	187	145	189	147	192	150	195	151	197	152	198
1·4	148	208	150	210	151	212	153	215	154	216	155	217
1·5	153	229	154	231	155	233	156	235	157	236	158	237
1·6	157	251	158	252	158	253	159	255	160	256	160	256
1·7	161	273	161	274	162	275	162	275	162	276	162	276
1·8	164	296	164	296	164	296	164	296	165	296	164	296
1·9	168	318	167	318	167	319	167	317	167	316	166	316
2	171	341	170	340	169	339	169	337	168	337	168	336
2·1	174	364	173	363	172	361	171	358	170	357	170	356
2·2	176	388	175	385	174	382	172	379	172	378	171	376
2·3	179	411	177	408	176	404	174	400	173	398	172	397
2·4	181	435	179	431	178	426	176	421	175	419	174	417
2·5	184	459	182	454	179	448	177	442	176	439	175	437
2·6	186	483	183	477	181	471	178	464	177	460	176	458
2·7	188	507	185	500	183	493	180	485	178	481	177	478
2·8	190	531	187	523	184	515	181	506	179	502	178	498
2·9	191	551	188	547	185	537	182	528	180	522	179	519
3	193	580	190	570	187	560	183	549	181	543	180	539

Table XXXII.—Bazin's and Kutter's Co-efficients.

$\sqrt{R}$	Bazin. $\gamma = \cdot 109$		Kutter. $N = \cdot 012$											
			1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	124	49·6	63·2	25·3	66·9	26·7	71·4	28·5	77·4	30·9	81·2	32·5	83·7	33·5
·45	127	57·1	68·9	31·1	72·6	32·6	77·1	34·7	83·1	37·4	86·9	39·1	89·2	40·3
·5	130	64·8	74·3	37·2	78	39	82·5	41·2	88·4	44·2	92·1	46	94·6	47·3
·55	132	72·6	79·3	43·7	83·1	45·6	87·5	48	93·3	51·3	96·9	53·2	99·2	54·5
·6	133	80	84·1	50·5	87·8	52·6	92·1	55·2	97·8	58·7	101	60·7	103	62
·65	135	87·7	88·6	57·6	92·2	59·9	96·4	62·6	102	66·3	105	68·3	107	69·7
·7	136	95·4	92·9	65	96·2	67·5	101	70·3	106	74	109	76·1	111	77·6
·8	139	111	101	80·7	104	83·2	108	86·3	113	90·1	115	92·3	117	93·7
·9	141	126	108	97·2	111	99·8	114	103	119	107	121	109	123	110
1	142	142	114	114	117	117	120	120	124	124	126	126	127	127
1·1	144	145	120	132	123	135	125	138	129	141	130	143	132	145
1·2	144	173	126	151	128	153	130	156	133	159	134	161	135	162
1·3	145	189	131	170	132	172	134	175	137	177	138	179	139	180
1·4	146	205	135	189	137	191	138	193	140	196	141	197	142	198
1·5	147	220	140	209	141	211	142	212	143	214	144	216	144	216
1·6	148	236	144	230	144	231	145	232	146	233	146	234	147	234
1·7	148	252	147	250	147	251	148	251	148	252	149	252	149	253
1·8	149	267	151	271	151	271	151	271	151	271	151	271	151	271
1·9	149	283	154	292	153	292	153	291	153	290	153	290	153	290
2	149	299	157	314	156	312	156	311	155	310	155	309	154	308
2·1	150	301	160	335	159	334	158	331	157	329	156	328	156	327
2·2	150	330	162	357	161	354	160	352	159	349	158	347	157	346
2·3	150	346	165	379	163	376	162	372	160	368	159	366	159	365
2·4	151	362	167	401	165	397	164	393	162	388	161	385	160	384
2·5	151	378	169	423	167	418	165	413	163	408	162	405	161	403
2·6	151	393	171	446	169	440	167	434	164	427	163	424	162	421
2·7	151	408	173	468	171	462	168	455	166	447	164	443	163	440
2·8	152	424	175	491	173	483	170	476	167	467	165	462	164	460
2·9	152	440	177	514	174	505	171	497	168	487	166	482	165	479
3	152	456	179	536	176	527	173	518	169	507	167	501	166	498
3·1	152	472												
3·2	152	487												
3·3	152	503												
3·4	153	519												
3·5	153	535												
3·6	153	550												
3·7	153	566												
3·8	153	582												
3·9	153	598												
4	153	614												

Bazin's co-efficients for higher values of $\sqrt{R}$.

$\sqrt{R} =$	5·0	7·0	8·0
$C =$	154	155	155

Table XXXIII.—Bazin's and Kutter's Co-efficients.

$\sqrt{R}$	Bazin. $\gamma = \cdot 290$		Kutter. $N = \cdot 013$											
			1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	91·8	36·7	56·7	22·7	60	24	64	25·6	69·4	27·8	72·8	29·1	75·2	30·1
·5	99·9	50	66·9	33·5	70·3	35·1	74·3	37·2	79·7	39·8	83	41·5	85·3	42·6
·6	106	63·8	76·0	45·6	79·3	47·6	83·3	50	88·4	53	91·5	54·9	93·6	56·2
·7	115	78	84·2	58·9	87·3	61	91·1	63·8	95·9	67·1	98·8	69·1	101	70·5
·8	116	92·6	91·6	73·3	94·6	75·7	98	78·4	102	81·9	105	84	107	85·4
·9	119	107	98·3	88·4	101	90·9	104	93·7	108	97·3	110	99·4	112	101
1	122	122	104	104	107	107	110	110	113	113	115	115	117	117
1·1	125	137	110	121	112	123	115	126	118	129	119	131	121	133
1·2	127	152	115	138	117	140	119	143	122	146	123	148	124	149
1·3	129	168	120	156	121	158	123	160	125	163	127	164	127	165
1·4	131	183	124	174	126	176	127	172	129	180	129	181	130	182
1·5	132	198	128	193	129	194	130	196	132	197	132	198	133	199
1·6	133	213	132	211	133	212	133	214	134	215	135	216	135	216
1·7	135	229	136	231	136	231	136	232	137	232	137	233	137	233
1·8	136	244	139	250	139	250	139	250	139	250	139	250	139	250
1·9	137	259	142	270	142	269	142	269	141	268	141	268	141	268
2	138	275	145	290	144	289	144	288	143	286	143	286	142	285
2·1	138	290	148	310	147	309	146	307	145	305	145	304	144	303
2·2	139	306	150	331	149	328	148	326	147	323	146	321	146	320
2·3	140	320	153	351	151	348	150	345	148	341	147	339	147	338
2·4	141	346	155	372	154	368	152	364	150	360	149	357	148	355
2·5	141	353	157	393	155	388	153	384	151	378	150	375	149	373
2·6	142	369	159	414	157	409	155	403	153	397	151	393	150	391
2·7	142	384	161	435	159	429	156	422	154	415	152	411	151	409
2·8	143	400	163	457	161	450	158	442	155	434	153	429	152	427
2·9	143	415	165	478	162	470	159	462	156	453	154	448	153	444
3	144	431	167	500	164	491	161	482	157	471	155	466	154	462
3·1	144	447												
3·2	144	462												
3·3	145	478												
3·4	145	493												
3·5	145	509												
3·6	146	525												
3·7	146	540												
3·8	146	556												
3·9	147	572												
4	147	588												

Bazin's co-efficients for higher values of $\sqrt{R}$.

$\sqrt{R}=$	4·5	5·0	6·0	7·0	8·0
$C=$	148	149	150	151	152

TABLE XXXIV.—KUTTER'S CO-EFFICIENTS ($N = \cdot 015$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	46·8	18·7	49·4	19·8	52·7	21·1	57·1	22·9	60	24	62	24·8
·5	55·5	27·8	58·3	29·2	61·6	30·8	66·1	33	68·9	34·4	70·8	35·4
·6	63·4	38	66·1	39·7	69·4	41·7	73·8	44·3	76·4	45·9	78·3	47·9
·7	70·4	49·3	73	51·2	76·2	53·4	80·3	56·2	82·8	58	84·6	59·2
·8	77·1	61·7	79·4	63·7	82·5	66	86·3	69·1	88·6	70·9	90·1	72·1
·9	83·1	74·8	85·4	76·8	88·1	79·3	91·5	82·4	93·6	84·2	94·9	85·4
1	88·6	88·6	90·6	90·6	93·1	93·1	96·1	96·1	97·9	97·9	99·1	99·1
1·1	93·6	103	95·5	105	97·7	107	100	110	102	112	103	113
1·2	98·3	118	99·9	120	102	122	104	125	105	126	106	127
1·3	103	134	104	135	106	137	107	140	109	141	109	142
1·4	107	149	108	150	109	153	111	155	111	156	112	157
1·5	111	166	111	166	112	168	113	170	114	171	114	172
1·6	114	182	115	183	115	184	116	185	116	186	117	187
1·7	117	199	118	200	118	201	118	201	119	202	119	202
1·8	120	217	120	217	120	217	121	217	121	217	121	217
1·9	123	234	123	234	123	233	123	233	122	233	122	232
2	126	252	126	251	125	250	125	249	124	248	124	248
2·1	129	270	128	269	127	267	126	265	126	264	125	263
2·2	131	288	130	286	129	284	128	281	127	280	127	279
2·3	133	307	132	304	131	301	129	298	129	296	128	294
2·4	136	326	134	322	133	318	131	314	130	312	129	310
2·5	138	344	136	340	134	336	132	331	131	328	130	326
2·6	140	363	138	358	136	353	134	347	132	342	131	342
2·7	142	382	140	377	137	371	135	364	133	360	133	358
2·8	143	402	141	395	139	388	136	381	134	376	133	374
2·9	145	421	143	414	140	406	137	397	135	393	134	390
3	147	440	144	432	141	424	138	414	136	409	135	405

TABLE XXXV.—BAZIN'S AND KUTTER'S CO-EFFICIENTS.

$\sqrt{R}$	Bazin. $\gamma=$ ·833		Kutter. $N=$ ·017											
			1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	51·1	20·4	39·6	15·9	41·8	16·7	44·5	17·8	48·2	19·3	50·5	20·2	52·3	20·9
·5	59·1	29·6	47·2	23·6	49·5	24·7	52·3	26·1	56·1	28	58·3	29·2	60·1	30
·6	66·1	39·7	54·2	32·5	56·5	33·9	59·2	35·5	62·9	37·8	65·1	39·1	66·8	40·1
·7	71·9	50·3	60·5	42·4	62·7	43·9	65·4	45·8	69	48·3	71	49·7	72·6	50·8
·8	77·1	61·7	66·4	53·1	68·5	54·8	71·1	56·9	74·3	59·4	76·2	60·9	77·7	62·2
·9	81·7	73·5	71·8	64·6	73·7	66·4	76·1	68·5	79·1	71·2	80·7	72·7	82·1	73·9
1	85·9	85·9	76·7	76·7	78·6	78·6	80·7	80·7	83·3	83·3	84·8	84·8	86	86
1·1	89·6	98·6	81·4	89·5	83	91·2	84·8	93·3	87·2	95·9	88·5	97·3	89·5	98·4
1·2	93·1	112	85·7	103	87·1	104	88·7	106	90·7	109	91·7	110	92·6	111
1·3	96·1	126	89·7	117	90·8	118	92·2	120	93·9	122	94·7	123	95·5	124
1·4	98·8	138	93·4	131	94·4	132	95·4	134	96·8	136	97·4	136	98·1	137
1·5	101	152	96·9	145	97·6	146	98·5	148	99·4	149	99·9	150	100	151
1·6	104	166	100	160	101	161	101	162	102	163	102	164	103	164
1·7	106	180	103	176	104	176	104	177	104	177	104	177	105	178
1·8	108	194	106	191	106	191	106	191	106	191	106	191	106	191
1·9	110	208	109	207	109	207	109	206	108	206	108	205	108	205
2	111	223	112	223	111	222	111	221	110	220	110	219	110	219
2·1	113	237	114	240	113	238	113	237	112	235	111	234	111	233
2·2	114	251	116	256	116	254	115	252	113	250	113	248	112	247
2·3	116	266	119	273	118	270	116	268	115	264	114	262	114	261
2·4	117	281	121	290	120	287	118	283	116	279	115	277	115	276
2·5	118	296	123	307	121	303	120	299	118	294	117	291	116	290
2·6	119	310	125	324	123	320	121	315	119	309	118	306	117	304
2·7	120	325	127	341	125	336	123	331	120	324	119	321	118	319
2·8	121	340	128	359	126	353	124	347	121	339	120	335	119	333
2·9	122	355	130	377	128	370	125	363	122	355	121	350	120	348
3	123	370	132	394	129	387	126	379	123	370	122	365	121	362
3·1	124	385	133	412	130	404	128	395	124	385	122	380	122	377
3·2	125	400	135	430	132	421	129	412	125	401	123	394	122	391
3·3	126	415	136	448	133	439	130	428	126	416	124	409	123	406
3·4	127	430	137	467	134	456	131	444	127	431	125	424	124	420
3·5	127	446	139	485	135	473	132	461	128	447	126	439	124	435
3·6	128	460	140	503	136	491	133	477	129	463	126	454	125	450
3·7	129	476	141	522	137	508	134	494	129	478	127	469	126	464
3·8	129	491	142	540	138	526	134	510	130	493	127	484	126	479
3·9	130	506	143	559	139	544	135	527	131	509	128	499	127	494
4	130	521	144	577	140	561	136	544	131	525	129	514	127	508

Bazin's co-efficients for higher values of R $\left\{\begin{array}{lllll} \sqrt{R}= {}^{4\cdot5}5 & 6 & 7 & 8 \\ C= {}^{133}135 & 138 & 141 & 143. \end{array}\right.$

TABLE XXXVI.—KUTTER'S CO-EFFICIENTS ($N = \cdot 020$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	32	12·8	33·6	13·4	35·7	14·3	38·7	15·5	40·6	16·2	41·9	16·8
·5	38·3	19·2	40·2	20·1	42·3	21·2	45·3	22·7	47·3	23·6	48·6	24·3
·6	44·2	26·5	46	27·6	48·2	28·9	51·2	30·7	53·1	31·9	54·4	32·6
·7	49·6	34·7	51·5	36	53·6	37·5	56·4	39·5	58·2	40·8	59·5	41·6
·8	54·7	43·8	56·3	45·1	58·4	46·7	61·1	48·9	62·8	50·3	64	51·2
·9	59·4	53·4	60·9	54·9	62·9	56·6	65·4	58·8	66·9	60·2	68	61·2
1	63·7	63·7	65·2	65·2	66·9	66·9	69·2	69·2	70·6	70·6	71·5	71·5
1·1	67·8	74·6	69·1	76	70·7	77·7	72·7	79·9	73·8	81·2	74·7	82·3
1·2	71·6	85·9	72·8	87·4	74·1	89	75·8	91	76·9	92·2	77·6	93·1
1·3	75·2	97·7	76·2	99	77·3	101	78·8	102	79·6	104	80·2	104
1·4	78·6	110	79·4	111	80·3	112	81·4	114	82·1	115	82·6	116
1·5	81·7	123	82·3	123	83·1	125	83·9	126	84·4	127	84·8	127
1·6	84·8	136	85·2	136	85·6	137	86·2	138	86·6	139	86·8	139
1·7	87·6	149	87·8	149	88	150	88·4	150	88·5	151	88·6	151
1·8	90·2	162	90·3	163	90·3	163	90·3	163	90·3	163	90·4	163
1·9	92·8	176	92·7	176	92·4	176	92·2	175	92	175	92	175
2	95·2	190	94·8	190	94·4	189	93·9	188	93·6	187	93·5	187
2·1	97·5	205	97	204	96·3	202	95·6	201	95·1	200	94·8	199
2·2	99·7	219	99	218	98·1	216	97·1	214	96·6	212	96·1	211
2·3	102	234	101	232	99·8	230	98·5	227	97·8	225	97·4	224
2·4	104	249	103	246	101	243	99·9	240	99	238	98·5	236
2·5	106	264	104	261	103	257	101	253	100	251	99·6	249
2·6	108	280	106	276	104	271	102	266	101	263	101	262
2·7	109	295	108	290	106	285	104	280	102	276	102	274
2·8	111	310	109	305	107	300	105	293	103	289	102	287
2·9	112	326	110	320	108	314	106	306	104	302	103	300
3	114	342	112	336	109	328	107	320	105	315	104	312
3·1	116	358	113	351	111	343	108	334	106	328	105	325
3·2	117	374	114	366	112	357	109	347	107	342	106	338
3·3	118	390	116	382	113	372	109	361	110	355	106	351
3·4	120	407	117	397	114	386	110	375	108	368	107	364
3·5	121	423	118	413	115	401	111	388	109	381	108	377
3·6	122	439	119	428	116	416	112	402	110	394	108	390
3·7	123	456	120	444	116	431	112	416	110	408	109	403
3·8	124	472	121	460	117	445	113	430	111	421	109	416
3·9	125	489	122	475	118	460	114	444	111	435	110	429
4	127	506	123	491	119	475	114	458	112	448	110	442

Table XXXVII.—Bazin's and Kutter's Co-efficients.

$\sqrt{R}$	Bazin. $\gamma=1{\cdot}54$		Kutter. $N=\cdot0225$											
			1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	32·8	13·1	27·4	11	28·8	11·5	30·5	12·2	33	13·2	34·6	13·8	35·5	14·2
·5	38·6	19·3	33	16·5	34·5	17·3	36·3	18·2	38·8	19·4	40·5	20·3	41·5	20·7
·6	44·2	26·5	38·2	22·9	39·7	23·8	41·6	25	44·1	26·5	45·7	27·4	46·7	28
·7	49·2	34·4	43	30·1	44·5	31·2	46·4	32·5	48·9	34·2	50·4	35·3	51·3	35·9
·8	53·9	43·1	47·5	38	49·0	39·1	50·7	40·6	53·1	42·5	54·5	43·6	55·4	44·3
·9	58	52·2	51·7	46·5	53·1	47·8	54·8	49·3	56·9	51·2	58·3	52·5	59	53·1
1	61·9	61·9	55·7	55·7	57	57	58·5	58·5	60·5	60·5	61·7	61·7	62·3	62·3
1·1	65·7	72·3	59·4	65·3	60·5	66·6	61·9	68·1	63·7	70·1	64·7	71·2	65·3	71·8
1·2	69	82·8	63	75·6	63·9	76·7	65·1	78·1	66·6	79·9	67·5	81	68	81·6
1·3	72·1	93·7	66·2	86·1	67·1	87·2	68·1	88·5	69·3	90·1	70·1	91·1	70·5	91·7
1·4	74·9	105	69·3	97	70	98	70·8	99·1	71·9	101	72·5	102	72·8	102
1·5	77·6	116	72·3	109	72·8	109	73·4	110	74·2	111	74·7	112	74·9	112
1·6	80·3	129	75·1	120	75·5	121	75·9	121	76·4	122	76·7	123	76·8	123
1·7	82·6	140	77·7	132	77·9	133	78·1	133	78·4	133	78·6	134	78·6	134
1·8	84·9	153	80·2	144	80·2	144	80·3	145	80·3	145	80·3	145	80·3	145
1·9	86·9	165	82·6	157	82·5	157	82·3	156	82·1	156	81·9	156	81·9	156
2	88·9	178	84·9	170	84·5	169	84·2	168	83·7	167	83·5	167	83·3	167
2·1	90·9	191	87·1	183	86·6	152	86	181	85·3	179	84·9	178	84·7	172
2·2	92·5	204	89·1	196	88·5	195	87·7	193	86·8	191	86·2	190	85·9	189
2·3	94·3	217	91·1	210	90·2	208	89·3	205	88·1	203	87·5	201	87·1	200
2·4	95·8	230	93	223	92	221	90·8	218	89·5	215	88·7	213	88·3	212
2·5	97·4	244	94·8	237	93·7	234	92·3	231	90·7	227	89·8	225	89·3	223
2·6	99	257	96·6	251	95·2	248	93·7	244	91·9	239	90·9	236	90·3	235
2·7	100	271	98·2	265	96·7	261	95	257	93	251	91·9	248	91·3	247
2·8	102	284	99·8	279	98·2	275	96·3	270	94·1	264	92·8	260	92·2	258
2·9	103	298	101	294	99·6	289	97·5	283	95·1	276	93·7	272	93	270
3	104	312	103	308	101	303	98·6	296	96·1	288	94·6	284	93·8	281
3·1	105	326	104	323	102	317	99·7	309	97	301	95·4	296	94·6	293
3·2	106	340	106	338	103	331	101	323	97·9	313	96·2	308	95·4	305
3·3	107	354	107	353	105	345	102	336	98·7	326	97	320	96·1	317
3·4	109	369	108	368	106	359	103	350	99·5	338	97·7	332	96·7	329
3·5	110	383	110	383	107	374	104	363	100	351	98·4	344	97·4	341
3·6	110	397	111	398	108	388	105	377	101	364	99	356	98	353
3·7	111	411	112	414	109	403	106	390	102	376	99·6	369	98·6	365
3·8	112	426	113	429	110	417	106	404	102	389	100	381	99·1	377
3·9	113	440	114	445	111	432	107	418	103	402	101	393	99·7	389
4	114	455	115	460	112	446	108	432	104	415	101	406	100	401

Bazin's co-efficients for higher values of R $\begin{cases} \sqrt{R}=4{\cdot}5 & 5 \quad 6 \quad 7 \quad 8 \\ C=117 & 121 \quad 125 \quad 129 \quad 132. \end{cases}$

TABLE XXXVIII.—BAZIN'S AND KUTTER'S CO-EFFICIENTS.

$\sqrt{R}$	Bazin. $\gamma=2{\cdot}35$		Kutter. $N={\cdot}025$											
			1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	23·2	92·8	23·9	9·6	25	9·95	26·5	10·6	28·6	11·4	30	12	30·9	12·4
·5	27·6	13·8	28·9	14·4	30·2	15	31·7	15·9	33·9	17	35·3	17·7	36·3	18·2
·6	32	19·2	33·5	20·1	34·9	20·8	36·4	21·8	38·6	23·2	40	24	41	24·6
·7	36	25·2	37·9	26·5	39·2	27·4	40·7	28·5	42·9	30	44·2	30·9	45·2	31·6
·8	40	32	42	33·6	43·2	34·6	44·7	35·8	46·8	37·4	48	38·4	48·9	39·1
·9	43·6	39·2	45·8	41·2	46·9	42·3	48·4	43·6	50·3	45·3	51·5	46·4	52·3	47·1
1	47·1	47·1	49·4	49·4	50·5	50·5	51·8	51·8	53·6	53·6	54·6	54·6	55·4	55·4
1·1	50·2	55·2	52·8	58·1	53·8	59·2	55	60·5	56·6	62·3	57·5	63·3	58·2	64
1·2	53·2	63·8	56	67·3	57	68·3	58	69·6	59·3	71·2	60·1	72·1	60·7	72·8
1·3	56	72·8	59·1	76·8	59·8	77·8	60·7	78·9	61·9	80·5	62·6	81·4	63	81·9
1·4	58·8	82·3	62	86·8	62·6	87·7	63·3	88·6	64·3	90	64·8	90·7	65·2	91·3
1·5	61·3	92	64·7	97·1	65·2	97·8	65·8	98·7	66·5	99·8	66·9	100	67·2	101
1·6	63·8	102	67·3	108	67·7	108	68	109	68·5	110	68·8	110	69	110
1·7	66·1	112	69·8	119	70	119	70·2	119	70·4	120	70·6	120	70·7	120
1·8	68·3	123	72·2	130	72·2	130	72·2	130	72·3	130	72·3	130	72·3	130
1·9	70·3	134	74·4	141	74·3	141	74·1	141	73·9	140	73·8	140	73·8	140
2	72·2	144	76·6	153	76·3	153	76	152	75·6	151	75·3	151	75·1	150
2·1	74·2	156	78·7	165	78·2	164	77·7	163	77·1	162	76·7	161	76·4	160
2·2	76·1	167	80·6	177	80·0	176	79·3	175	78·5	173	78	172	77·7	171
2·3	77·9	179	82·5	190	81·8	188	80·9	186	79·8	184	79·2	182	78·8	181
2·4	79·6	191	84·3	202	83·4	200	82·4	198	81·1	195	80·4	193	79·9	192
2·5	81·2	203	86·1	215	85·0	213	83·8	210	82·3	206	81·5	204	80·9	202
2·6	82·8	215	87·7	228	86·5	225	85·1	221	83·5	217	82·5	215	81·9	213
2·7	84·2	227	89·3	241	87·9	238	86·4	233	84·5	228	83·5	226	82·8	224
2·8	85·6	240	90·9	255	89·3	250	87·6	245	85·6	240	84·4	236	83·7	234
2·9	86·9	252	92·4	268	90·7	263	88·8	256	86·6	251	85·3	247	84·5	245
3	88·1	264	93·8	281	92	276	89·9	270	87·5	263	86·2	259	85·3	256
3·1	89·4	277	95·2	295	93·2	289	91	282	88·4	274	87	270	86	267
3·2	90·7	290	96·5	309	94·4	302	92	294	89·3	286	87·7	281	86·7	277
3·3	91·9	303	97·8	323	95·6	315	93	307	90·1	297	88·5	292	87·4	288
3·4	93·1	317	99	337	96·7	329	94	320	90·9	309	89·2	303	88·1	300
3·5	94·2	330	100	351	97·7	342	94·9	332	91·7	321	89·9	315	88·7	311
3·6	95·3	343	101	365	98·7	355	95·8	345	92·4	333	90·5	326	89·3	322
3·7	96·2	356	103	379	99·7	389	96·6	358	93·1	344	91·1	337	89·9	333
3·8	97·2	369	104	394	101	383	97·5	370	93·8	356	91·7	349	90·4	344
3·9	98·2	383	105	408	102	396	98·2	383	94·4	368	92·3	360	91	355
4	99·2	397	106	423	103	410	99	396	95	380	92·8	371	91·5	366

Bazin's co-efficients for higher values of $R \begin{cases} \sqrt{R} = 4{\cdot}5 \quad 5 \quad 6 \quad 7 \quad 8 \\ C = 103 \quad 107 \quad 113 \quad 118 \quad 122. \end{cases}$

TABLE XXXIX.—KUTTER'S CO-EFFICIENTS ($N = \cdot 0275$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	21·2	8·5	22·2	8·9	23·4	9·4	25·2	10·1	26·4	10·5	27·2	10·9
·5	25·6	12·8	26·7	13·3	28	14	29·9	15	31·2	15·6	32	16
·6	29·8	17·9	30·9	18·5	32·3	19·4	34·2	20·5	35·5	21·3	36·3	21·8
·7	33·8	23·7	34·9	24·5	36·3	25·4	38·1	26·7	39·3	27·5	40·2	28·1
·8	37·5	30	38·6	30·9	39·9	31·9	41·7	33·4	42·8	34·3	43·6	34·9
·9	41	36·9	42·1	37·9	43·2	39	45	40·5	46	41·4	46·8	42·1
1	44·4	44·4	45·3	45·3	46·5	46·5	48	48	49	49	49·6	49·6
1·1	47·5	52·2	48·4	53·2	49·4	54·4	50·8	55·9	51·7	56·8	52·3	57·5
1·2	50·5	60·6	51·3	61·5	52·2	62·6	53·4	64·1	54·1	64·9	54·6	65·6
1·3	53·3	69·3	54	70·2	54·8	71·2	55·8	72·5	56·4	73·4	56·9	73·9
1·4	56	78·4	56·6	79·2	57·2	80·1	58	81·2	58·5	82	58·9	82·4
1·5	58·6	87·9	59	88·5	59·5	89·3	60·1	90·2	60·5	90·8	60·8	91·2
1·6	61	97·7	61·4	98·1	61·7	98·7	62·1	99·4	62·4	99·8	62·5	100
1·7	63·4	108	63·5	108	63·7	108	63·9	109	64·1	109	64·2	109
1·8	65·6	118	65·6	118	65·6	118	65·7	118	65·7	118	65·7	118
1·9	67·8	129	67·6	128	67·5	128	67·3	128	67·2	128	67·1	128
2	69·8	140	69·5	136	69·2	138	68·8	138	68·6	137	68·5	137
2·1	71·7	151	71·3	150	70·9	149	70·3	148	69·9	147	69·7	146
2·2	73·6	162	73·1	161	72·4	159	71·6	158	71·1	157	70·9	156
2·3	75·4	174	74·8	172	73·9	170	72·9	168	72·4	167	72	166
2·4	77·2	185	76·3	183	75·4	181	74·2	178	73·6	177	73·1	175
2·5	78·8	197	77·8	195	76·7	192	75·4	188	74·6	187	74·1	185
2·6	80·4	209	79·3	206	78	203	76·5	199	75·6	197	75	195
2·7	82	221	80·8	218	79·3	214	77·5	209	76·6	207	75·9	205
2·8	83·5	234	82	230	80·5	225	78·6	220	77·6	217	76·8	215
2·9	84·9	246	83·4	242	81·6	237	79·5	231	78·4	227	77·6	225
3	86·3	259	84·6	254	82·7	248	80·4	241	79·2	238	78·4	235
3·1	87·6	272	85·8	266	83·8	260	81·3	252	80	248	79·1	245
3·2	88·9	285	86·9	278	84·8	271	82·2	263	80·7	258	79·8	255
3·3	90·2	298	88·1	291	85·8	283	83	274	81·5	269	80·5	266
3·4	91·4	311	89·2	303	86·7	295	83·8	285	82·2	279	81·1	276
3·5	92·5	324	90·2	316	87·6	307	84·5	296	82·8	290	81·8	286
3·6	93·7	337	91·2	328	88·5	319	85·3	307	83·5	301	82·3	296
3·7	94·8	351	92·2	341	89·3	330	85·9	318	84·1	311	82·9	307
3·8	95·8	364	93·1	354	90·1	342	86·6	329	84·7	322	83·4	317
3·9	96·9	378	94	367	90·9	355	87·3	340	85·2	333	84	328
4	97·9	392	94·9	380	91·6	367	87·9	352	85·8	343	84·5	338

Table XL.—Bazin's and Kutter's Co-efficients.

$\sqrt{R}$	Bazin. $\gamma=3{\cdot}17$		Kutter. $N=\cdot030$													
			1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	17·9	71·6	19	7·6	19·8	7·9	20·9	8·4	22·4	9	23·5	9·4	24·2	9·7
·5	21·5	10·8	23	11·5	24	12	25·1	12·6	26·7	13·4	27·8	13·9	28·6	14·3
·6	25·2	15·1	26·8	16·1	27·8	16·7	29	17·4	30·7	18·4	31·8	19·1	32·5	19·5
·7	28·6	20	30·5	21·3	31·4	22	32·6	22·8	34·3	24	35·3	24·7	36·1	25·3
·8	31·9	25·5	33·9	27·1	34·8	27·8	36	28·8	37·6	30·1	38·6	30·9	39·3	31·4
·9	34·9	31·4	37·1	33·4	38·1	34·3	39·1	35·2	40·6	36·5	41·6	37·4	42·2	38
1	37·8	37·8	40·2	40·2	41·1	41·1	42·1	42·1	43·5	43·5	44·3	44·3	44·9	44·9
1·1	40·6	44·7	43·1	47·4	43·9	48·3	44·8	49·3	46·1	50·7	46·8	51·5	47·4	52·1
1·2	43·3	52	45·9	55·1	46·6	56	47·4	56·9	48·5	58·2	49·2	59	49·7	59·6
1·3	45·8	59·5	48·6	63·1	49·1	64	49·9	64·9	50·8	66	51·4	66·8	51·8	67·3
1·4	48·2	67·5	51·1	71·5	51·5	72·2	52·2	73·1	52·9	74·1	53·4	74·8	53·7	75·2
1·5	50·5	75·8	53·5	80·3	53·9	80·8	54·3	81·5	54·9	82·4	55·2	82·8	55·5	83·3
1·6	52·8	84·5	55·8	89·3	56·1	89·7	56·4	90·2	56·8	90·9	57	91·2	57·2	91·5
1·7	55	145	58	98·6	58·1	98·8	58·3	99·1	58·5	99·5	58·7	99·8	58·7	99·8
1·8	57	103	60·1	108	60·2	108	60·2	109	60·2	108	60·2	108	60·2	108
1·9	59	112	62·2	118	62·1	118	61·9	118	61·8	117	61·6	117	61·6	117
2	61	122	64·1	128	63·9	128	63·6	127	63·2	126	63	126	62·9	126
2·1	62·8	132	66	139	65·6	138	65·2	137	64·6	136	64·3	135	64·1	135
2·2	64·6	142	67·8	149	67·3	149	66·7	147	66	145	65·5	144	65·3	144
2·3	66·3	152	69·5	160	68·9	158	68·1	157	67·2	155	66·7	153	66·3	153
2·4	67·9	163	71·2	171	70·4	169	69·5	167	68·4	164	67·8	163	67·4	162
2·5	69·5	174	72·8	182	71·8	180	70·8	177	69·6	174	68·8	172	68·3	171
2·6	71·1	185	74·3	193	73·4	191	72·1	188	70·7	184	69·8	182	69·3	180
2·7	72·5	196	75·7	204	74·7	202	73·3	198	71·7	194	70·8	191	70·2	190
2·8	73·8	207	77·2	216	76	213	74·5	209	72·7	204	71·7	201	71	199
2·9	75·2	218	78·6	228	77·2	224	75·6	219	73·6	213	72·5	210	71·8	208
3	76·5	230	80	240	78·5	235	76·6	230	74·5	224	73·3	220	72·6	218
3·1	77·9	241	81·3	252	79·6	247	77·7	241	75·4	234	74·1	230	73·3	227
3·2	79·2	253	82·5	264	80·7	258	78·7	252	76·2	244	74·9	240	74	237
3·3	80·3	265	83·7	276	81·8	270	79·6	263	77	254	75·6	250	74·6	246
3·4	81·5	277	84·9	289	82·8	282	80·5	274	77·8	265	76·3	259	75·3	256
3·5	82·6	289	86	301	83·9	294	81·4	285	78·6	275	76·9	269	75·9	266
3·6	83·8	302	87·1	314	84·9	307	82·3	296	79·3	285	77·6	279	76·5	275
3·7	84·7	313	88·2	326	85·8	317	83·1	308	79·9	296	78·2	289	77	285
3·8	85·8	326	89·3	339	86·7	330	83·9	319	80·6	306	78·8	299	77·6	295
3·9	86·8	339	90·3	352	87·6	342	84·7	330	81·2	317	79·3	309	78·1	305
4	87·8	351	91·2	365	88·5	354	85·4	342	81·9	327	79·9	320	78·6	314

Bazin's co-efficients for higher values of $R\left\{\begin{array}{llllll}\sqrt{R}= & 4{\cdot}5 & 5 & 6 & 7 & 8 \\ C= & 92 & 97 & 103 & 108 & 113.\end{array}\right.$

TABLE XLI.—KUTTER'S CO-EFFICIENTS ($N = \cdot 035$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	15·6	6·3	16·3	6·5	17·1	6·84	18·3	7·3	19·1	7·6	19·7	7·9
·5	19	9·5	19·8	9·9	20·7	10·4	21·9	11	22·8	11·4	23·4	11·7
·6	22·3	13·4	13·1	13·8	24	14·4	25·3	15·2	26·2	15·7	26·8	16·1
·7	25·4	17·8	26·2	18·3	27·1	19	28·3	19·8	29·2	20·4	29·9	20·9
·8	28·3	22·7	29·1	23·2	30	24	31·3	25	32·1	25·7	32·7	26·2
·9	31·1	28	31·8	28·6	32·7	29·4	33·9	30·5	34·7	31·2	35·3	31·8
1	33·8	33·8	34·4	34·4	35·3	35·3	36·4	36·4	37·1	37·1	37·6	37·6
1·1	36·4	40	37	40·7	37·7	41·5	38·8	42·7	39·4	43·3	39·8	43·8
1·2	38·8	46·6	39·4	47·2	40	48	40·9	49·1	41·5	49·8	41·9	50·3
1·3	41·1	53·5	41·6	54·1	42·2	54·9	43	55·9	43·5	56·6	43·8	56·9
1·4	43·4	60·8	43·8	61·3	44·3	62	44·9	62·9	45·3	63·4	45·6	63·8
1·5	45·6	68·3	45·9	68·8	46·2	69·3	46·7	70·1	47	70·5	47·2	70·8
1·6	47·6	76·2	47·8	76·6	48·1	77	48·4	77·4	48·6	77·8	48·8	78·1
1·7	49·6	84·3	49·7	84·5	49·9	84·8	50·1	85·2	50·1	85·2	50·2	85·3
1·8	51·5	92·7	51·6	92·8	51·6	92·9	51·6	92·9	51·6	92·9	51·6	92·9
1·9	53·4	101	53·3	101	53·2	101	53	101	52·9	101	52·9	101
2	55·1	110	55	110	54·7	109	54·4	109	54·2	108	54·1	108
2·1	56·9	119	56·5	119	56·2	118	55·7	117	55·4	116	55·2	116
2·2	58·5	129	58·1	128	57·6	127	57	125	56·6	125	56·3	124
2·3	60·1	138	59·6	137	58·9	136	58·2	134	57·7	133	57·4	132
2·4	61·6	148	61	146	60·2	145	59·3	142	58·7	141	58·4	140
2·5	63·1	158	62·4	156	61·5	154	60·4	151	59·7	149	59·3	148
2·6	64·5	168	63·6	166	62·7	163	61·4	160	60·7	158	60·2	157
2·7	65·9	178	64·9	175	63·8	172	62·4	169	61·6	166	61	165
2·8	67·3	188	66·2	185	64·9	182	63·3	177	62·4	175	61·8	173
2·9	68·6	199	67·4	195	66	191	64·2	186	63·2	183	62·6	182
3	69·8	209	68·5	206	67	201	65·1	195	64	192	63·3	190
3·1	71	220	69·6	216	68	211	66	205	64·8	201	64	198
3·2	72·2	231	70·7	226	68·9	221	66·8	214	65·5	210	64·7	207
3·3	73·4	242	71·7	237	69·8	230	67·5	223	66·2	219	65·4	216
3·4	74·5	253	72·8	247	70·7	240	68·3	232	66·9	228	66	224
3·5	75·6	265	73·7	258	71·6	251	69	242	67·5	236	66·6	233
3·6	76·6	276	74·6	269	72·4	261	69·7	251	68·1	245	67·2	242
3·7	77·7	287	75·6	281	73·2	271	70·4	260	68·7	254	67·7	251
3·8	78·7	299	76·5	290	74	281	71	270	69·3	263	68·2	259
3·9	79·6	311	77·4	302	74·7	291	71·6	279	69·9	273	68·8	268
4	80·6	322	78·1	313	75·4	302	72·2	289	70·4	282	69·2	277

TABLE XLII.—MANNING'S CO-EFFICIENTS.

$\sqrt{R}$	Values of Kutter's N.												
	·009	·010	·011	·012	·013	·015	·017	·020	·0225	·025	·0275	·030	·035
·4	121	109	98	91	81	73	64	55	49	44	40	36	31
·5	131	118	106	98	91	79	69	59	52	47	43	39	34
·6	140	125	113	104	97	84	72	63	56	50	45	42	35
·7	147	132	119	110	102	88	80	66	59	53	48	44	38
·8	153	138	124	115	106	92	81	70	61	55	50	46	39
·9	159	143	129	120	111	96	84	72	64	57	52	48	41
1·0	165	149	134	124	114	99	87	74	66	59	54	50	43
1·1	170	153	138	128	118	102	90	77	68	61	56	51	44
1·2	176	158	142	132	122	106	93	79	70	63	58	53	45
1·3	180	162	146	135	125	108	95	81	72	65	59	54	46
1·4	185	167	150	139	128	111	98	84	74	67	61	56	48
1·5	190	170	154	142	131	114	101	86	76	68	62	57	49
1·6	194	173	157	145	134	116	103	87	78	70	63	58	50
1·7	198	178	161	149	137	119	105	89	79	71	65	59	51
1·8	201	180	163	151	140	121	108	91	81	73	66	61	52
1·9	205	184	166	154	142	123	109	92	82	74	67	62	53
2·0	208	187	169	156	144	125	110	94	83	75	68	62	54
2·1	212	190	171	159	147	127	113	95	85	76	69	64	54
2·2	215	193	174	161	149	129	114	97	86	78	70	65	55
2·3	218	196	177	163	151	131	116	98	87	79	71	66	56
2·4	221	199	179	166	153	133	117	100	88	80	72	66	57
2·5	224	202	182	168	155	135	119	101	90	81	73	67	58
2·6	227	204	184	170	157	136	120	102	91	82	74	68	58
2·7	230	207	186	172	159	138	122	104	92	83	75	69	59
2·8	233	209	189	174	161	140	123	105	93	84	76	70	60
2·9	235	212	191	177	163	141	125	106	94	85	77	71	60
3·0	238	214	193	179	165	143	126	107	95	86	78	71	61
3·1	241	217	195	180	167	144	128	108	96	87	79	72	62
3·2	243	219	197	183	168	146	129	110	97	88	80	73	63
3·3	246	221	199	184	170	147	130	111	98	88	81	74	63
3·4	248	223	201	186	172	149	132	112	99	89	81	74	64
3·5	251	226	203	188	174	150	133	113	100	90	82	75	65
3·6	253	228	205	190	175	152	134	114	101	91	83	76	65
3·7	255	230	207	192	177	153	135	115	102	92	84	77	66
3·8	258	232	209	193	178	154	137	116	103	93	84	77	66
3·9	260	234	211	195	180	156	138	117	104	94	85	78	67
4·0	262	236	212	197	181	157	139	118	105	94	86	79	67

TABLES OF SECTIONAL DATA.

Rectangular and Trapezoidal Sections.

For a bed-width intermediate to those given it is only necessary, in order to find A, to multiply D by the difference in width and add or subtract the result. Thus, for bed 43 ft., slope ½ to 1, and depth 3·75 ft., $A = 175{\cdot}8 - 3{\cdot}75 \times 2 = 168{\cdot}3$: $\sqrt{R}$ changes so slowly that the correct figure can be interpolated by inspection. For the above section it is 1·81.

Widths outside the range of the tables.—To find $\sqrt{R}$ for a width W and depth D, look out $\sqrt{R}$ for width $\frac{W}{4}$ and depth $\frac{D}{4}$ and multiply by 2, or for $\frac{W}{9}$ and $\frac{D}{9}$ and multiply by 3. Interpolations can also be made on this principle. For instance, the figures for a bed of 12·5 feet can be found from those for a 50-feet bed.

For side-slopes of 4 *to* 3 *and* 3 *to* 4.—A and $\sqrt{R}$ are the same respectively as for a rectangular section and a ½ to 1 section of the same mean width. Thus for a channel of bed 21 feet, side-slopes 4 to 3, and depth 3 feet, the mean width is 25 feet, and $A = 75$, $\sqrt{R} = 1{\cdot}56$. For a bed-width of 11 feet, side-slopes 3 to 4, and depth 4 feet, the mean width is 14 feet, which is the same as for a channel with bed 12 feet, side-slopes ½ to 1, and depth 4 feet. $A = 56$ and $\sqrt{R} = 1{\cdot}64$. These rules can be conveniently applied when the mean widths are whole numbers. For other cases interpolations can be used.

For streams of very shallow section (W very great in proportion to D) $\sqrt{R}$ is nearly independent of the ratio of the side-slopes, and depends practically on the mean width only.

Table XLIII.—Sectional Data for Open Channels.

Rectangular Sections.

Depth of Water.	Bed 1 foot.		Bed 2 feet.		Bed 3 feet.		Bed 4 feet.		Bed 5 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	·5	·5	1	·56	1·5	·61	2	·63	2·5	·65
·75	·75	·55	1·5	·66	2·25	·71	3	·74	3·75	·76
1	1	·58	2	·71	3	·77	4	·82	5	·85
1·25	1·25	·6	2·5	·74	3·75	·83	5	·88	6·25	·91
1·5	1·5	·61	3	·78	4·5	·87	6	·93	7·5	·97
1·75	1·75	·62	3·5	·8	5·25	·9	7	·97	8·75	1·01
2	2	·63	4	·82	6	·93	8	1	10	1·05
2·25	2·25	·64	4·5	·83	6·75	·95	9	1·03	11·25	1·09
2·5	2·5	·65	5	·84	7·5	·97	10	1·05	12·5	1·12
2·75	2·75	·65	5·5	·86	8·25	·99	11	1·08	13·75	1·14
3	3	·66	6	·87	9	1	12	1·1	15·	1·17
3·25	...	...	6·5	·87	9·75	1·01	13	1·11	16·25	1·19
3·5	...	...	7	·88	10·5	1·02	14	1·13	17·5	1·21
3·75	...	...	7·5	·89	11·25	1·03	15	1·14	18·75	1·23
4	...	...	8	·89	12	1·04	16	1·15	20	1·24
4·25	...	...	...	...	12·75	1·05	17	1·17	21·25	1·25
4·5	...	...	...	...	13·5	1·07	18	1·19	22·5	1·27
4·75	...	...	...	...	14·25	1·07	19	1·19	23·75	1·28
5	...	...	...	...	15	1·07	20	1·2	25	1·29

TABLE XLIII.—*Continued.* (*Rectangular.*)

Depth of Water.	Bed 6 feet.		Bed 7 feet.		Bed 8 feet.		Bed 10 feet.		Bed 12 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	3	·65	3·5	·66	4	·67	5	·67	6	·68
·75	4·5	·78	5·25	·79	6	·8	7·5	·81	9	·82
1	6	·87	7	·88	8	·8	10	·91	12	·93
1·25	7·5	·94	8·75	·96	10	·83	12·5	1	15	1·02
1·5	9	1	10·5	1·03	12	1·04	15	1·07	18	1·1
1·75	10·5	1·05	12·25	1·08	14	1·1	17·5	1·14	21	1·17
2	12	1·1	14	1·13	16	1·15	20	1·2	24	1·22
2·25	13·5	1·13	15·75	1·17	18	1·2	22·5	1·25	27	1·28
2·5	15	1·17	17·5	1·21	20	1·24	25	1·29	30	1·33
2·75	16·5	1·2	19·25	1·24	22	1·28	27·5	1·33	33	1·37
3	18	1·23	21	1·27	24	1·31	30	1·37	36	1·41
3·25	19·5	1·25	22·75	1·3	26	1·34	32·5	1·4	39	1·45
3·5	21	1·28	24·5	1·32	28	1·37	35	1·43	42	1·48
3·75	22·5	1·3	26·25	1·35	30	1·39	37·5	1·46	45	1·52
4	24	1·31	28	1·37	32	1·41	40	1·49	48	1·55
4·25	25·5	1·33	29·75	1·39	34	1·44	42·5	1·52	51	1·58
4·5	27	1·34	31·5	1·4	36	1·46	45	1·54	54	1·6
4·75	28·5	1·36	33·25	1·42	38	1·47	47·5	1·56	57	1·63
5	30	1·37	35	1·44	40	1·49	50	1·58	60	1·65
5·25	31·5	1·39	36·75	1·45	42	1·51	52·5	1·6	63	1·67
5·5	33	1·39	38·5	1·46	44	1·52	55	1·62	66	1·69
5·75	34·5	1·4	40·25	1·48	46	1·54	57·5	1·64	69	1·71
6	36	1·41	42	1·49	48	1·55	60	1·65	72	1·73
6·25	...	...	...	...	...	...	62·5	1·67	75	1·75
6·5	...	...	...	...	...	...	65	1·68	78	1·77
6·75	...	...	...	...	...	...	67·5	1·69	81	1·78
7	...	...	...	...	...	...	70	1·71	84	1·8
7·25	...	...	...	...	...	...	72·5	1·72	87	1·81
7·5	...	...	...	...	...	...	75	1·73	90	1·83
7·75	...	...	...	...	...	...	77·5	1·74	93	1·84
8	...	...	...	...	...	...	80	1·75	96	1·85

TABLE XLIII.—*Continued.* (*Rectangular.*)

Depth of Water.	Bed 14 feet.		Bed 16 feet.		Bed 18 feet.		Bed 20 feet.		Bed 25 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	7	·68	8	·69	9	·73	10	·69	12·5	·68
·75	10·5	·82	12	·83	13·5	·83	15	·84	18·8	·84
1	14	·94	16	·94	18	·95	20	·95	25	·96
1·25	17·5	1·03	20	1·04	22·5	1·05	25	1·05	31·3	1·07
1·5	21	1·12	24	1·12	27	1·13	30	1·14	37·5	1·16
1·75	24·5	1·18	28	1·2	31·5	1·21	35	1·22	43·8	1·24
2	28	1·25	32	1·27	36	1·28	40	1·29	50	1·31
2·25	31·5	1·3	36	1·33	40·5	1·34	45	1·36	56·3	1·38
2·5	35	1·36	40	1·38	45	1·4	50	1·41	62·5	1·44
2·75	38·5	1·4	44	1·43	49·5	1·45	55	1·47	68·8	1·5
3	42	1·45	48	1·48	54	1·5	60	1·52	75	1·56
3·25	45·5	1·49	52	1·52	58·5	1·55	65	1·57	81·3	1·61
3·5	49	1·53	56	1·56	63	1·59	70	1·61	87·5	1·65
3·75	52·5	1·56	60	1·6	67·5	1·63	75	1·65	93·8	1·7
4	56	1·6	64	1·63	72	1·66	80	1·69	100	1·75
4·25	59·5	1·63	68	1·67	76·5	1·7	85	1·73	106·3	1·78
4·5	63	1·66	72	1·7	81	1·73	90	1·76	112·5	1·82
4·75	66·5	1·68	76	1·73	85·5	1·76	95	1·79	118·8	1·82
5	70	1·71	80	1·76	90	1·79	100	1·83	125	1·89
5·25	73·5	1 73	84	1·78	94·5	1·82	105	1·86	131·3	1·92
5·5	77	1·76	88	1·8	99	1·85	110	1·89	137·5	1·95
5·75	80·5	1·78	92	1·83	103·5	1·87	115	1·91	143·8	1·99
6	84	1·8	96	1·85	108	1·9	120	1·94	150	2·02
6·25	87·5	1·82	100	1·87	112·5	1·92	125	1·96	156·3	2·04
6·5	91·	1·84	104	1·89	117	1·94	130	1·98	162·5	2·07
6·75	94·5	1·85	108	1·91	121·5	1·96	135	2·01	168·8	2·09
7	98	1·87	112	1·93	126	1·98	140	2·03	175	2·11
7·25	101·5	1·89	116	1·95	130·5	2	145	2·05	181·3	2·14
7·5	105	1·9	120	1·97	135	2·02	150	2·07	187·5	2·17
7·75	108·5	1·92	124	1·98	139·5	2·04	155	2·09	193·3	2·19
8	112	1·93	128	2	144	2·06	160	2·11	200	2·21
8·25	...	...	...	...	148·5	2·07	165	2·13	206·3	2·23
8·5	...	..	...	...	153	2·09	170	2·14	212·5	2·25
8·75	...	...	...	...	157·5	2·11	175	2·16	218·8	2·27
9	...	...	...	...	162	2·12	180	2·18	225	2·29
9·25	...	...	...		166·5	2·14	185	2·19	231·3	2·31
9·5	...	...	...	...	171	2·15	190	2·21	237·5	2·32
9·75	...	...	...	...	175·5	2·16	195	2·22	243·8	2·34
10	...	...	...	...	180	2·18	200	2·24	250	2·36

TABLE XLIII.—*Continued.* (*Rectangular.*)

Depth of Water.	Bed 30 feet.		Bed 35 feet.		Bed 40 feet.		Bed 50 feet.		Bed 60 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	30	·97	35	·97	40	·98	50	·98	60	·98
1·5	35	1·17	52·5	1·18	60	1·18	75	1·19	90	1·2
2	60	1·33	70	1·34	80	1·35	100	1·36	120	1·37
2·25	67·5	1·39	78·8	1·41	90	1·42	112·5	1·44	135	1·45
2·5	75	1·46	87·5	1·48	100	1·49	125	1·51	150	1·52
2·75	82·5	1·53	96·3	1·54	110	1·56	137·5	1·57	165	1·59
3	90	1·58	105	1·6	120	1·62	150	1·64	180	1·65
3·25	97·5	1·63	113·8	1·66	130	1·67	162·5	1·7	195	1·71
3·5	105	1·68	122·5	1·71	140	1·73	175	1·75	210	1·77
3·75	112·5	1·73	131·3	1·75	150	1·76	187·5	1·81	225	1·83
4	120	1·78	140	1·78	160	1·83	200	1·86	240	1·88
4·25	127·5	1·82	148·8	1·85	170	1·87	212·5	1·91	255	1·93
4·5	135	1·86	157·5	1·89	180	1·92	225	1·95	270	1·98
4·75	142·5	1·9	166·3	1·93	190	1·96	237·5	2	285	2·03
5	150	1·94	175	1·99	200	2	250	2·04	300	2·07
5·25	157·5	1·97	183·8	2·01	210	2·04	262·5	2·08	315	2·11
5·5	165	2·01	192·5	2·04	220	2·08	275	2·12	330	2·16
5·75	172·5	2·04	201·3	2·08	230	2·11	287·5	2·16	345	2·2
6	180	2·07	210	2·11	240	2·15	300	2·2	360	2·24
6·25	187·5	2·1	218·8	2·15	250	2·18	312·5	2·24	375	2·27
6·5	195	2·13	227·5	2·18	260	2·22	325	2·27	390	2·31
6·75	202·5	2·16	236·3	2·21	270	2·25	337·5	2·31	405	2·35
7	210	2·18	245	2·24	280	2·28	350	2·34	420	2·38
7·25	217·5	2·21	253·8	2·26	290	2·31	362·5	2·37	435	2·42
7·5	225	2·24	262·5	2·29	300	2·34	375	2·4	450	2·45
7·75	232·5	2·26	271·3	2·32	310	2·37	387·5	2·43	465	2·48
8	240	2·28	280	2·34	320	2·39	400	2·46	480	2·51
8·25	247·5	2·31	288·8	2·37	330	2·42	412·5	2·49	495	2·54
8·5	255	2·33	297·5	2·39	340	2·44	425	2·52	510	2·57
8·75	262·5	2·35	306·3	2·42	350	2·47	437·5	2·55	525	2·6
9	270	2·37	315	2·44	360	2·49	450	2·57	540	2·63
9·25	277·5	2·39	323·8	2·46	370	2·52	462·5	2·6	555	2·66
9·5	285	2·41	332·5	2·48	380	2·54	475	2·62	570	2·69
9·75	292·5	2·43	341·3	2·5	390	2·56	487·5	2·65	585	2·71
10	300	2·45	350	2·52	400	2·58	500	2·67	600	2·74
10·5	...	...	...	...	...	...	525	2·72	630	2·79
11	...	...	...	...	...	...	550	2·76	660	2·84
11·5	...	...	...	...	...	...	575	2·81	690	2·88
12	...	...	...	...	...	...	600	2·85	720	2·93

TABLE XLIII.—*Continued.* (*Rectangular.*)

Depth of Water.	Bed 70 feet.		Bed 80 feet.		Bed 90 feet.		Bed 100 feet.		Bed 120 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	70		80		90		100		120	
1·5	105		120		135		150		180	
2	140		160		180		200		240	
2·25	157·5		180		202·5		225		270	
2·5	175		200		225		250		300	
2·75	192·5		220		247·5		275		330	
3	210		240		270		300		360	
3·25	227·5		260		292·5		325		390	
3·5	245		280		315		350		420	
3·75	262·5		300		337·5		375		450	
4	280		320		360		400		480	
4·25	297·5		340		382·5		425		510	
4·5	315		360		405		450		540	
4·75	332·5		380		427·5		475		570	
5	350		400		450		500		600	
5·25	367·5		420		472·5		525		630	
5·5	385		440		495		550		660	
5·75	402·5		460		517·5		575		690	
6	420		480		540		600		720	
6·25	437·5		500		562·5		625		750	
6·5	455		520		585		650		780	
6·75	472·5		540		607·5		675		810	
7	490		560		630		700		840	
7·25	507·5		580		652·5		725		870	
7·5	525		600		675		750		900	
7·75	542·5		620		697·5		775		930	
8	560		640		720		800		960	
8·25	577·5		660		742·5		825		990	
8·5	595		680		765		850		1020	
8·75	612·5		700		787·5		875		1050	
9	630		720		810		900		1080	
9·25	647·5		740		832·5		925		1110	
9·5	665		760		855		950		1140	
9·75	682·5		780		877·5		975		1170	
10	700		800		900		1000		1200	
10·5	735		840		945		1050		1260	
11	770		880		990		1100		1320	
11·5	805		920		1035		1150		1380	
12	840		960		1080		1200		1440	

Table XLIV.—Sectional Data for Open Channels.

Trapezoidal Sections—Side-slopes ½ to 1.

Depth of Water.	Bed 1 foot.		Bed 2 feet.		Bed 3 feet.		Bed 4 feet.		Bed 5 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet. ·5	·63	·54	1·13	·60	1·63	·63	2·13	·64	2·63	·65
·75	1·03	·62	1·78	·69	2·53	·73	3·28	·76	4·03	·77
1	1·5	·68	2·5	·77	3·5	·82	4·5	·85	5·5	·87
1·25	2·03	·73	3·28	·83	4·53	·88	5·78	·92	7·03	·95
1·5	2·63	·78	4·13	·88	5·63	·94	7·13	·98	8·63	1·02
1·75	3·28	·82	5·03	·92	6·78	·99	8·53	1·04	10·28	1·08
2	4	·86	6	·96	8	1·03	10	1·09	12	1·13
2·25	4·78	·89	7·03	1	9·28	1·07	11·53	1·13	13·78	1·17
2·5	5·63	·92	8·13	1·03	10·63	1·11	13·13	1·17	15·63	1·21
2·75	6·53	·95	9·28	1·07	12·03	1·15	14·78	1·21	17·53	1·25
3	7·5	·99	10·5	1·1	13·5	1·18	16·5	1·24	19·5	1·29
3·25	...	...	11·78	1·13	15·03	1·21	18·28	1·27	21·53	1·33
3·5	...	...	13·13	1·16	16·63	1·24	20·13	1·30	23·63	1·36
3·75	...	...	14·53	1·18	18·28	1·27	22·03	1·33	25·78	1·39
4	...	...	16	1·21	20	1·29	24	1·36	28	1·42
4·25	...	...	...	...	21·78	1·32	26·03	1·39	30·28	1·45
4·5	...	...	...	...	23·63	1·35	28·13	1·41	32·63	1·47
4·75	...	...	...	...	25·53	1·37	30·28	1·44	35·03	1·5
5	...	...	...	...	27·5	1·39	32·5	1·46	37·5	1·52

TABLE XLIV.—*Continued.* ($\frac{1}{2}$ to 1.)

Depth of Water.	Bed 6 feet.		Bed 7 feet.		Bed 8 feet.		Bed 9 feet.		Bed 10 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet										
·5	3·13	·66	3·63	·67	4·13	·67	4·63	·68	5·13	·68
·75	4·78	·78	5·53	·79	6·28	·8	7·03	·81	7·78	·81
1	6·5	·89	7·5	·9	8·5	·91	9·5	·92	10·5	·93
1·25	8·28	·97	9·53	·99	10·78	1	12·03	1·01	13·28	1·02
1·5	10·13	1·04	11·63	1·06	13·13	1·07	14·63	1·09	16·13	1·1
1·75	12·03	1·1	13·78	1·12	15·53	1·14	17·28	1·16	19·03	1·17
2	14	1·16	16	1·18	18	1·2	20	1·22	22	1·23
2·25	16·03	1·21	18·28	1·23	20·53	1·25	22·78	1·28	25·03	1·29
2·5	18·13	1·25	20·63	1·28	23·13	1·3	25·63	1·33	28·13	1·34
2·75	20·28	1·29	23·03	1·32	25·78	1·35	28·53	1·38	31·28	1·39
3	22·5	1·33	25·5	1·36	28·5	1·39	31·5	1·42	34·5	1·44
3·25	24·78	1·37	28·03	1·4	31·28	1·43	34·53	1·46	37·78	1·48
3·5	27·13	1·4	30·63	1·44	34·13	1·47	37·63	1·5	41·13	1·52
3·75	29·23	1·43	33·28	1·47	37·03	1·5	40·78	1·54	44·53	1·56
4	32	1·46	36	1·5	40	1·54	44	1·57	48	1·59
4·25	34·53	1·49	38·78	1·53	43·03	1·57	47·28	1·6	51·53	1·63
4·5	37·13	1·52	41·63	1·56	46·13	1·6	50·63	1·63	55·13	1·66
4·75	39·78	1·55	44·53	1·59	49·28	1·63	54·03	1·66	58·78	1·69
5	42·5	1·57	47·5	1·62	52·5	1·65	57·5	1·69	62·5	1·72
5·25	45·28	1·6	50·53	1·65	55·78	1·68	61·03	1·72	65·28	1·75
5·5	48·13	1·62	53·63	1·67	59·13	1·71	64·63	1·74	70·13	1·77
5·75	51·03	1·65	56·78	1·69	62·53	1·73	68·28	1·77	74·03	1·8
6	54	1·67	60	1·71	66	1·76	72	1·79	78	1·82
6·25	...	...	...	...	...	...	...	...	82·03	1·85
6·5	...	...	...	...	...	...	...	...	86·12	1·87
6·75	..	...	...	...	...	...	...	...	90·28	1·9
7	...	...	...	...	...	...	...	...	94·5	1·92
7·25	...	...	...	...	...	...	...	...	98·78	1·94
7·5	...	...	...	...	...	...	...	...	103·1	1·96
7·75	...	...	...	...	...	...	...	...	107·53	1·98
8	...	...	...	...	...		...	...	112	2

TABLE XLIV.—*Continued.* (½ to 1.)

Depth of Water.	Bed 12 feet.		Bed 14 feet.		Bed 16 feet.		Bed 18 feet.		Bed 20 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	6·1	·68	7·1	·69	8·1	·69	9·1	·69	10·13	·69
·75	9·3	·82	10·8	·83	12·3	·83	13·8	·84	15·28	·84
1	12·5	·94	14·5	·94	16·5	·95	18·5	·96	20·5	·96
1·25	15·8	1·03	18·3	1·05	20·8	1·05	23·3	1·06	25·8	1·06
1·5	19·1	1·12	22·1	1·13	25·1	1·14	28·1	1·15	31·1	1·15
1·75	22·5	1·19	26	1·2	29·5	1·22	33·3	1·23	36·5	1·23
2	26	1·26	30	1·27	34	1·29	38	1·3	42	1·31
2·25	29·5	1·32	34	1·33	38·5	1·35	43	1·37	47·5	1·38
2·5	33·1	1·37	38·1	1·39	43·1	1·41	48·1	1·43	53·1	1·44
2·75	36·8	1·42	42·3	1·45	47·8	1·47	53·3	1·49	58·8	1·5
3	40·5	1·47	46·5	1·5	52·5	1·52	58·5	1·54	64·5	1·55
3·25	44·3	1·52	50·8	1·55	57·3	1·57	63·8	1·59	70·3	1·6
3·5	48·1	1·56	55·1	1·59	62·1	1·61	69·1	1·64	76·1	1·65
3·75	52	1·6	59·5	1·63	67	1·66	74·5	1·68	82	1·7
4	56	1·64	64	1·67	72	1·7	80	1·72	88	1·74
4·25	60	1·67	68·5	1·71	76	1·74	84·5	1·76	94	1·79
4·5	64·1	1·7	73·1	1·74	82·1	1·78	91·1	1·8	100·1	1·83
4·75	68·3	1·74	77·8	1·78	87·3	1·81	96·8	1·84	106·3	1·86
5	72·5	1·77	82·5	1·81	92·5	1·84	102·5	1·87	112·5	1·9
5·25	76·8	1·8	87·3	1·84	97·8	1·88	108·3	1·91	118·8	1·94
5·5	81·1	1·83	92·1	1·87	103·1	1·91	114·1	1·94	125·1	1·97
5·75	85·5	1·86	97	1·9	108·5	1·94	120	1·97	131·5	2
6	90	1·88	102	1·93	114	1·97	126	2	138	2·03
6·25	94·5	1·91	107	1·96	119·5	2	132	2·03	144·5	2·06
6·5	99·1	1·93	112·1	1·98	125·1	2·02	138·1	2·06	151·1	2·09
6·75	103·8	1·96	117·3	2·01	130·8	2·05	144·3	2·09	157·8	2·12
7	108·5	1·98	122·5	2·03	136·5	2·08	150·5	2·11	164·5	2·15
7·25	113·3	2·01	127·8	2·06	142·3	2·11	156·8	2·14	171·3	2·18
7·5	118·1	2·03	133·1	2·08	148·1	2·13	163·1	2·17	178·1	2·2
7·75	123	2·05	138·5	2·1	154	2·15	169·5	2·19	185	2·23
8	128	2·07	144	2·12	160	2·17	176	2·21	192	2·25
8·25	...	...	...	...	...	...	182·5	2·24	199	2·28
8·5	...	...	...	...	...	...	191·2	2·26	208·2	2·3
8·75	...	...	...	...	...	...	195·8	2·28	213·3	2·32
9	...	...	...	...	...	...	202·5	2·3	220·5	2·34
9·25	...	...	...	...	...	...	209·3	2·33	227·8	2·37
9·5	...	...	...	...	...	...	216·1	2·35	235·1	2·39
9·75	...	...	...	...	...	...	223	2·37	242·5	2·41
10	...	...	...	...	...	...	230	2·39	250	2·43

TABLE XLIV.—*Continued.* (½ to 1.)

Depth of Water.	Bed 25 feet.		Bed 30 feet.		Bed 35 feet.		Bed 40 feet.		Bed 45 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	25·5	·97	30·5	·97	35·5	·98	40·5	·98	45·5	·98
1·5	38·6	1·17	46·1	1·18	53·6	1·18	61	1·19	68·6	1·19
2	52	1·33	62	1·34	72	1·35	82	1·36	92	1·36
2·25	58·8	1·4	70	1·41	81·3	1·42	92·5	1·43	103·8	1·44
2·5	65·6	1·46	78·1	1·48	90·6	1·49	103·2	1·5	115·6	1·51
2·75	72·5	1·52	86·3	1·54	100	1·56	113·8	1·57	127·5	1·58
3	79 5	1·58	94·5	1·6	109·5	1·62	124·5	1·63	139·5	1·64
3·25	86·5	1·64	102·8	1·66	119	1·63	135·3	1·69	151·5	1·7
3·5	93·6	1·69	111·1	1·71	123·6	1·73	146·1	1·75	163·6	1·76
3·75	100·8	1·74	119·5	1·76	138·3	1·79	157	1·8	175·8	1·82
4	108	1·78	128	1·81	148	1·84	168	1·85	188	1·87
4·25	115·3	1·83	136·5	1·86	157·8	1·89	179	1·9	200·3	1·92
4·5	122·6	1·87	145·1	1·9	167·6	1·93	190·1	1·95	212·6	1·96
4·75	130	1·91	153·8	1·95	177·5	1·97	201·3	2	225	2·01
5	137·5	1·95	162·5	1·99	187·5	2·01	212·5	2·04	237·5	2·06
5·25	145	1·99	171·3	2·03	197·5	2·05	223·8	2·08	250	2·1
5·5	152·6	2·02	180·1	2·06	207·6	2·1	235·1	2·12	262·6	2·14
5·75	160·3	2·06	189	2·1	217·8	2·14	246·5	2·16	275·3	2·18
6	168	2·09	198	2·14	228	2·17	258	2·2	288	2·22
6·25	175·8	2·12	207	2·17	238·3	2·21	269·5	2·24	300·8	2·26
6·5	183·6	2·15	216·1	2·2	248·6	2·24	281·1	2·27	313·6	2·3
6·75	191·6	2·19	225·3	2·24	259·1	2·28	292·8	2·31	326·6	2·34
7	199·5	2·22	234·5	2·27	269·5	2·31	304·5	2·34	339·5	2·37
7·25	207·5	2·25	243·8	2·3	280	2·34	316·3	2·37	352·5	2·4
7·5	215·6	2·27	253·1	2·33	290·6	2·37	328·1	2·4	365·6	2·43
7·75	223·8	2·3	262·5	2·36	301·3	2·4	340	2·44	378·8	2·47
8	232	2·33	272	2·38	312	2·43	352	2·47	392	2·5
8·25	240·3	2·36	281·5	2·41	322·8	2·46	364	2·5	405·3	2·53
8·5	248·6	2·38	291·1	2·44	333·6	2·49	376·1	2·52	418·6	2·56
8·75	257	2·4	300·8	2·47	344·6	2·52	388·3	2·55	432·1	2·59
9	265·5	2·43	310·5	2·49	355·5	2·54	400·5	2·58	445·5	2·62
9·25	274·1	2·45	320·3	2·52	366·6	2·57	412·8	2·61	459·1	2·64
9·5	282·6	2·47	330·1	2·54	377·6	2·59	425·1	2·63	472·6	2·67
9·75	291·3	2·5	340	2·57	388·8	2·62	437·5	2·66	486·3	2·7
10	300	2·52	350	2·59	400	2·64	450	2·69	500	2·72
10·5	...	...	...	...	...	...	475·1	2·74	527·6	2·78
11	...	...	...	...	...	...	500·5	2·78	555·5	2·83
11·5	...	...	...	...	...	...	526·1	2·83	583·6	2·87
12	...	...	...	...	...	...	552	2·87	612	2·92

TABLE XLIV.—*Continued.* (½ to 1.)

Depth of Water.	Bed 50 feet.		Bed 60 feet.		Bed 70 feet.		Bed 80 feet.		Bed 90 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	51·5	·98	60·5	·99	70·5	·99	80·5	·99	90·5	·99
1·5	76·1	1·19	91·1	1·2	106·1	1·21	121·1	1·21	136·1	1·21
2	102	1·37	122	1·38	142	1·38	162	1·38	182	1·39
2·25	115	1·45	137·5	1·46	160	1·46	182·5	1·47	205	1·47
2·5	128·1	1·52	153·1	1·53	178·1	1·54	203·1	1·54	228·1	1·54
2·75	141·3	1·59	168·8	1·6	196·3	1·61	224·8	1·61	252·3	1·62
3	154·5	1·65	184·5	1·66	214·5	1·67	244·5	1·68	274·5	1·68
3·25	167·8	1·71	200·3	1·73	232·8	1·74	265·3	1·74	297·8	1·75
3·5	181·1	1·77	216·1	1·78	251·1	1·8	286·1	1·8	321·1	1·81
3·75	194·5	1·83	232	1·84	269·5	1·86	307	1·86	344·5	1·87
4	208	1·88	248	1·9	288	1·91	328	1·92	368	1·93
4·25	221·5	1·93	264	1·96	306·5	1·96	349	1·98	391·5	1·99
4·5	235·1	1·98	280·1	2	325·1	2·02	370·1	2·03	415·1	2·04
4·75	248·8	2·03	296·3	2·05	343·8	2·07	391·3	2·08	438·8	2·09
5	262·5	2·07	312·5	2·1	362·5	2·11	412·5	2·13	462·5	2·14
5·25	276·3	2·12	328·8	2·15	381·3	2·16	433·8	2·18	486·3	2·19
5·5	290·1	2·16	345·1	2·18	400·1	2·2	455·1	2·22	510·1	2·23
5·75	304	2·2	361·5	2·23	419	2·25	476·5	2·26	534	2·28
6	318	2·24	378	2·27	438	2·29	498	2·31	558	2·32
6·25	332	2·28	394·5	2·31	457	2·33	519·5	2·35	582	2·37
6·5	346·1	2.32	411·1	2·35	476·1	2·37	541·1	2·39	606·1	2·41
6·75	360·3	2·36	427·8	2·39	495·3	2·41	562·8	2·43	630·3	2·45
7	374·5	2·39	444·5	2·42	514·5	2·45	584·5	2·47	654·5	2·49
7·25	388·8	2·43	461·3	2·46	533·8	2·49	606·3	2·51	678·8	2·53
7·5	403·1	2·46	478·1	2·5	553·1	2·52	628·1	2·55	703·1	2·57
7·75	417·5	2·49	495	2·53	572·5	2·56	650	2·59	727·5	2·6
8	432	2·52	512	2·56	592	2·6	672	2·62	752	2·64
8·25	446·5	2·55	529	2·59	611·5	2·63	694	2·66	776·5	2·68
8·5	461·1	2·58	546·1	2·63	631·1	2·66	716·1	2·69	701·1	2·71
8·75	475·8	2·61	563·3	2·66	650·8	2·7	738·3	2·73	825·8	2·75
9	490·5	2·64	580·5	2·69	670·5	2·73	760·5	2·76	850·5	2·78
9·25	505·3	2·67	597·8	2·72	690·3	2·76	782·8	2·79	875·3	2·81
9·5	520·1	2·7	615·1	2·75	710·1	2·79	805·1	2·82	900·1	2·84
9·75	535	2·73	632·5	2·78	730	2·82	827·5	2·85	925	2·88
10	550	2·76	650	2·81	750	2·85	850	2·88	950	2·91
10·5	580·1	2·81	685·1	2·86	790·1	2·91	895·1	2·94	1000	2·97
11	610·5	2·86	720·5	2·92	830·5	2·96	940·5	3	1050	3·03
11·5	641·1	2·91	756·1	2·97	871·1	3·02	986·1	3·05	1101	3·08
12	672	2·96	792	3·02	912	3·07	1032	3·11	1152	3·14

TABLE XLIV.—*Continued.* (½ to 1.)

Depth of Water.	Bed 100 feet.		Bed 120 feet.		Bed 140 feet.		Bed 160 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.								
1	100·5	·99	120·5	·99	140·5	·99	160·5	·99
1·5	151·1	1·21	181·1	1·21	211·1		241·1	1·4
2	202	1·39	242	1·39	282	1·4	322	1·56
2·25	227·5	1·47	272·5	1·47				
2·5	253·1	1·55	303·1	1·55	353·1	1·56	403·1	
2·75	278·8	1·62	333·8	1·62	388·8	1·63	443·8	1·63
3	304·5	1·69	364·5	1·69	424·5	1·7	484·5	1·7
3·25	330·3	1·76	395·3	1·76	460·3	1·77	525·3	1·77
3·5	356·1	1·82	426·1	1·82	496·1	1·83	566·1	1·83
3·75	382	1·88	457	1·88	532	1·89	607	1·9
4	408	1·94	488	1·94	568	1·95	648	1·96
4·25	434	1·99	519	2	604	2·01	689	2·02
4·5	460·1	2·04	550·1	2·05	640·1	2·06	730·1	2·07
4·75	486·3	2·1	581·3	2·11	676·3	2·12	771·3	2·13
5	512·5	2·15	612·5	2·16	712·5	2·17	812·5	2·18
5·25	538·8	2·2	643·8	2·21	748·8	2·22	853·8	2·23
5·5	565·1	2·24	675·1	2·25	785·1	2·26	895·1	2·28
5·75	591·5	2·29	706·5	2·3	821·5	2·31	936·5	2·32
6	618	2·33	738	2·35	858	2·36	978	2·37
6·25	644·5	2·38	769·5	2·4	894·5	2·41	1020	2·42
6·5	671·1	2·42	801·1	2·44	931·1	2·45	1061	2·46
6·75	697·8	2·46	832·8	2·48	967·8	2·5	1103	2·51
7	724·5	2·5	864·5	2·52	1005	2·54	1145	2·55
7·25	751·3	2·54	896·3	2·56	1041	2·58	1186	2·59
7·5	778·1	2·58	928·1	2·6	1078	2·62	1228	2·63
7·75	805	2·62	960	2·64	1115	2·66	1270	2·68
8	832	2·66	992	2·68	1152	2·7	1312	2·72
8·25	859	2·69	1024	2·72	1189	2·74	1354	2·76
8·5	886·1	2·73	1056	2·75	1226	2·78	1396	2·79
8·75	913·3	2·76	1088	2·79	1263	2·82	1438	2·83
9	940·5	2·8	1121	2·83	1301	2·85	1481	2·86
9·25	967·8	2·83	1153	2·86	1338	2·89	1523	2·9
9·5	995·1	2·86	1185	2·89	1375	2·92	1565	2·93
9·75	1023	2·9	1118	2·93	1413	2·96	1608	2·97
10	1050	2·93	1250	2·96	1450	2·99	1650	3
10·5	1105	2·99	1315	3·03	1525	3·05	1735	3·07
11	1161	3·05	1381	3·09	1601	3·12	1821	3·13
11·5	1216	3·11	1446	3·15	1676	3·18	1906	3·2
12	1272	3·17	1512	3·21	1752	3·24	1992	3·27

Table XLV.—Sectional Data for Open Channels

Trapezoidal Sections—Side-slopes 1 to 1.

Depth of Water.	Bed 1 foot.		Bed 2 feet.		Bed 3 feet.		Bed 4 feet.		Bed 5 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	·75	·577	1·25	·605	1·75	·629	2·25	·645	2·75	·655
·75	1·31	·652	2·06	·707	2·81	·741	3·56	·763	4·31	·779
1	2	·723	3	·788	4	·828	5	·856	6	·875
1·25	2·81	·787	4·06	·856	5·31	·901	6·56	·933	7·81	·956
1·5	3·75	·846	5·25	·917	6·75	·965	8·25	1	9·75	1·03
1·75	4·81	·899	6·56	·971	8·31	1·02	10·06	1·06	11·81	1·09
2	6	·95	8	1·02	10	1·08	12	1·12	14	1·15
2·25	7·31	·996	9·56	1·07	11·81	1·12	14·06	1·17	16·31	1·2
2·5	8·75	1·04	11·25	1·11	13·75	1·17	16·25	1·21	18·75	1·25
2·75	10·32	1·08	13·06	1·16	15·81	1·21	18·56	1·26	21·31	1·29
3	12	1·13	15	1·2	18	1·25	21	1·3	24	1·33
3·25	...	...	17·06	1·24	20·31	1·29	23·56	1·34	26·81	1·37
3·5	...	...	19·25	1·27	22·75	1·33	26·25	1·38	29·75	1·41
3·75	...	...	21·56	1·31	25·31	1·36	29·06	1·41	32·81	1·45
4	...	...	24	1·34	28	1·4	32	1·45	36	1·49
4·25	...	..	...	...	30·81	1·43	35·06	1·48	39·31	1·52
4·5	...	...	...	...	33·75	1·47	38·25	1·51	42·75	1·55
4·75	...	...	...	...	36·81	1·5	41·56	1·54	46·32	1·59
5	...	...	...	...	40	1·53	45	1·58	50	1·62

TABLE XLV.—*Continued.* (1 to 1.)

Depth of Water.	Bed 6 feet.		Bed 7 feet.		Bed 8 feet.		Bed 9 feet.		Bed 10 feet.	
	A	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$
Feet.										
·5	3·25	·662	3·75	·667	4·25	·672	4·63	·667	3·25	·678
·75	5·06	·781	5·81	·798	6·56	·805	7·03	·795	8·06	·815
1	7	·891	8	·902	9	·911	10	·919	11	·926
1·25	9·06	·975	10·31	·989	11·56	1	12·81	1·01	14·06	1·02
1·5	11·25	1·05	12·75	1·07	14·25	1·08	15·75	1·09	17·25	1·1
1·75	13·56	1·11	15·31	1·13	17·06	1·15	18·81	1·16	20·56	1·17
2	16	1·17	18	1·19	20	1·21	22	1·23	24	1·24
2·25	18·56	1·23	20·81	1·25	23·06	1·27	25·31	1·28	27·56	1·29
2·5	21·23	1·28	23·75	1·3	26·25	1·32	28·75	1·33	31·25	1·35
2·75	24·06	1·32	26·81	1·35	29·56	1·37	32·31	1·39	35·06	1·40
3	27	1·37	30	1·39	33	1·42	36	1·44	39	1·45
3·25	30·06	1·41	33·31	1·43	35·56	1·44	39·81	1·48	43·06	1·5
3·5	33·25	1·45	36·75	1·47	40·25	1·51	43·75	1·52	47·25	1·54
3·75	36·56	1·48	40·31	1·51	44·06	1·54	47·81	1·56	51·56	1·58
4	40	1·52	44	1·55	48	1·58	52	1·6	56	1·62
4·25	43·56	1·56	47·81	1·59	52·06	1·61	56·31	1·64	60·56	1·66
4·5	47·25	1·59	51·75	1·62	56·25	1·65	60·75	1·67	65·25	1·69
4·75	51·06	1·62	55·81	1·65	60·56	1·68	65·31	1·71	70·06	1·73
5·5	55	1·65	60	1·68	65	1·71	70	1·74	75	1·76
5·25	59·06	1·68	64·31	1·72	69·56	1·75	74·81	1·77	80·06	1·8
5	63·25	1·71	68·75	1·75	74·25	1·78	79·75	1·8	85·25	1·83
5·75	67·57	1·74	73·32	1·78	79·07	1·81	84·82	1·83	90·57	1·86
6	77	1·77	78	1·8	84	1·83	90	1·86	96	1·89
6·25	...	...	...	...	...	...	95·31	1·89	101·56	1·92
6·5	...	...	...	...	...	...	100·7	1·92	107·2	1·94
6·75	. .	...	...	...	...		106·3	1·94	113·05	1·97
7	...	...	...	...	...	...	112	1·97	119	2
7·25	...	...	...	...	...	...	117·8	2	125·05	2·03
7·5	...	...	...	...	...	...	123·8	2·02	131·3	2·06
7·75	...	. .	...	...	...	...	129·8	2·05	137·55	2·08
8	...	...	...	...	...	...	136	2·07	144	2·11

TABLE XLV.—*Continued.* (1 to 1.)

Depth of Water.	Bed 12 feet.		Bed 14 feet.		Bed 16 feet.		Bed 18 feet.		Bed 20 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	6·25	·682	7·37	·683	8·37	·686	9·25	·477	10·25	·692
·75	9·56	·823	11·34	·824	12·84	·828	14·06	·694	15·56	·839
1	13	·936	15	·944	17	·95	19	·955	21	·959
1·25	16·56	1·03	19·06	1·04	21·56	1·05	24·06	1·06	26·56	1·06
1·5	20·25	1·12	23·25	1·13	26·25	1·14	29·25	1·15	32·25	1·15
1·75	24·06	1·19	27·56	1·21	31·06	1·22	34·56	1·23	38·06	1·24
2	28	1·26	32	1·28	36	1·29	40	1·3	44	1·31
2·25	32·06	1·32	36·56	1·34	41·06	1·35	45·56	1·37	50·06	1·38
2·5	36·25	1·38	41·25	1·4	46·25	1·42	51·25	1·43	56·25	1·44
2·75	40·56	1·43	46·06	1·45	51·56	1·47	57·06	1·49	62·56	1·5
3	45	1·48	51	1·51	57	1·53	63	1·54	69	1·56
3·25	49·56	1·53	56·06	1·56	62·56	1·58	69·06	1·59	75·56	1·61
3·5	54·25	1·57	61·25	1·6	68·25	1·62	75·25	1·64	82·25	1·66
3·75	59·06	1·62	66·56	1·65	74·06	1·67	81·56	1·69	89·06	1·71
4	64	1·66	72	1·69	80	1·71	88	1·73	96	1·75
4·25	69·06	1·7	77·56	1·73	86·06	1·75	94·56	1·77	103·1	1·79
4·5	74·25	1·73	83·25	1·77	92·25	1·79	101·3	1·81	110·3	1·84
4·75	79·56	1·77	89·06	1·8	98·56	1·83	108·1	1·85	117·6	1·88
5	85	1·8	95	1·84	105	1·87	115	1·89	125	1·91
5·25	90·56	1·84	101·1	1·87	111·6	1·9	122·1	1·93	132·6	1·95
5·5	96·25	1·87	107·3	1·91	118·3	1·94	129·3	1·96	140·3	1·99
5·75	102·1	1·9	113·6	1·94	125·1	1·97	136·6	2	148·1	2·02
6	108	1·93	120	1·97	132	2	144	2·03	156	2·05
6·25	114·1	1·96	126·6	2	139·1	2·03	151·6	2·06	164·1	2·09
6·5	120·2	1·99	133·3	2·03	146·3	2·06	159·3	2·09	172·3	2·12
6·75	126·6	2·02	140·1	2·06	153·6	2·09	167·1	2·12	180·6	2·15
7	133	2·05	147	2·09	161	2·12	175	2·15	189	2·18
7·25	139·6	2·07	154·1	2·11	168·6	2·15	183·1	2·18	197·6	2·21
7·5	146·3	2·1	161·3	2·14	176·3	2·18	191·3	2·21	206·3	2·24
7·75	153·1	2·13	168·6	2·17	184·1	2·21	199·6	2·24	215·1	2·27
8	160	2·15	176	2·19	192	2·23	208	2·26	224	2·29
8·25		...	...	...	200·1	2·26	216·6	2·29	233·1	2·32
8·5	...	...	...	...	208·3	2·28	225·3	2·32	242·3	2·35
8·75	...	...	...	...	216·6	2·31	234·1	2·34	251·6	2·37
9	...	...	...	...	225	2·33	243	2·37	261	2·4
9·25	...	...	...	...	233·6	2·35	252·1	2·39	270·6	2·42
9·5	...	...	...	...	242·3	2·38	261·3	2·41	280·3	2·45
9·75	...	...	...	...	251·1	2·4	270·6	2·44	290·1	2·47
10	...	...	...	...	260	2·42	280	2·46	300	2·49

TABLE XLV.—*Continued.* (1 to 1.)

Depth of Water.	Bed 25 feet.		Bed 30 feet.		Bed 35 feet.		Bed 40 feet.		Bed 45 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	26	·966	31	·976	36	·976	41	·978	46	·981
1·5	39·75	1·17	47·25	1·18	54·75	1·18	62·25	1·19	69·75	1·19
2	54	1·33	64	1·34	74	1·35	84	1·36	94	1·36
2·25	61·31	1·4	72·56	1·41	83·81	1·42	95·06	1·43	106·3	1·44
2·5	68·75	1·46	81·25	1·47	93·75	1·49	106·3	1·5	118·8	1·51
2·75	76·31	1·53	90·06	1·54	103·8	1·56	117·6	1·57	131·3	1·58
3	84	1·58	99	1·6	114	1·62	129	1·63	144	1·64
3·25	91·81	1·64	108·1	1·66	124·3	1·68	140·6	1·69	156·8	1·7
3·5	99·75	1·69	117·3	1·77	134·8	1·73	152·3	1·75	169·8	1·76
3·75	107·8	1·74	126·6	1·77	145·3	1·79	164·1	1·8	182·8	1·81
4	116	1·79	136	1·81	156	1·84	176	1·85	196	1·87
4·25	124·3	1·83	145·6	1·86	166·8	1·88	188·1	1·9	209·3	1·92
4·5	132·8	1·88	155·3	1·91	177·8	1·93	200·3	1·95	222·8	1·96
4·75	141·3	1·92	165·1	1·95	188·8	1·97	212·6	1·99	236·3	2·01
5	150	1·96	175	1·99	200	2·02	225	2·04	250	2·06
5·25	158·8	1·97	185·1	2·03	211·3	2·06	237·6	2·08	263·8	2·1
5·5	167·8	2·03	195·3	2·07	222·8	2·1	250·3	2·12	277·8	2·14
5·75	176·8	2·07	205·6	2·11	234·3	2·14	263·1	2·16	291·8	2·18
6	186	2·11	216	2·15	246	2·18	276	2·2	306	2·22
6·25	195·3	2·14	226·6	2·18	257·8	2·21	289·1	2·24	320·3	2·26
6·5	204·8	2·17	237·3	2·21	269·8	2·25	302·3	2·28	334·8	2·3
6·75	214·3	2·2	248·1	2·25	281·8	2·28	315·6	2·31	349·3	2·34
7	224	2·24	259	2·28	294	2·32	329	2·34	364	2·37
7·25	233·8	2·27	270·1	2·31	306·3	2·35	342·6	2·37	378·8	2·41
7·5	243·8	2·3	281·3	2·34	318·8	2·38	356·3	2·41	393·8	2·44
7·75	253·8	2·33	292·6	2·37	331·3	2·41	370·1	2·45	408·8	2·47
8	264	2·35	304	2·4	344	2·44	384	2·48	424	2·5
8·25	274·4	2·38	315·6	2·43	356·9	2·47	398·1	2·51	489·4	2·54
8·5	284·8	2·41	327·3	2·46	369·8	2·5	412·3	2·54	454·8	2·57
8·75	295·4	2·44	339·1	2·49	382·9	2·53	426·6	2·57	470·4	2·6
9	306	2·46	351	2·52	396	2·56	441	2·6	486	2·62
9·25	316·9	2·49	363·1	2·54	409·4	2·59	455·6	2·62	501·9	2·66
9·5	327·8	2·51	375·3	2·57	422·8	2·61	470·3	2·65	517·8	2·68
9·75	338·9	2·54	387·6	2·6	436·4	2·64	485·1	2·68	533·9	2·71
10	350	2·56	400	2·62	450	2·67	500	2·71	550	2·74
10·5	...	...	...	...	...	...	530·3	2·76	582·8	2·79
11	...	...	...	...	...	...	561	2·81	616	2·85
11·5	...	...	...	...	...	..	593·3	2·86	650·8	2·9
12	...	...	...	...	...	...	624	2·91	684	2·94

TABLE XLV.—*Continued.* (1 to 1.)

Depth of Water.	Bed 50 feet.		Bed 60 feet.		Bed 70 feet.		Bed 80 feet.		Bed 90 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	51	·982	61	·985	71	·987	81	·989	91	·99
1·5	77·25	1·19	92·25	1·2	107·3	1·2	...	...	...	...
2	104	1·37	124	1·39	144	1·35	164	1·38	184	1·39
2·25	117·6	1·44	140·1	1·45	162·6	1·46	185·1	1·46	207·6	1·47
2·5	131·3	1·52	156·3	1·53	181·3	1·53	206·3	1·54	231·3	1·54
2·75	145·1	1·58	172·6	1·6	200·1	1·6	227·6	1·61	255·1	1·61
3	159	1·65	189	1·66	219	1·67	249	1·68	279	1·68
3·25	173·1	1·71	205·6	1·72	238·1	1·73	270·6	1·74	303·1	1·75
3·5	187·3	1·77	222·3	1·78	257·3	1·79	292·3	1·8	327·3	1·81
3·75	201·6	1·82	239·1	1·84	276·6	1·85	314·1	1·86	351·6	1·87
4	216	1·88	256	1·9	296	1·91	336	1·92	376	1·93
4·25	230·6	1·93	273·1	1·95	315·6	1·96	358·1	1·97	400·6	1·98
4·5	245·3	1·98	290·3	2	335·3	2·01	380·3	2·03	425·3	2·03
4·75	260·1	2·03	307·6	2·05	355·1	2·06	402·6	2·08	450·1	2·09
5	275	2·07	325	2·1	375	2·11	425	2·13	475	2·14
5·25	290·1	2·12	342·6	2·14	395·1	2·16	447·6	2·17	500·1	2·18
5·5	305·3	2·16	360·3	2·18	415·3	2·2	470·3	2·22	525·3	2·23
5·75	320·6	2·2	378·1	2·23	435·6	2·25	493·1	2·26	550·6	2·28
6	336	2·24	396	2·27	456	2·29	516	2·31	576	2·32
6·25	351·6	2·28	414·1	2·31	476·6	2·33	539·1	2·35	601·6	2·36
6·5	367·3	2·32	433·3	2·35	497·3	2·37	562·3	2·39	627·3	2·41
6·75	383·1	2·35	450·6	2·39	518·1	2·41	585·6	2·43	653·1	2·45
7	399	2·39	469	2·42	539	2·45	609	2·47	679	2·49
7·25	415·1	2·43	487·6	2·46	560·1	2·49	632·6	2·51	705·1	2·53
7·5	437·3	2·46	506·3	2·5	581·3	2·52	656·3	2·55	731·3	2·56
7·75	447·6	2·49	525·1	2·53	602·6	2·56	680·1	2·58	757·6	2·6
8	464	2·53	544	2·57	624	2·6	704	2·62	784	2·64
8·25	480·6	2·56	563·1	2·6	645·6	2·63	728·1	2·65	810·6	2·67
8·5	497·3	2·59	582·3	2·63	667·3	2·66	752·3	2·69	837·3	2·71
8·75	514·1	2·62	601·6	2·66	689·1	2·7	776·6	2·72	864·1	2·74
9	531	2·65	621	2·7	711	2·73	801	2·76	891	2·78
9·25	548·1	2·68	640·6	2·73	733·1	2·76	825·6	8·79	918·1	2·81
9·5	565·3	2·71	660·3	2·76	755·3	2·79	850·3	2·82	945·3	2·84
9·75	582·6	2·74	680·1	2·79	777·6	2·82	875·1	2·85	972·6	2·88
10	600	2·77	700	2·82	800	2·85	900	2·88	1000	2·91
10·5	635·3	2·82	740·3	2·87	845·3	2·91	950·3	2·94	1055	2·97
11	671	2·87	781	2·93	891	2·97	1001	3	1111	3·03
11·5	708·3	2·93	823·3	2·98	938·3	3·02	1053	3·06	1168	3·09
12	744	2·98	864	3·03	984	3·08	1104	3·11	1224	3·14

TABLE XLV.—*Continued.* (1 to 1.)

Depth of Water.	Bed 100 feet.		Bed 120 feet.		Bed 140 feet.		Bed 160 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.								
1	101	·991	121	·992	141	·993	161	·994
2	204	1·39	224	1·39	284	1·4	324	1·4
2·25	230·1	1·47	275·1	1·48	320·1	1·47	360·1	1·48
2·5	256·3	1·55	306·3	1·55	356·3	1·56	406·3	1·56
2·75	282·6	1·62	337·6	1·63	392·6	1·63	447·6	1·64
3	309	1·69	369	1·7	429	1·7	489	1·7
3·25	335·6	1·75	400·6	1·76	465·6	1·77	530·6	1·77
3·5	362·3	1·82	432·3	1·82	502 3	1·83	572·3	1·84
3·75	389·1	1·88	464·1	1·89	539·1	1·89	614·1	1·9
4	416	1·99	496	1·94	576	1·95	656	1·96
4·25	443·1	1·99	528·1	2	613·1	2·01	698·1	2·01
4·5	470·3	2·04	560·3	2·05	650·3	2·06	740·3	2·07
4·75	497·6	2·09	592·6	2·11	687·6	2·12	782·6	2·12
5	525	2·15	625	2·16	725	2·17	825	2·18
5·25	552·6	2·19	657·6	2·21	762·6	2·22	867·6	2·23
5·5	580·3	2·24	690·3	2·26	800·3	2·27	910·3	2·28
5·75	608·1	2·29	723·1	2·3	838·1	2·32	953·1	2·33
6	636	2·33	756	2·35	876	2·36	996	2·37
6·25	664·1	2·38	789·1	2·39	914·1	2·41	1039	2·41
6·5	692·3	2·42	822·3	2·44	952·3	2·45	1082	2·46
6·75	720·6	2·46	855·6	2·48	990·6	2·5	1126	2·51
7	749	2·5	889	2·52	1029	2·54	1169	2·55
7·25	777·6	2 54	922·6	2·56	1068	2·58	1213	2·59
7·5	806·3	2·58	956·3	2·6	1106	2·62	1256	2·63
7·75	835·1	2·62	990·1	2·64	1145	2·66	1300	2·67
8	864	2·65	1024	2 68	1184	2·7	1344	2·71
8·25	893·1	2·69	1058	2·72	1223	2·74	1386	2·75
8·5	922·3	2·73	1092	2·75	1262	2·77	1432	2·79
8·75	951·6	2·76	1127	2·79	1302	2·81	1477	2·83
9	981	2·8	1161	2·83	1341	2·85	1521	2·86
9·25	1011	2·83	1196	2·86	1381	2·88	1566	2·9
9·5	1040	2·86	1230	2·89	1420	2·92	1610	2·94
9·75	1070	2·9	1265	2·93	1460	2·95	1655	2·97
10	1100	2·93	1300	2·96	1500	2·99	1700	3·01
10·5	1160	2·99	1370	3·03	1580	3·05	1790	3·07
11	1221	3·05	1441	3·09	1661	3·12	1881	3·14
11·5	1282	3·11	1512	3·15	1642	3·18	1972	3·2
12	1344	3·17	1584	3·21	1824	3·25	2064	3·26

Table XLVI.—Sectional Data for Open Channels.

Trapezoidal Sections—Side-slopes $1\frac{1}{2}$ to 1.

Depth of Water.	Bed 1 feet.		Bed 2 feet.		Bed 3 feet.		Bed 4 feet.		Bed 5 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	·87	·56	1·38	·6	1·88	·63	2·38	·64	2·875	·64
·75	1·59	·65	2·34	·71	3·09	·73	3·84	·76	4·59	·77
1	2·5	·74	3·5	·79	4·5	·83	5·5	·85	6·5	·87
1·25	3·59	·81	4·84	·86	6·09	·9	7·34	·93	8·59	·95
1·5	4·48	·87	6·37	·93	7·87	·97	9·37	1	10·87	1·02
1·75	6·34	·93	8·09	·99	9·84	1·03	11·59	1·06	13·34	1·09
2	8	·99	10	1·04	12	1·08	14	1·12	16	1·15
2·25	9·84	1·04	12·09	1·09	14·34	1·14	16·59	1·17	18·84	1·2
2·5	11·87	1·09	14·37	1·14	16·87	1·19	19·37	1·22	21·87	1·25
2·75	14·09	1·14	16·84	1·19	19·59	1·23	22·34	1·27	25·09	1·3
3	16·5	1·18	19·50	1·23	22·5	1·28	25·5	1·31	28·5	1·34
3·25	...	...	22·34	1·28	25·6	1·32	28·84	1·36	32·09	1·39
3·5	...	...	25·37	1·32	28·87	1·36	32·37	1·4	35·87	1·43
3·75	...	..	28·6	1·36	32·34	1·4	36·09	1·44	39·84	1·47
4	...	...	32	1·39	36	1·44	40	1·47	44	1·51
4·25	...	...	...	...	39·84	1·48	44·09	1·51	48·34	1·53
4·5	...	...	...	...	43·87	1·51	48·37	1·55	52·87	1·58
4·75	...	...	...	...	48·09	1·55	52·84	1·58	57·59	1·61
5	...	...	...	...	52·5	1·58	57·5	1·62	62·5	1·64

TABLE XLVI.—*Continued.* ($1\frac{1}{2}$ to 1.)

Depth of Water.	Bed 6 feet.		Bed 7 feet.		Bed 8 feet.		Bed 9 feet.		Bed 10 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	3·37	·66	3·87	·67	4·37	·67	4·88	·68	5·38	·68
·75	5·34	·78	6·09	·79	6·84	·8	7·59	·81	8·34	·81
1	7·5	·89	8·5	·89	9·5	·9	10·5	·91	11·5	·92
1·25	9·84	·97	11·09	·98	12·34	·99	13·59	1	14·84	1·01
1·5	12·37	1·04	13·87	1·06	15·37	1·07	16·88	1·08	18·38	1·09
1·75	15·09	1·11	16·84	1·12	18·59	1·14	20·34	1·15	22·09	1·16
2	18	1·17	20	1·18	22	1·2	24	1·22	26	1·23
2·25	21·09	1·23	23·34	1·24	25·59	1·26	27·84	1·28	30·09	1·29
2·5	24·37	1·28	26·87	1·3	29·37	1·31	31·88	1·33	34·38	1·34
2·75	27·84	1·33	30·59	1·35	33·34	1·36	36·09	1·38	38·84	1·39
3	31·5	1·37	34·5	1·39	37·5	1·41	40·5	1·43	43·5	1·44
3·25	35·34	1·41	38·59	1·44	41·84	1·46	45·09	1·48	48·34	1·49
3·5	39·37	1·45	42·87	1·48	46·37	1·5	49·88	1·52	53·38	1·54
3·75	43·59	1·49	47·34	1·52	51·09	1·54	54·84	1·56	58·59	1·58
4	48	1·53	52	1·56	56	1·58	60	1·6	64	1·62
4·25	52·59	1·57	56·54	1·59	61·09	1·62	65·34	1·64	69·59	1·66
4·5	57·37	1·6	61·87	1·63	66·37	1·65	70·88	1·68	75·38	1·7
4·75	62·34	1·64	67·09	1·66	71·84	1·69	76·59	1·71	81·34	1·74
5	67·5	1·67	72·5	1·7	77·5	1·72	82·5	1·75	87·5	1·77
5·25	72·84	1·71	78·09	1·73	88·34	1·76	88·59	1·78	93·84	1·8
5·5	78·37	1·74	83·87	1·77	89·37	1·79	94·87	1·81	100·4	1·83
5·75	84·09	1·77	89·84	1·8	95·59	1·83	101·34	1·85	107·1	1·87
6	90	1·81	96	1·83	102	1·85	108	1·88	114	1·9
6·25	...	...	...	...	...	...	114·8	1·91	121·1	1·93
6·5	...	...	...	...	...	...	121·9	1·94	128·4	1·96
6·75	...	...	...	...	...	...	129·1	1·97	135·9	1·99
7	...	...	...	...	...	...	136·5	2	143·5	2·02
7·25	...	...	...	...	...	...	144·1	2·03	151·4	2·05
7·5	...	...	...	...	...	...	151·9	2·05	159·4	2·07
7·75	...	...	...	...	...	...	159·8	2·08	167·6	2·1
8	...	...	...	...	...	...	168	2·11	176	2·13

TABLE XLVI.—*Continued.* (1½ to 1.)

Depth of Water.	Bed 12 feet.		Bed 14 feet.		Bed 16 feet.		Bed 18 feet.		Bed 20 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
·5	6·37	·68	7·37	·68	8·37	·69	...	...	...	...
·75	9·84	·82	11·34	·82	12·84	·83	14·34	·83	15·8	·84
1	13·5	·93	15·5	·93	17·5	·94	19·5	·95	21·5	·95
1·25	17·34	1·02	19·84	1·04	22·34	1·04	24·84	1·05	27·34	1·05
1·5	21·38	1·11	24·37	1·12	27·37	1·13	30·37	1·14	33·37	1·15
1·75	25·59	1·18	29·09	1·2	32·59	1·21	36·09	1·22	39·59	1·23
2	30	1·25	34	1·26	38	1·28	42	1·29	46	1·3
2·25	34·59	1·31	39·09	1·33	43·59	1·34	48·09	1·36	52·59	1·37
2·5	39·38	1·37	44·37	1·39	49·37	1·4	54·37	1·42	59·37	1·43
2·75	44·34	1·42	49·84	1·44	55·34	1·46	60·84	1·48	66·34	1·49
3	49·5	1·47	55·5	1·5	61·5	1·51	67·50	1·53	73·5	1·54
3·25	54·84	1·52	61·34	1·55	67·84	1·56	74·34	1·58	80·84	1·6
3·5	60·38	1·57	67·37	1·59	74·37	1·61	81·37	1·63	88·37	1·65
3·75	66·09	1·61	73·59	1·64	81·09	1·66	88·59	1·68	96·09	1·69
4	72	1·65	80	1·68	88	1·7	96	1·72	104	1·73
4·25	78·09	1·69	86·59	1·72	95·09	1·74	103·6	1·76	112·1	1·78
4·5	84·38	1·73	93·37	1·76	102·4	1·78	111·4	1·8	120·4	1·82
4·75	90·84	1·76	100·3	1·79	109·8	1·82	119·3	1·84	128·8	1·86
5	97·5	1·8	107·5	1·83	117·5	1·86	127·5	1·88	137·5	1·9
5·25	104·3	1·83	114·8	1·86	125·3	1·89	135·8	1·91	146·3	1·94
5·5	111·4	1·87	122·4	1·9	133·4	1·93	144·4	1·95	155·4	1·97
5·75	118·6	1·9	130·1	1·93	141·6	1·96	153·1	1·98	164·6	2·01
6	126	1·94	138	1·97	150	2	182	2·02	174	2·04
6·25	133·6	1·96	146·1	2	158·6	2·03	171·1	2·05	183·6	2·08
6·5	141·4	2	154·4	2·03	167·4	2·06	180·4	2·09	193·4	2·11
6·75	149·4	2·02	162·9	2·06	176·4	2·09	189·9	2·12	203·4	2·14
7	157·5	2·05	171·5	2·09	185·5	2·12	199·5	2·15	213·5	2·17
7·25	165·9	2·08	180·4	2·12	194·9	2·15	209·4	2·18	223·9	2·2
7·5	174·4	2·11	189·5	2·15	204·4	2·18	219·4	2·21	234·4	2·23
7·75	183·1	2·14	198·6	2·17	214·1	2·21	229·6	2·24	245·1	2·26
8	192	2·17	208	2·2	224	2·24	240	2·27	256	2·28
8·25	...	...	...	...	...	...	250·6	2·3	267·1	2·31
8·5	...	...	...	...	...	...	261·4	2·32	278·4	2·34
8·75	...	...	...	...	...	...	272·3	2·34	289·8	2·37
9	...	...	...	...	...	...	283·5	2·37	301·5	2·4
9·25	...	...	...	...	...	...	294·8	2·4	313·3	2·42
9·5	...	...	...	...	...	...	306·4	2·42	325·4	2·45
9·75	...	...	...	...	...	...	318·1	2·45	337·6	2·47
10	...	...	...	...	...	...	330	2·47	350	2·5

TABLE XLVI.—*Continued.* ($1\frac{1}{2}$ to 1.)

Depth of Water.	Bed 25 feet.		Bed 30 feet.		Bed 35 feet.		Bed 40 feet.		Bed 45 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	26·5	·96	31·5	·97	36·5	·97	41·5	·98	46·5	·98
1·5	40·88	1·16	48·38	1·18	55·88	1·18	63·38	1·18	70·88	1·19
2	56	1·32	66	1·33	76	1·34	86	1·35	96	1·36
2·25	63·84	1·39	75·09	1·4	86·34	1·41	97·59	1·42	108·8	1·43
2·5	71·88	1·45	84·37	1·47	96·88	1·48	109·4	1·49	121·9	1·5
2·75	80·09	1·51	93·84	1·53	107·6	1·55	121·3	1·56	135·1	1·57
3	88·5	1·57	103·6	1·59	118·5	1·61	133·5	1·62	148·5	1·63
3·25	87·09	1·63	113·3	1·65	129·6	1·67	145·8	1·68	162·1	1·69
3·5	105·9	1·68	123·4	1·7	140·9	1·72	158·4	1·73	175·9	1·75
3·75	114·8	1·73	133·6	1·75	152·3	1·77	171·1	1·79	189·8	1·8
4	124	1·78	144	1·8	164	1·82	184	1·84	204	1·85
4·25	133·3	1·82	154·6	1·85	175·8	1·87	197·1	1·89	218·3	1·9
4·5	142·9	1·86	165·4	1·89	187·9	1·91	210·4	1·93	232·9	1·95
4·75	152·6	1·9	176·3	1·93	200·1	1·96	223·8	1·98	247·6	2
5	162·5	1·94	187·5	1·97	212·5	2	237·5	2·03	262·5	2·04
5·25	172·6	1·98	198·8	2·01	225·1	2·04	251·3	2·07	277·6	2·08
5·5	182·9	2·02	210·4	2·05	237·9	2·08	265·4	2·11	292·9	2·13
5·75	193·3	2·06	222	2·09	250·8	2·12	279·6	2·15	308·3	2·16
6	204	2·09	234	2·13	264	2·16	294	2·18	324	2·2
6·25	214·8	2·13	246·1	2·16	277·3	2·2	308·6	2·22	339·8	2·24
6·5	225·9	2·16	258·4	2·2	290·9	2·23	323·4	2·26	356	2·28
6·75	237·1	2·19	270·9	2·23	304·6	2·27	338·4	2·29	372·1	2·32
7	248·5	2·22	283·5	2·27	318·5	2·3	353·5	2·33	388·5	2·35
7·25	260·1	2·25	296·4	2·3	332·6	2·33	368·9	2·36	405·1	2·38
7·5	271·9	2·29	309·4	2·33	346·9	2·36	384·4	2·39	421·9	2·42
7·75	283·8	2·31	322·6	2·36	361·3	2·39	400·1	2·43	438·8	2·45
8	296	2·34	336	2·39	376	2·42	416	2·46	456	2·48
8·25	308·4	2·37	349·6	2·42	390·9	2·45	432·1	2·49	473·4	2·51
8·5	320·9	2·4	363·4	2·45	405·9	2·48	448·4	2·52	490·9	2·55
8·75	333·6	2·43	377·3	2·48	421·1	2·51	464·8	2·55	508·6	2·58
9	346·5	2·46	391·5	2·5	436·5	2·54	481·5	2·58	526·5	2·61
9·25	359·6	2·48	405·8	2·53	452·1	2·57	498·3	2·61	544·6	2·64
9·5	372·9	2·51	420·4	2·56	467·9	2·6	515·4	2·64	562·9	2·66
9·75	386·4	2.53	435·1	2·58	483·9	2·63	532·5	2·66	581·3	2·69
10	400	2·56	450	2·61	500	2·65	550	2·69	600	2·72
10·5	...	...	...	...	...	...	585·4	2·74	637·9	2·77
11	...	...	...	...	...	...	621·5	2·79	676·5	2·83
11·5	...	...	...	...	...	...	658·4	2·84	715·9	2·83
12	...	...	...	...	...	...	696	2·89	756	2·93

TABLE XLVI.—*Continued.* ($1\frac{1}{2}$ to 1.)

Depth of Water.	Bed 50 feet.		Bed 60 feet.		Bed 70 feet.		Bed 80 feet.		Bed 90 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet.										
1	51·5	·98	61·5	·98	71·5	·98	81·5	...	91·5	...
1·5	78·38	1·19	91·13	1·18	108·4	1·19	123·4	...	138·4	...
2	106	1·36	126	1·37	146	1·37	166	...	186	...
2·25	120·1	1·44	142·6	1·45	165·1	1·45	187·6	...	210·1	...
2·5	134·4	1·51	159·4	1·52	184·4	1·53	209·4	...	234·4	...
2·75	148·8	1·58	176·3	1·59	203·8	1·6	231·3	...	258·8	...
3	163·5	1·64	193·5	1·65	223·5	1·66	253·5	...	283·5	...
3·25	178·3	1·7	210·8	1·71	243·3	1·73	275·8	...	308·3	...
3·5	193·4	1·76	228·4	1·77	263·4	1·79	298·4	...	333·4	...
3·75	208·6	1·81	246·1	1·83	283·6	1·84	321·1	...	358·6	...
4	224	1·86	264	1·88	304	1·9	344	...	384	...
4·25	239·6	1·92	282·1	1·94	324·6	1·95	367·1	...	409·6	...
4·5	255·4	1·96	300·4	1·99	345·4	2	390·4	...	435·4	...
4·75	271·3	2·01	318·8	2·03	366·3	2·05	413·8	...	461·3	...
5	287·5	2·05	337·5	2·08	387·5	2·1	437·5	...	487·5	...
5·25	303·8	2·1	356·3	2·12	408·8	2·14	461·3	...	513·8	...
5·5	320·4	2·14	375·4	2·17	430·4	2·19	485·4	...	540·4	...
5·75	337·1	2·18	394·6	2·21	452·1	2·23	509·6	...	567·1	...
6	354	2·22	414	2·25	474	2·27	534	...	594	...
6·25	371·1	2·26	433·6	2·29	496·1	2·32	558·6	...	621·1	...
6·5	388·4	2·3	453·4	2·33	518·4	2·36	583·4	...	648·4	...
6·75	405·9	...	473·4	...	540·9	...	608·4	...	675·9	...
7	423·5	2·37	493·5	2·4	563·5	2·43	633·5	...	703·5	...
7·25	441·4	...	513·9	...	586·4	...	658·9	...	731·4	...
7·5	459·4	2·44	534·4	2·47	609·4	2·51	684·4	...	759·4	...
7·75	477·6	2·47	555·1	...	632·6	2·54	710·1	...	787·6	...
8	496	2·5	576	2·54	656	2·57	736	...	816	...
8·25	514·6	...	597·1	...	679·6	...	762·1	...	844·6	...
8·5	533·4	2·57	618·4	2·61	703·4	2·64	788·4	...	873·4	...
8·75	552·3	2·6	639·8	2·64	727·3	...	814·8	...	902·3	...
9	571·5	2·63	661·5	2·67	757·5	2·71	841·5	...	931·5	...
9·25	590·8	...	683·3	2·7	775·8	2·74	868·3	...	960·8	...
9·5	610·4	2·69	705·4	2·73	800·4	2·77	895·4	...	990·4	...
9·75	630	2·72	727·5	2·76	825	2·8	922·5	...	1020	...
10	650	2·75	750	2·79	850	2·83	950	...	1050	...
10·5	690·4	2·8	795·4	2·85	900·4	2·89	1005	...	1110	...
11	731·5	2·86	841·5	2·9	951·5	2·94	1062	...	1172	...
11·5	773·4	2·91	888·4	2·96	1003	3	1118	...	1233	...
12	816	2·96	936	3·01	1056	3·05	1176	...	1296	...

Table XLVII.—Sectional Data for Oval Sewers. (Art. 3.)

Metropolitan Ovoid.

Dimensions.	Full.		Two-thirds full.		One-third full.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
1′ 0″ × 1′ 6″	1·15	·54	·76	·56	·28	·45
1′ 2″ × 1′ 9″	1·16	·58	1·03	·61	·39	·49
1′ 4″ × 2′ 0″	2·04	·62	1·34	·65	·51	·53
1′ 6″ × 2′ 3″	2·58	·66	1·7	·69	·64	·56
1′ 8″ × 2′ 6″	3·19	·69	2·1	·73	·79	·59
1′ 10″ × 2′ 9″	3·86	·73	2·54	·76	·96	·62
2′ 0″ × 3′ 0″	4·59	·76	3·02	·79	1·14	·64
2′ 2″ × 3′ 3″	5·39	·79	3·55	·83	1·33	·67
2′ 4″ × 3′ 6″	6·25	·82	4·12	·86	1·55	·69
2′ 6″ × 3′ 9″	7·18	·85	4·72	·88	1·78	·72
2′ 8″ × 4′ 0″	8·17	·88	5·38	·92	2·02	·74
2′ 10″ × 4′ 3″	9·22	·91	6·07	·95	2·28	·76
3′ 0″ × 4′ 6″	10·34	·93	6·8	·97	2·56	·79
3′ 2″ × 4′ 9″	11·52	·96	7·58	1	2·85	·81
3′ 4″ × 5′ 0″	12·76	·98	8·4	1·03	3·16	·83
3′ 6″ × 5′ 3″	14·07	1·01	9·26	1·05	3·48	·85
3′ 8″ × 5′ 6″	15·41	1·03	10·16	1·08	3·82	·87
3′ 10″ × 5′ 9″	16·88	1·06	11·11	1·1	4·17	·89
4′ 0″ × 6′ 0″	18·38	1·08	12·09	1·12	4·54	·91
4′ 2″ × 6′ 3″	19·94	1·1	13·12	1·15	4·93	·93
4′ 4″ × 6′ 6″	21·57	1·12	14·19	1·17	5·33	·95
4′ 6″ × 6′ 9″	23·26	1·14	15·31	1·19	5·75	·96
4′ 8″ × 7′ 0″	25·01	1·16	16·46	1·21	6·19	·98
4′ 10″ × 7′ 3″	26·83	1·18	17·66	1·23	6·64	1
5′ 0″ × 7′ 6″	28·71	1·2	18·9	1·26	7·1	1·02
5′ 2″ × 7′ 9″	30·67	1·22	20·18	1·28	7·58	1·03
5′ 4″ × 8′ 0″	32·67	1·24	21·5	1·3	8·08	1·05
5′ 6″ × 8′ 3″	34·74	1·26	22·86	1·32	8·59	1·07

Table XLVIII.—Sectional Data for Oval Sewers. (Art. 3.)

Hawksley's Ovoid.

Transverse Diameter.	Full.		Two-thirds full.		One-third full.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
1′ 0″	1	·53	·67	·56	·26	·44
1′ 2″	1·36	·57	·91	·6	·35	·48
1′ 4″	1·77	·61	1·19	·64	·46	·51
1′ 6″	2·24	·64	1·51	·68	·58	·54
1′ 8″	2·77	·68	1·87	·72	·71	·57
1′ 10″	2·35	·71	2·25	·75	·86	·6
2′ 0″	3·98	·74	2·69	·79	1·03	·63
2′ 2″	4·67	·77	3·14	·82	1·21	·66
2′ 4″	5·42	·8	3·66	·85	1·4	·68
2′ 6″	6·22	·83	4·2	·88	1·61	·7
2′ 8″	7·08	·86	4·77	·91	1·83	·72
2′ 10″	7·89	·89	5·38	·94	2·06	·74
3′ 0″	8·97	·91	6·04	·96	2·31	·77
3′ 2″	9·98	·94	6·73	·99	2·58	·79
3′ 4″	11·06	·96	7·46	1·02	2·85	·81
3′ 6″	12·2	·98	8·22	1·04	3·15	·83
3′ 8″	13·38	1·01	9	1·07	3·45	·85
3′ 10″	14·63	1·03	9·87	1·09	3·78	·87
4′ 0″	15·93	1·05	10·74	1·11	4·11	·89
4′ 2″	17·28	1·07	11·66	1·14	4·46	·91
4′ 4″	18·69	1·09	12·57	1·16	4·82	·93
4′ 6″	20·18	1·12	13·6	1·18	5·20	·94
4′ 8″	21·68	1·14	14·62	1·2	5·59	·96
4′ 10″	23·25	1·16	15·68	1·22	6	·98
5′ 0″	24·89	1·18	16·79	1·24	6·42	1
5′ 2″	26·57	1·2	17·92	1·27	6·86	1·01
5′ 4″	28·32	1·21	19·1	1·29	7·31	1·03
5′ 6″	30·11	1·23	20·26	1·31	7·76	1·04
5′ 8″	31·56	1·25	51·5	1·33	8·24	1·06
5′ 10″	33·87	1·27	22·84	1·34	8·74	1·07
6 0″	35·84	1·29	24·17	1·36	9·25	1·09

TABLE XLIX.—SECTIONAL DATA FOR OVAL SEWERS. (Art. 3.)

Jackson's Peg-top Section.

Dimensions.	Full.		Two-thirds full.		One-third full.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
1′ 0″×1′ 6″	1·039	·52	·646	·53	·242	·44
1′ 2″×1′ 9″	1·414	·56	·88	·57	·33	·47
1′ 4″×2′ 0″	1·846	·6	1·148	·61	·431	·5
1′ 6″×2′ 3″	2·337	·63	1·453	·65	·545	·53
1′ 8″×2′ 6″	2·885	·67	1·793	·68	·65	·56
1′ 10″×2′ 9″	3·491	·7	2·115	·72	·813	·59
2′ 0″×3′ 0″	4·154	·73	2·583	·75	·969	·62
2′ 2″×3′ 3″	4·874	·76	3·032	·78	1·136	·64
2′ 4″×3′ 6″	5·654	·79	3·516	·81	1·319	·67
2′ 6″×3′ 9″	6·491	·82	4·034	·84	1·513	·69
2′ 8″×4′ 0″	7·385	·84	4·593	·86	1·722	·71
2′ 10″×4′ 3″	8·337	·87	5·184	·89	1·943	·73
3′ 0″×4′ 6″	9·347	·89	5·813	·92	2·179	·76
3′ 2″×4′ 9″	10·41	·92	6·478	·94	2·427	·78
3′ 4″×5′ 0″	11·54	·94	7·172	·97	2·602	·8
3′ 6″×5′ 3″	12·72	·97	7·912	·99	2·967	·82
3′ 8″×5′ 6″	13·96	·99	8·461	1·01	3·254	·84
3′ 10″×5′ 9″	15·26	1·01	9·492	1·03	3·556	·85
4′ 0″×6′ 0″	16·62	1·03	10·33	1·06	3·874	·87
4′ 2″×6′ 3″	18·03	1·06	11·22	1·08	4·201	·89
4′ 4″×6′ 6″	19·5	1·08	12·13	1·1	4·542	·91
4′ 6″×6′ 9″	21·03	1·1	13·08	1·12	4·903	·93
4′ 8″×7′ 0″	22·62	1·12	14·07	1·14	5·274	·94
4′ 10″×7′ 3″	24·26	1·14	15·09	1·16	5·653	·96
5′ 0″×7′ 6″	25·96	1·16	16·14	1·18	6·054	·98
5′ 2″×7′ 9″	27·72	1·18	17·24	1·2	6·46	·99
5′ 4″×8′ 0″	29·54	1·19	18·37	1·22	6·844	1·01
5′ 6″×8′ 3″	31·42	1·21	19·54	1·24	7·321	1·02
5′ 8″×8′ 6″	33·35	1·23	20·74	1·26	7·77	1·04
5′ 10″×8′ 9″	35·34	1·25	21·98	1·28	8·234	1·05
6′ 0″×9′ 0″	37·39	1·27	23·25	1·3	8·718	1·07

Table L.—Ratios of Combined Length of Two Side-slopes to Depth of Water.

Side-slope	$= \frac{1}{2}$ to 1	$\frac{3}{4}$ to 1	1 to 1	$1\frac{1}{3}$ to 1	$1\frac{1}{2}$ to 1	2 to 1	$2\frac{1}{2}$ to 1	3 to 1
Ratio	=2·236	2·5	2·828	3·333̇	3·606	4·472	5·385	6·325

These ratios can be used for calculating R for channels outside the range of tables xliii.-xlvi.

Table La.—Circular Channels partly full. (Art. 6).

The Diameter of the Channel is supposed to be 1.

Depth of Water.	Angle subtended by Wet Portion of Border.	Relative Values of A	Relative Values of $\sqrt{R}$
Feet. ·25	120°	·196	·767
·5	180°	·5	1
·75	240°	·804	1·1
1	360°	1	1

For actual values of A and $\sqrt{R}$ see table xxiii., page 142.

CHAPTER VII

OPEN CHANNELS—VARIABLE FLOW

[For preliminary information see chapter ii. articles 10 to 14 and 17 to 21]

SECTION I.—BENDS AND ABRUPT CHANGES

1. **Bends.**—The loss of head at a change in direction in an open stream is, as in the case of a pipe, greater for an elbow than for a bend. The formula for loss of head at a bend arrived at by observations on the Mississippi is $H=\frac{V^2 \sin^2 \theta}{134}$ where θ is the angle subtended by the bend. This takes no account of the radius. In a bend of 90° the loss of head by this formula is $\cdot 48\frac{V^2}{2g}$. Generally a single bend with ordinary velocities causes little heading-up, but if a stream has a long succession of bends their cumulative effect may be considerable. It is practically the same as that of an increase of roughness, and may be allowed for by taking a lower value of the co-efficient C. How far the loss of head at a bend depends on the radius of the bend is not known. (Cf. chap. v. art. 4.)

At a bend there is a 'set of the stream' towards the concave bank, the greatest velocity being near that bank; and there is a raising of the water-level there, so that the surface has a transverse slope (Fig. 117). There is also a deepening near the concave bank and a shoaling at the opposite one, but this is not all due to the direct action of centrifugal force. The high-water level at the concave bank, due to centrifugal force, gives a greater pressure and tends to cause a transverse current from the concave towards the convex bank. This tendency is, in the greater part of the cross-section, resisted by the centrifugal force. But the water near the bed and sides has a low velocity, the centrifugal

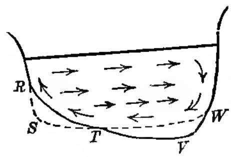


FIG. 117.

force is therefore smaller, and transverse flow occurs. Solid material is thus rolled towards the convex bank, and it accumulates there because the velocity is low. To compensate for the low-level current towards the convex bank there are high-level currents towards the concave bank. The directions of the currents are shown by the arrows on Fig. 117. In Fig. 118 the dotted line shows the direction of the strongest surface current and the arrows the currents near the bed. This explanation is due to Thomson, and has been confirmed by him experimentally. When the channel is of masonry or even very hard soil the deepening *TVW* cannot occur, but the bank *RST* may still be formed, the material for it being brought down by the stream. The greatest velocity is still on the side next the concave bank.

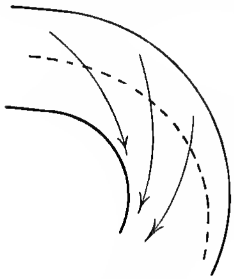


Fig. 118.

As the transverse current and transverse surface-slope cannot commence or end abruptly there is a certain length in which they vary. In this length the radius of curvature of the bend and the form of the cross-section also tend to vary. This can often be seen in plans of river-bends, the curvature being less sharp towards the ends. This principle has been adopted in constructing river training-walls, and it appears to be sound as tending against any abruptness in the change of section. For training-walls to remove bars at the mouth of the Mississippi it has been proposed to construct, instead of two walls, only one wall having a curve concave to the stream. The success of this plan would appear to depend on whether the curve is sharp enough to ensure the stream keeping close to the wall and not going off in another direction.

The sectional area of a stream may be less at a bend than in straight reaches, especially when the channel is hard, so that the stream cannot excavate a hollow to compensate for the silt-bank; but the surface-width is often greatest at bends, and in constructing training-walls the width between the walls is sometimes increased at bends. In the silt clearances of some tortuous canals in India it was once the custom to remove the silt *RST*, the dotted line showing the section of the cleared channel in the straight reaches. No allowance was made for the hollow *TVW*. A silt-bank so removed quickly forms again. Its removal is equivalent to the digging of a hole or recess in the bed.

When once a stream has assumed a curved form, be it ever so slight, the tendency is for the bend to increase. The greater velocity and greater depth near the concave bank react on each other, each inducing the other. The concave bank is undermined, becomes vertical owing to scour of the bed, cracks, falls in, and is washed away. The bend may go on increasing as indicated by the dotted lines in Fig. 119, a deposit of silt occurring at the convex bank, so that the width of the stream remains tolerably constant. Some of the large Indian rivers flowing through alluvial soil sometimes cut away, at bends, hundreds of acres of land, together with the trees, crops, and villages standing thereon. Works to check the erosion would cost many times as much as the value of the property to be saved. When a bend has formed in a channel previously straight, the stream at the lower end of the bend, by setting against the bank, tends to cause another bend of the opposite kind to the first. Thus the tendency is for the stream to become tortuous, and while the tortuosity is slight the length, and therefore the slope and velocity, are little affected; but the action may continue until the increase in the length of the stream materially flattens the slope, and the consequent reduction in velocity causes erosion to cease. Or the stream during a flood may find, along the chord of a bend, a direct route, with of course a steeper slope. Scouring a channel along this route it straightens itself, and its action then commences afresh.

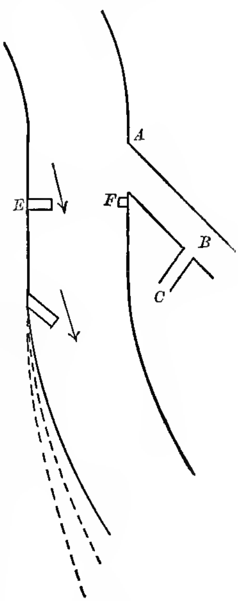


Fig. 119.

2. Changes of Section.—An 'obstruction' is anything causing an abrupt decrease of area in a part of the cross-section of a stream such as a pier or spur. There may or may not be a decrease in the sectional area of the stream as a whole. There is a tendency to scour alongside an obstruction owing to the increased velocity, and downstream of it owing to the eddies. When a spur is constructed for the purpose of deflecting a stream or checking erosion of the bank, the scour near the end of the spur may be very severe, even though there may be very little contraction of the stream as a whole. If the bed is soft the spur may be undermined. A continuous lining of the bank with protective material is not open to such an attack. Similarly a hole may be formed alongside of and downstream of a bridge-pier. The hole may work back to the upstream side of the obstruction, though there is little original tendency to scour there.

When an obstruction reaches up to the surface, or nearly up to it, there is a heaping-up of the water on its upstream side due to the checking of the velocity. In the eddy downstream of an obstruction the water-level is depressed. The changes of water-level and velocity are local; that is, they do not necessarily extend across the stream, and they are independent of the effects of any general change—supposing such to occur—in the sectional area of the stream. Their amounts cannot be calculated, but they often have to be recognised. They should be avoided in observing water-levels where accuracy is required, as for instance when finding the surface-slope. The discharge of a branch will be increased by a spur or obstruction just below it, and decreased by one just above it. On some irrigation canals in India, where the velocity is high and the channel of boulders, the cultivators sometimes run out small spurs below their water-course heads in order to obtain more water.

An obstruction causes a 'set of the stream,' that is a strong current, as shown by the arrows in Fig. 119; but the distance to which such a current extends depends entirely on its impetus, and is not usually great.[1] If a spur is merely intended to cause slack water or silt deposit on its own side of the stream several short spurs will do as well as one long one, but when the object is to cause a stream to set against the opposite bank the spurs may have to be very long.

In a short deep recess in the bed or bank of a stream or downstream of an obstruction, if it is large enough to cause dead water, there is generally a rapid deposit of silt, but not where strong eddies occur.

When an obstruction causes material reduction of the section of the stream the velocity past it is increased, and the scour may be excessive, both from the high velocity past it and (if there is a subsequent expansion of the stream) the eddies downstream of it. Thus a partly formed dam EF (Fig. 119) is, unless the gap is quickly closed, liable to be destroyed by the stream, and so is any structure which reduces the water-way. In order to lessen scour of the banks downstream of contracted water-ways the channel is sometimes widened out so as to form a basin in which the eddies exhaust themselves.

[1] See Notes at end of chapter.

3. Bifurcations and Junctions.—The general effects of these have been stated in chapter ii. (art. 20). In an irrigation distributary constructed in India the velocity was exceptionally high, and it was found that the discharges of some narrow masonry outlets, taking off from the distributary at right angles, were so small that it became necessary to rebuild them at a smaller angle. On the other hand, it was once the custom to build the heads of the distributaries themselves at an angle of 45° with the canal, but they are now built at right angles. The velocity in the canal is 2 or 3 feet per second, and that in the distributary less. A slight fall into the distributary is not objectionable. A skew head is suitable in cases where loss of head is not permissible.

When there is a bend in the main stream importance is sometimes attached to the set of the stream as affecting the supply in a branch taking off on the concave bank. The velocity in the branch is that due to its slope and to the depth of water in it. The advantage possessed by the branch as compared with one on the opposite bank is the greater depth of water, owing to velocity of approach. This advantage is small except in the case of a sharp bend and a high velocity.[1] A river about 20 feet deep was eroding the concave bank at a bend. An attempt was made to divert it by a straight cut, about a mile long, across the bend. Owing to the high level of the sub-soil water, the cut could only be dug down to about 2 feet below the water-level of the river. The slope of the cut was about one-and-a-half times that of the river, but owing to the small depth of water the velocity was low, and the cut, or at least its upper part, rapidly silted up. The reason given for its failure was that its head was not so placed as to catch the set of the stream at the bend next above. This set might have given an inch or two more water, and the cut might have taken a few days longer to silt up.

In river diversion works spurs are sometimes used to 'drive the river' down a branch channel. A spur may make the current set against the branch head (art. 2), but unless the spur is so long

[1] There is also the advantage—very slight unless the velocity is high—due to the higher water level at the concave bank.

as to greatly contract the water-way, the rise of water-level will not be great except in cases of very high velocities, and the river will continue to distribute itself according to the discharging capacities of the two branches. It is only by closing or thoroughly obstructing one branch or enlarging the other that the stream can be forced to alter its distribution of discharge.

At a junction of one stream with another there are the usual eddies and inequalities in the water-level, all depending as before on the sharpness of the angle and on the velocity. When the main stream is not much larger than the tributary, the latter may cause a set of the current against the opposite bank and erode it.

4. Relative Velocities in Cross-section.—In every case of abrupt contraction in a stream there are (chap. ii. art. 21) eddies which extend back to the point where the fall in the surface begins. Upstream of these eddies the distribution of the velocities in the cross-section is not affected. In the case of a pier, even a wide one, in the middle of a straight uniform stream, the maximum velocity remains in mid-stream till just before the pier is reached. If a plank or gate obstructs the upper portion of a stream from side to side, the surface velocities are affected for only a short distance upstream. A spur or sudden decrease of width causes slack water for only a short distance. In all these cases the state of the flow further upstream, as far as regards the distribution of the velocities, is precisely the same as if no obstruction existed. In the case of a weir visual evidence is wanting, but by analogy the same law holds good.

Section II.—Variable Flow in a Uniform Channel

(General Description)

5. Breaks in Uniformity.—Variable flow may be caused by a change in slope (Figs. 16 and 17, pp. 24 and 25) or in roughness (Figs. 120 and 121), by a debouchure into a pond or river (Figs. 122 and 123), by a weir (Figs. 124 and 125), by a change in width (Figs. 126 and 127), or in bed-level (Figs. 128 and 129). Heading-up may be caused by a local contraction or submerged weir

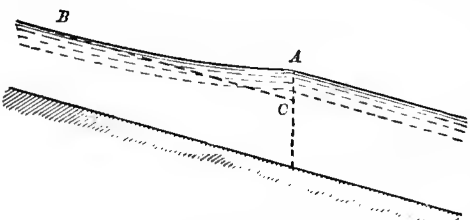


Fig. 120.

(Fig. 130), but the analogous case of a local enlargement has no effect. A change of hydraulic radius seldom occurs without a change of sectional area, and it need not therefore be considered as a separate case. A bend generally causes some degree of heading-up.[1] In each case the line *BC* is the 'natural water-surface' of the upper reach, that is, the surface as it would have been if no change had occurred. The profiles of the water-surface touch the natural surface at points far upstream. Above

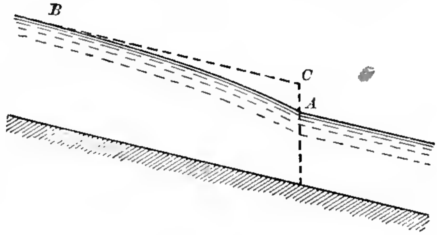


FIG. 121.

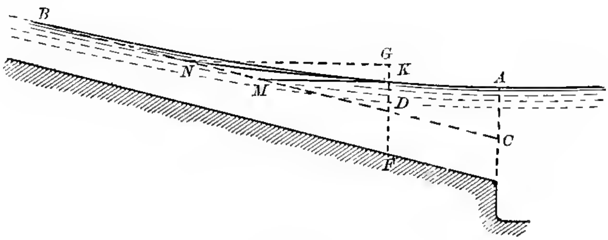


FIG. 122

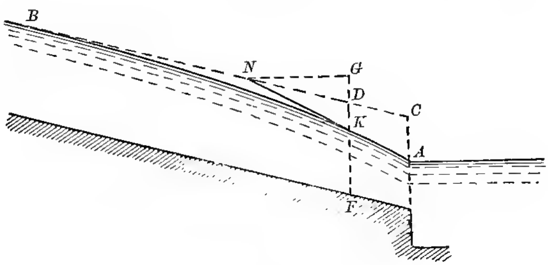


FIG. 123.

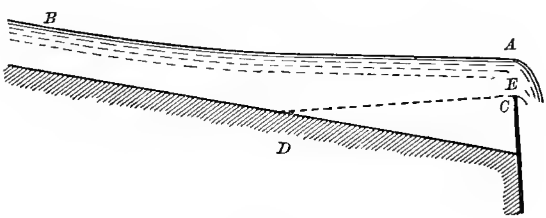


FIG. 124.

[1] It may be slight or even inappreciable.

these points the flow is uniform if the reach extends far enough. In heading-up there is a tendency to silt, and in drawing-down to scour.

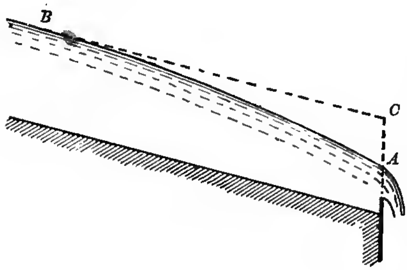


Fig. 125.

In the cases shown in Figs. 126 to 130 there are abrupt changes in the sectional area, falls in the surface when the area decreases, and perhaps rises where it increases (chap. ii. arts. 18 and 19). In Figs. 124 and 125 the weir formula gives the discharge having reference to the surface above the local fall, which therefore need not be considered. In the other cases there are no abrupt changes in section, and therefore no local changes in level.

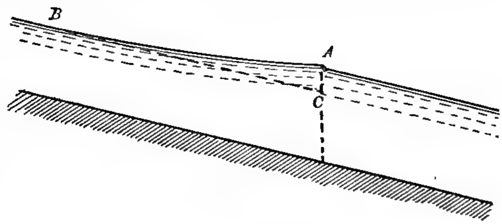


Fig. 126.

A change of one kind may be combined with another so that the change of water-level is altered or suppressed. For instance, the changes of roughness may be accompanied by changes in slope, so that the water-level in the lower reach is at *C* and the flow is uniform, but any local falls or rises due to abrupt change of section (Figs. 126 to 130) will remain. The rises are generally, however, negligible, and the falls are much reduced if the changes are not actually sudden (chap. ii. art. 21).

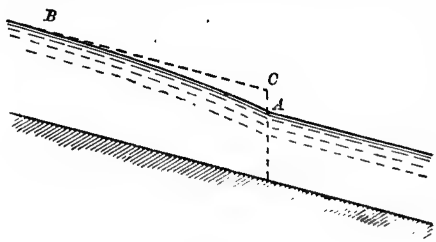


Fig. 127.

In all cases, whatever, the upstream level has to accommodate itself to the downstream level. The water-level in the lower reach or pond or on the crest of the fall is known or can be ascertained. The local fall or rise, if any, must be found, and there will be heading-up or drawing-down or neither in the reach above, according as the level found is above or below or equal to the natural level in that reach.[1]

When the variable flow extends upstream to a point where there is another break in uniformity the flow in the reach is said to be 'variable throughout.' If the bed of the reach is level, or slopes upward (Figs. 135 and 136, p. 240), the flow must be variable throughout, however long the reach may be and the surface convex upward.

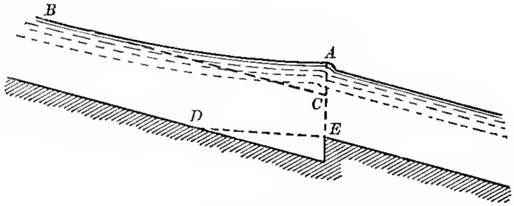


FIG. 128.

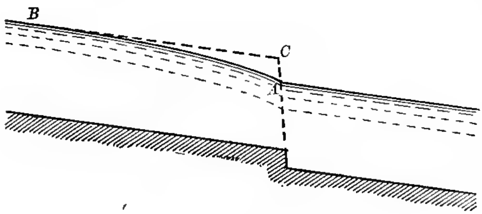


FIG. 129.

In a uniform channel let CD (Fig. 131) be a 'flume' of the same section as the rest of the channel, but of smoother material. If the flume extended upstream far enough the water-surface would be CGH. Actually it will be CGL, GL being a curve of drawing-down. The height DG will generally be very small, and no appreciable change in the velocity will be caused, but if surface-slope observations are made a serious error may occur if the upstream point of observation falls at M. The slope required is $ECDL$, that actually observed is EM. Often a flume has vertical sides, and is of a different section to the rest of the channel. If the change is made gradually there may possibly be no interference with the straight line of the water-surface, the smaller

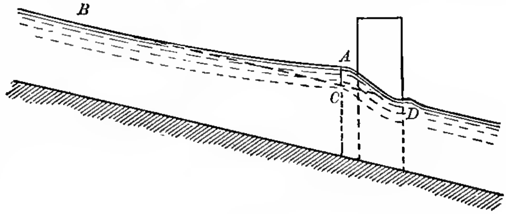


FIG. 130.

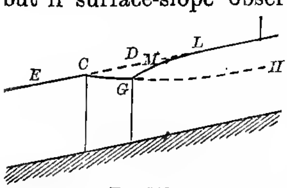


FIG. 131.

[1] See Appendix D.

sectional area and hydraulic radius of the flume compensating for its smoother material. But this is not likely to be the case exactly. If the change of section is abrupt there will be a change in the water-level at the entrance of the flume. In the Roorkee Hydraulic Experiments observations were made in a masonry aqueduct 900 feet long in the Ganges Canal. The surface-slope, instead of being observed within the aqueduct, was obtained from points lying far outside it in the earthen channel, and the results of the experiments, so far as concerns the relation between slope and velocity in masonry channels, were vitiated.[1]

6. **Bifurcations and Junctions.**—A bifurcation or junction may cause variable flow upstream of it. At a junction let Q_1 and Q_2 be the discharges of the two tributaries. The flow in the main stream is uniform, and its water-level is that corresponding to the discharge Q_1+Q_2. If the conditions of the debouchure of either tributary are such as to cause any local fall or rise, the amount of this must be estimated, and the water-level in the tributary just above the junction is then known. There will be heading-up or drawing-down or neither in the tributary, according as its natural water-level is below or above or equal to that so found. There may be heading-up in one tributary and drawing-down in the other.

At a bifurcation let Q be the discharge of the main stream. The flow in the branches is uniform. Assume discharges Q_1 and Q_2 for them—Q_1+Q_2 being equal to Q—and find their water-levels. Allow for any local fall or rise, and if the water-levels upstream of them are equal the assumed discharges Q_1 and Q_2 are correct, and the water-level found is that of the main stream. If they are not equal it is necessary to alter the quantities Q_1 and Q_2 and make a second trial. In the main stream there will be heading-up or drawing-down or neither, according as the water-level found is higher or lower than, or equal to, its natural water-level. If a stream flows out of a reservoir the flow will be uniform downstream of the fall in the surface (chap. iv. art. 15) which occurs at the head. If more than two streams meet or separate at one place the discharges Q_1, Q_2, Q_3, etc., must be considered, and the above processes adopted. The variable flow caused by a junction or bifurcation may be counteracted wholly or partly by any other cause, just as in the other instances of variable flow.

In a paper[2] on the designing of trapezoidal notches at canal falls it has been observed that a distributary usually takes off a short

[1] *Transactions, Society of Engineers*, 1886.

[2] *Punjab Irrigation Branch Paper*, No. 2.

distance above a fall, and that though the notch must obviously be able to pass the whole discharge when the distributary is closed, it has to be settled in each case whether the design of the notch should be such as to cause draw when the distributary is open or heading-up when it is closed. The question must occur with every distributary, and not only with those taking off above falls. If the canal is designed so as to give uniform flow with the distributary closed, then there must be draw when it is open. If there is uniform flow when the distributary is open, there must be heading-up when it is closed. The best arrangement depends on engineering considerations which need not be discussed here.

The opening of an escape or branch may cause scouring upstream of it. One method of freeing the upper reach of a canal from silt is to make an escape from a point some distance below its head leading back to the river. If there is a weir across the river the slope of the escape may be great. By opening the escape scour is caused in the canal, but this may cause some deposit in the canal downstream of the escape, unless it can be shut off when the escape is opened.

There were once to be seen in a large canal two gauges, one just above and the other just below the off-take of an escape channel. It was stated that the two gauges had been erected in order that, by noting the difference of their readings, the quantity of water passing down the escape could be estimated. Both gauges were carefully read, and copies of the readings sent to various officials. But when the escape was opened the water-level on the upper gauge fell practically as much as that on the lower one. Both gauges always read the same. The assistant in charge put up a temporary gauge half a mile upstream. This also fell when the escape was opened. The proper arrangement in such a case is to have one gauge in the canal below the escape and one in the escape. Again, some irrigators who wanted a new water-course were anxious that its off-take should be placed just above and not just below the off-take of an existing water-course. Practically it made no difference whether it was above or below. There was no sudden fall in the water-level of the canal. If a branch whose discharge is to be q is to be supplied from a channel whose discharge is Q, it is necessary first to find what the water-level in the channel will be when its discharge is $Q-q$, and then to design the branch so that it will obtain a discharge q with the water-level thus found.

7. Effect of Change in the Discharge.—An increase or decrease of the discharge is always accompanied by a rise or fall of the water level throughout every reach except at the points A (Figs. 122 and 123), where the stream enters or leaves a river or pond whose water-level is not affected by the alteration of discharge. It is clear, however, that for a given change of discharge the changes in the water-levels at two distant points may be very different from one another. In changes of slope, roughness, width, or bed-level, a change in the discharge causes no change in the character of the flow, that is, there is always heading-up or draw, whichever there was at first. In a local contraction there is always heading-up, and also with a weir, except when deeply drowned, if there is no fall in the bed. In the other cases (debouchures or weirs with falls) there will be heading-up if the supply falls low enough, and drawing-down if it rises high enough.

At a bifurcation, if the branches are such that the flow in the main stream is uniform with the average discharge, and if the beds of all three channels are at one level, the flow in the main stream will probably be nearly uniform with all discharges. At a junction a similar rule obtains only if the discharges of the tributaries vary in the same proportion.

Above a weir or a rise in the bed the water approaches the line DE (Figs 124 and 128) as the discharge is reduced, the tendency to silt increases, supposing the water to be silt-laden, and deposit will doubtless occur if the discharge falls low enough. A fall in the bed (Fig. 129) is converted into a clear 'fall' (Fig. 79, p. 99) at low supply, and in that case there will probably be scour or 'cutting back' owing to the high velocity.

8. Effects of Alterations in a Channel.—When a natural or artificial change occurs in a channel, such as deepening, widening, silting, the erection or removal of a structure, or the manipulation of a gate or sluice, the consequent change of water-level may extend upstream to a bifurcation and so affect the discharge. If the bifurcation is from a body of water whose level is not affected, the depth at the head of the channel remains constant, but the surface-slope alters, and with it the discharge; or a change in the channel may cause an alteration in the quantity of water lost by evaporation, percolation, or flooding, and so affect the discharge. But if the discharge of the channel is unaltered, the effect on the water-level and velocity caused by any change in the channel is wholly upstream. The building, for instance, of a weir in a stream ordinarily causes little difference to persons further down the stream as long as water is not permanently diverted.

In a discussion[1] on some oblique weirs erected in the Severn it is implied that the weirs caused a lowering of the flood-level and a deepening upstream. Above the weirs basins had been made by widening the channel, and the widening might, by itself, have caused some slight reduction in the flood-level, but not when a weir was added. It was not contended that the flood discharge at the weir was reduced. The water-level at D (Fig. 130) would therefore be the same as it was originally, and since there must always be some fall from A to D, the flood-level at A must have been raised. No deepening due to the weir could occur except close alongside a very oblique weir. (See also chap. iv. art. 18.)

Upstream of a place where changes occur a gauge-reading affords no proper indication of the discharge, and a discharge table, if it can be made at all, must be one of double entry, showing the discharge as depending not only on the gauge-reading, but on other conditions. If gates or shutters are worked there may be any number of water-surfaces corresponding to one discharge. An instance of this has already been given in the case of the flow upstream of an escape. Gauges are sometimes fixed in canals near their heads, and tables are made showing the discharges as depending on the gauge-reading. The deposit of silt in the heads alters the discharges, vitiates the tables, and destroys the utility of statistics based on the discharges obtained from them. Gauges ought to be placed below the reaches in which the deposits occur. The deposit of silt changes both the section and the slope, and it is next to impossible to allow for it by merely observing the depth at the gauge.

Sometimes masses of silt are said to travel down a stream. On the Western Jumna Canal there is a gauge at Jhind and another about twenty miles upstream. When the upper gauge is kept steady that at Jhind sometimes slowly rises, although no water is introduced in the intervening reaches. This has been ascribed to travelling masses of silt. What happens is that there is scour downstream of the upper gauge or silting downstream of the lower gauge, or both.

If a channel AB (Fig. 119) is drawn from a source whose water-level is not affected, and if, near the head of the channel, a branch BC is taken off, the discharge of the channel below B may be very little affected. A very slight lowering of the water-level at B increases the slope AB, and causes more water to be drawn in. The water-level in the channel may rise slightly at B (chap. ii. art. 20). A case occurred in which an engineer, wishing to

[1] *Minutes of Proceedings, Institution of Civil Engineers*, vol. lx.

reduce the supply in an overcharged canal, caused a breach to be made in the bank a short distance below its off-take from the river. He was surprised to find, that although a large volume of water passed out of the breach, there was no appreciable diminution of the canal discharge below the breach. In the case of an irrigation distributary which takes out of a canal, and has itself a number of water-courses taking out of it not far from its head, the discharge of the distributary may partly depend on whether the water-courses are open or not. (Cf. case of branched water-main, chap. v. art. 3.)

Let a straight cut be made across a bend in a uniform stream. The slope in the cut is increased and the longitudinal section is as in Fig. 132. If the discharge is unaltered the water-level at B is as before, and there is tendency to scour at A and to silt at B. The bed and water-surface tend to assume the positions shown by the dotted lines, and the probability of this occurring must be considered in making a cut. If it is desired to keep the water-level at A the same as before, the cut AB must be made smaller than the original channel, but the velocity in it will be greater, and there will therefore be a still greater tendency to scour. If the abandoned loop is left open the velocity in it will be greatly reduced, owing to the lower water-level at A, and at B will be further reduced by heading-up. It generally silts up.

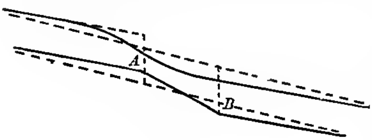


FIG. 132.

To increase the discharge of a channel ABC (Fig. 136, p. 266), supposed to be of shallow section, without enlarging it throughout, the plan involving least work is to alter the bed to DB. As D recedes from A the discharge increases, but so does the tendency to silt. (Cf. chap. vi. art. 2.)

9. Effect of a Weir or Raised Bed.—The tendency to silting, common to all cases of heading-up, may be somewhat enhanced in the case of a rise in the bed or a weir extending across a channel, because of the obstruction offered to rolling material. This however does not seem to be very great. The silt may form a long slope against the weir, and material may be rolled up the slope. Usually even this slope is not formed. Probably the eddies stir up the silt, and it is carried over.

The deposit occurring upstream of a rise or a weir has caused it to be supposed that there is a layer of still water upstream of and below the level of the crest. This idea is absolutely untenable. The general velocity undoubtedly decreases as the rise or weir is approached. This is due to the increasing section of the stream. If the water below DE (Figs. 124 and 128) were still the section would be decreasing. The same amount of heading-up might be caused by obstructions of other forms, but it has been shown, (art. 4) not only that the water upstream of them is moving, but that upstream of the eddies not even the distribution of the velocities is affected. The same is no doubt true of a rise or weir. If in a silt-bearing stream the water near the bed were still, there would be a rapid deposit of silt as there is in a short hollow or recess. But the contrary often happens. In some of the large canals in India the bed upstream of bridges has been scoured for miles, to a depth of perhaps two feet below the masonry floors of the bridges which are left standing up, and forming, in fact, submerged weirs. This alone shows the preposterous nature of the still-water theory.

The idea might have been supposed to be exploded, but for a somewhat recent case. In a paper on the Irrawaddy [1] it is stated that, if the discharges for the water-levels A, C, etc. (Fig. 133), are plotted, the discharge seems to become zero at E, which is level with a sand-bar four miles downstream, although the depth EG was 34 feet, and that 'this dead area of cross-section lying below the level of the bar regulating the discharge, exists on almost all rivers.' It is natural that the discharge should become zero at E. As the water-level falls the effect of the obstruction at F increases (art. 7), and the surface-slope becomes flatter. If the water-level ever fell to E the surface would be horizontal and the discharge zero. But the reduction of the discharge to zero is due to the flattening of the slope, and not to a portion of the section of the stream being still. If it were still it could never have been scoured out, or being in existence it would quickly silt up.

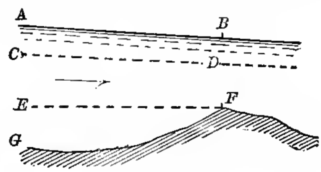


FIG. 133.

'Profile walls' are sometimes built across a channel at intervals. They are useful for showing the correct form of the cross-section, but will not prevent scour, unless built extremely close together. A single wall built at a point where the bed-slope becomes steeper will not prevent scour. If scour does occur, walls or weirs will of course stop it eventually.

[1] *Minutes of Proceedings, Institution of Civil Engineers*, vol. cxiii.

In clearing the silt from a canal it is often convenient to make the level of the cleared bed coincide with the level of a masonry bridge floor, but it is not a fact that any deeper clearance is useless. The deeper bed gives an increased discharge for the same water-level, and there is not necessarily a deposit of silt upstream of the raised floor. Similarly, there is no particular harm in omitting the clearance in any reach where, the depth of the deposit being small, say half a foot, it is troublesome to clear it.

Section III.—Variable Flow in a Uniform Channel

(Formulæ and Analysis)

10. Formulæ.—To find the length L between two points where the depths are D_1 and D_2 (Fig. 134) let S' be the bed-slope. Then

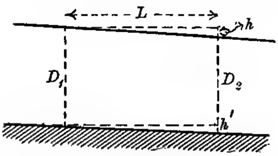


Fig. 134.

$$h = D_1 - D_2 + LS'.$$

And from equation 17, p. 22,

$$L = \frac{C^2 R}{V^2}(D_1 - D_2 + LS' + h_v)$$

or $V^2 L - C^2 RS'L = C^2 R(D_1 - D_2 + h_v)$.

Therefore $$L = \frac{C^2 R(D_1 - D_2 + h_v)}{V^2 - C^2 RS'} \quad . \; . \; . \; (74),$$

where C, R, and V have values suited to the mean section between the two points. The quantity h_v is nearly always small compared to $(D_1 - D_2)$. In heading-up $(D_1 - D_2)$ and $(V^2 - C^2 RS')$ are negative, so that in equation 74 both numerator and denominator are negative. In drawing-down the above quantities are positive.

To find the surface-slope S at any point, consider a point midway between the two sections, and suppose them very near together, so that the changes are very small. Let $V_1 - V_2 = v$, then $V_1^2 - V_2^2 = \left(V + \frac{v}{2}\right)^2 - \left(V - \frac{v}{2}\right)^2 = 2Vv$ and equation 17 becomes

$$h = \frac{V^2 L}{C^2 R} - \frac{Vv}{g} \quad . \; . \; . \; (75).$$

Let A be the sectional area and B the surface-width at the midway point. Let a be the difference in area in the length L. Then $Q = VA = \left(V + \frac{v}{2}\right)\left(A - \frac{a}{2}\right) = VA + \frac{vA}{2} - \frac{Va}{2} - \frac{va}{4}$,

neglecting the very small last term, $vA = Va$ or $v = \frac{aV}{A}$.

Therefore from equation 75, $h=\dfrac{V^2L}{C^2R}-\dfrac{V^2a}{gA}$. But $a=B(D_2-D_1)$ and if d is the mean depth in the cross-section, $A=Bd$.

Therefore $h=\dfrac{V^2L}{c^2R}-\dfrac{V^2}{g}\cdot\dfrac{D_2-D_1}{d}=\dfrac{V^2L}{C^2R}-\dfrac{V^2}{gd}(LS'-h)$

or
$$h\left(1-\frac{V^2}{gd}\right)=L\left(\frac{V^2}{C^2R}-\frac{V^2S'}{gd}\right).$$

Therefore
$$S=\frac{h}{L}=\frac{V^2}{C^2R}\cdot\frac{1-\dfrac{C^2RS'}{gd}}{1-\dfrac{V^2}{gd}} \quad . \; . \; . \; (76).$$

The difference between the bed-slope and the surface-slope is
$$S'-S=\frac{S'\left(1-\dfrac{V^2}{gd}\right)-V^2\left(\dfrac{1}{C^2R}-\dfrac{S'}{gd}\right)}{1-\dfrac{V^2}{gd}}=\frac{S'-\dfrac{V^2}{C^2R}}{1-\dfrac{V^2}{gd}} \quad . \; . \; . \; (77).$$

The fraction by which $\dfrac{V^2}{C^2R}$ is multiplied in equation 76 is the ratio of the surface-slope to what it would be in a uniform stream with the same velocity and hydraulic radius. This fraction may be written $\dfrac{1-\dfrac{V'^2}{gd}}{1-\dfrac{V^2}{gd}}$ where V' is the velocity in a uniform stream with the same values of C and R, but with a slope equal to the bed-slope. For ordinary depths and velocities the numerator is not much less than unity. In cases of heading-up the denominator is still nearer unity, but in drawing-down less so. In a stream of shallow section R is nearly as d and V is as $\dfrac{1}{d}$, so that, neglecting the above fraction S is for moderate changes in depth roughly as $\dfrac{1}{d^3}$. In order that the slope obtained by observing the water-levels at the ends of a reach may agree with the local slope at the centre of the reach, the sectional areas of the stream at the two ends of the reach must not differ, in ordinary cases, by more than 10 or 12 per cent.

Equation 76 establishes a direct connection between the depth at any cross-section and the surface-slope at that section, but not the connection between the depth or slope at any section and the position of the section. To find this, the profile must be worked out in short reaches (restricted as above as to length) by equation 71, or by a method which will be given below.

To find the length of a tangent from any point K (Figs. 122 and 123, p. 255) to N, where it meets the line of natural water-surface. Let D be the depth at K and D' the natural depth. Let $GN=x$, $GD=y$. Then $y=xS'$ and $y+D'-D=xS$.

Therefore $$D-D'=x(S'-S)$$

and $$x=\frac{D-D'}{S'-S}=(D-D')\frac{1-\frac{V^2}{gd}}{S'-\frac{V^2}{C^2R}} \quad . \quad . \quad . \quad (78).$$

When the bed is level or slopes upward (Figs. 135 and 136)

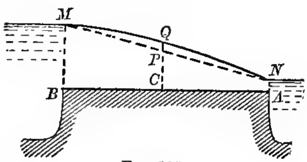


Fig. 135.

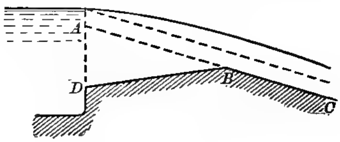


Fig. 136.

S' in equations 74 and 76 is zero or negative. In the former case

$$L=\frac{C^2R(D_1-D_2+h_v)}{V^2} \quad . \qquad (79)$$

and $$S=\frac{V^2}{C^2R}\cdot\frac{1}{1-\frac{V^2}{gd}} \quad . \qquad (80).$$

11. Standing Wave.—If a stream has a high velocity relatively to the depth of water in it V^2 may be greater than gd. Let heading-up occur in such a stream, so that V^2 becomes less than gd. Then the curve of heading up does not extend back till it touches the natural water-surface, but ends abruptly at a point A (Fig. 137). At this point $V^2=gd$, the denominator in equation 76 is zero, and the slope therefore infinite, that is, the water-surface is vertical, or a standing wave occurs. In order that the velocity may be sufficiently high, relatively to the depth, to produce a standing wave, the slope must be steep or the channel smooth. It is not necessary that there should be any variable flow except at the wave. The flow in both the upstream and downstream reaches may be uniform. Instances may be seen

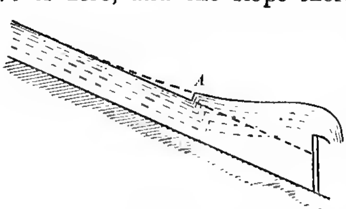


Fig. 137.

where a steep wooden trough tails into a pond or downstream of a sloping weir or contracted water-way. One occurs where the Amazon suddenly changes its slope. The quantity $\frac{V_1^2 - V_2^2}{2g}$ in equation 17 is greater than, and of opposite sign to the quantity, $\frac{V^2L}{c^2R}$. In order that V^2 or C^2RS may be greater than gd, S must be greater than $\frac{g}{C^2}$ assuming R and d to be equal. If C is 100, S must be more than ·0032.

At the foot of a rapid forming the left flank of the weir across the river Ravi at the head of the Bari Doab Canal the standing wave, when floods are passing, is 6 or 8 feet high, not counting the masses of broken water on the crest of the wave. Logs 6 feet in diameter brought down by the flood disappear into the wave.

The following statement shows some results observed by Bidone :—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
D_1	D_2	V_1	V_2	$\frac{V_1^2 - V_2^2}{2g}$	$D_2 - D_1$	Difference of Columns 5 and 6.	$\frac{(V_1 - V_2)^2}{2g}$
Feet. ·149	Feet. ·423	4·59	1·62	·287	Feet. ·274	·013	·137
·246	·739	6·28	2·09	·545	·493	·052	·273

Column 7 shows (chap. ii. art. 1) the head lost. This is small and is nothing like $\frac{(V_1 - V_2)^2}{2g}$ (chap. ii. art. 18), but it is much greater for the second case than for the first.

Let AB (Fig. 138) be a stream, and let it be desired to lower

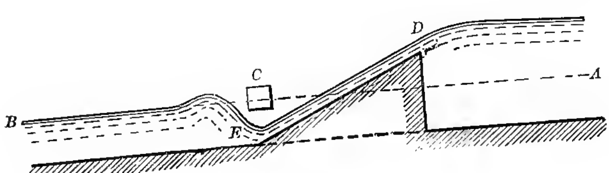


Fig. 138.

the water-level at E, say in order that floating logs or rafts may clear a structure C, or in order to allow of a drainage outfall into the stream. The object can be to some extent attained by heading up the stream and introducing a rapid *DE*. It is conceivable that some practical application of this principle might occur. (Cf. case of constricted pipe, chap. v. art. 7.)

A standing wave is also called a jump. The condition necessary for its existence is that upstream (Fig. 138A) $d_1 < \frac{V_1^2}{g}$, and that downstream $d_2 > \frac{V_2^2}{g}$. To find the height of the wave. Let the bed of the channel be horizontal and the width of the stream be unity. In a short time t let the mass $mnpq$ come to the position $m'n'p'q'$. The change of momentum is the difference between that of $mnn'm'$ and of $pqq'p'$. The force causing the change is the

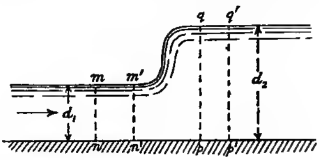


FIG. 138A.

difference between the pressures on mn and on qp. Equating the impulse and change of momentum,

$$W\left(d_1\frac{d_1}{2} - d_2\frac{d_2}{2}\right)t = \frac{W}{g}(d_2V_2^2 - d_1V_1^2)t$$

$$d_1^2 - d_2^2 = \frac{2}{g}(d_2V_2^2 - d_1V_1^2).$$

But $d_1V_1 = d_2V_2$ and $V_2^2 = V_1^2\frac{d_1^2}{d_2^2}$.

Substituting this value of V_2^2 in the above,

$$d_1^2 - d_2^2 = \frac{2}{g} \cdot V_1^2\left(\frac{d_1^2}{d_2} - d_1\right) = \frac{2}{g}V_1^2\left(\frac{d^2 - d_1d_2}{d_2}\right)$$

or

$$(d_1 - d_2)(d_1 + d_2) = 2(d_1 - d_2)\frac{d_1}{d_2} \cdot \frac{V_1^2}{g}.$$

Multiplying by $\frac{d_2}{d_1(d_1 - d_2)}$,

$$\frac{(d_1 + d_2)d_2}{d_1} = 2\frac{V_1^2}{g}.$$

Whence the quadratic

$$d_2^2 + d_1d_2 + \frac{d_1^2}{4} = 2d_1\frac{V_1^2}{g} + \frac{d_1^2}{4}.$$

Therefore

$$d_2+\frac{d_1}{2}=\sqrt{\frac{2d_1V_1^2}{g}+\frac{d_1^2}{4}}$$

$$d_2=\sqrt{\frac{2d_1V_1^2}{g}+\frac{d_1^2}{4}}-\frac{d_1}{2} \quad . \quad . \quad . \quad (80\text{A})$$

which gives d_2 in terms of d_1 and V_1.

The first term on the right in equation 80A is by far the greatest, and d_2 is more affected by change in V_1 than by change in d_1.

The stream of high velocity necessary to cause a standing wave can be produced not only by means of a steep or smooth channel, but by means of a falling sheet (Fig. 74, p. 95) or by the issue of the stream under a head (Fig. 63, p. 69). If the shoot there shown be supposed to have its water level raised, say by means of a weir, to the proper level, a standing wave will occur. The discharge, Q, is then independent of the level of the tail water. Further raising of the tail water alters the conditions and Q depends on $H_1 - H_2$ as usual. Experiments by Gibson[1] on flow under a sluice gate show

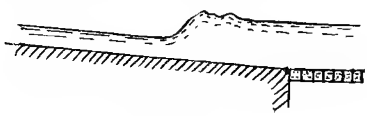


FIG. 138B.

that the height of the standing wave was nearly as given by equation 80A. Also that, if the tail water, instead of being raised, is lowered there is no standing wave, the water rising gradually (as at XY, Fig. 9, p. 14) a marked instance of the rises—frequently referred to in the present work—which may occur in variable flow. The surface is in this particular case convex upwards although the depth is increasing.

In any standing wave a large amount of energy is absorbed. There is much tumbling of the water and a general foamy condition. There is, of course, loss of head. Otherwise the preceding proof would not be needed and the increase in pressure head would be equal to the decrease in velocity head. Immediately below the jump the water is raised to a level higher than that of the water downstream of it (Fig. 138B), but this 'superelevation' is not dealt with in the above formula. It will be again mentioned below.

It has been seen that upstream of a standing wave $V_1^2 > gd$ or $\frac{V_1^2}{2g} > \frac{d}{2}$. That is, the velocity at mid-depth—and this is nearly the

[1] *Min. Proc. Inst. C.E.*, vol. cxcvii.

mean velocity of the stream—is greater than would be attained if the stream issued from an orifice under a head equal to the half depth. Let $V_c^2 = gd_c$. Then V_c is the critical velocity with reference to a depth d_c, that is the least velocity which can cause a standing wave when the upstream depth is d_c.

For given values of d_1 and V_1 a standing wave will be formed only when d_2 satisfies equation 80A. If, from any cause operating in the downstream reach, d_2 is increased and the bed of the upstream channel is sloping, the jump shifts to a point further upstream. It shifts downstream if the downstream water level is lowered. If—as in the case shown in Fig. 138—it cannot shift further downstream, the jump is imperfect and there is great disturbance, waves and broken water. The same thing may occur when there is no well-defined channel downstream but merely a pond.

A jump, as above remarked, absorbs a very large amount of energy, and the best method of preventing a large stream, issuing say from a sluice, from doing damage is to construct a rapid and cause the jump to occur.[1] The design of the rapid should be such that the jump will occur at a suitable place and in a complete form. The report just quoted describes new experiments made with standing waves, d_1 ranging up to ·22 foot, V_1 up to 14·9 feet per second, and d_2 up to 1·15 foot. The jump can occur even when $\frac{d_1}{d_c}$ is as low as ·025, but the position of the jump is then uncertain.

On rapids constructed in connection with irrigation works in Burma the head on the crest may be 3 feet to 11 feet. The slope of the rapid is generally about 1 in 15. It has been seen (chap. iv., art. 15) that the depth of water on the crest may possibly be about the critical depth, d_c. As the water flows down the slope its velocity further increases and its depth decreases. At N (Fig. 80c, p. 109) let the depth be d_c. In this particular case the surface is concave upwards although the depth is decreasing. At M the depth is the natural depth and the flow has become uniform. N in Kutter's[2] formula being known, the lengths of the curves can be calculated as explained in article 12 and the profile of the whole water surface obtained. But experiments on large existing rapids are first required in order to see what the exact conditions are. It will then be easier to design other rapids on correct principles.

[1] State of Ohio. The Miami Conservancy District, Technical Reports. Part iii. Dayton, Ohio.

[2] Probably about ·020 for a rapid pitched with boulders.

The water level below the rapid being known, the position of the standing wave can be found.

The slopes of the rapids are of boulders. The channel below the rapid is protected by pitching (Fig. 138B), but not for any great length. A rapid should be so designed as to reduce the action on the pitching and channel. This is less the higher up the jump occurs and the lower the velocity downstream of the jump. If a rapid is roughened d_1 is increased, and V_1 is reduced. Since d_2 depends more on V_1 than on d_1, therefore d_2 is reduced—that is, the jump occurs higher up than before. On any given rapid an increase in the discharge causes a rise in the downstream water-level and the jump occurs higher up. The jump should be complete for all except low discharges. Let the slope of a rapid be produced so as to bring the crest further upstream with a reduced depth of water on it and let the length of the crest be increased so that the discharge is as before. The downstream water level is as before. The jump occurs higher up. V_1, d_1, and d_2 are all reduced. Another plan is to splay out the side walls so as to gradually increase the width of the rapid from the crest downwards. If the slope of a rapid is steepened, V_1 is increased and d_1 reduced; d_2 is increased. The jump occurs at a relatively greater distance from the crest and actually nearer to the channel. The action is more violent, and the rapid, though shorter, must be built more strongly.

The superelevation at the jump is due to air and water being intimately mixed so as to form a homogeneous mass not so heavy as water alone. If the total depth at the wave is H_w and d_c is the critical depth, then $H_w = Kd_c$. In the experiments referred to above, K was found to be as follows:—

$\frac{d_1}{d_c}=$	·2	·25	·3	·35	·4	·5	·6	·7	·8
K =	12·6	8·1	5·7	4·3	3·3	2·3	1·7	1·4	1·2

This information is useful for finding the height of the side walls.

12. The Surface-curve.—In any given channel with a given discharge there is only one curve of heading-up and one of drawing-down, whatever the cause of the variable flow may be. If the cause operating at A (Figs. 122 and 123) be removed and another cause introduced, say at K, making the water-level at K as before, the curve BK is the same as before. The water in the reach BK is only concerned with accommodating itself to the water-level at K, and not with the question how that water-level has been caused. If the surface-curve is once found, it will not have to be found again for any lesser change of water-level, but only a part of the same curve used. Theoretically the curve extends to an infinite distance upstream, approaching indefinitely near to the line BC, which is an asymptote of the curve. Practically the curve extends to a limited distance beyond which no change in the natural water-surface is perceptible. The less the ratio of KD to KF the greater is the relative length of the curve BK. If the discharge of the channel is altered, the curve is entirely changed, and no part of it is the same as any part of the original curve. If the natural water-level is higher than before, a change of the same amount as before will cause a smaller ratio of KD to KF, and therefore a longer curve. The greater the relative area of that part of the cross-section of a stream which lies over the side-slopes of the channel, the more rapidly does the section change with change of water-level, the more, therefore, does the surface-slope at K differ from the natural slope, and the less the length of the curve. The length of the curve is of course less the steeper the bed-slope.

The curves for heading up are far more important than those for drawing down. Heading up is frequently caused by weirs or obstructions or by swollen tributaries or flood-water entering a stream, and the effect at upstream points is often important. Drawing down is far less frequent, and when it occurs is generally of less consequence.

In all cases met with in long uniform channels the curve is concave upwards when the depth is increasing and convex upwards when it is decreasing. But when the depth is less than d_c the rule is reversed, as stated in art. 11.

13. Method of finding Surface-curve.—To obviate the tedious process of working out length by length, and obtain a direct approximation to the surface-curve, one or two methods have been used. An old rule, given by Neville for cases of heading-up, is that the total length of the curve BK (Fig. 122, p. 255) is 1·5 to 1·9 times the length of the horizontal line KM. This is only an approximation, or rather guess, of the very roughest kind, and it gives no idea of the form of the curve, that is, of the depths at intermediate points. For an imaginary case in which the bed-width is infinite, the sides vertical, and the co-efficient C constant for all depths, an equation to the curve can be found by integration

It is far too complicated for practical use, but certain tables have been based on it. Such tables, owing to the wholly imaginary condi-

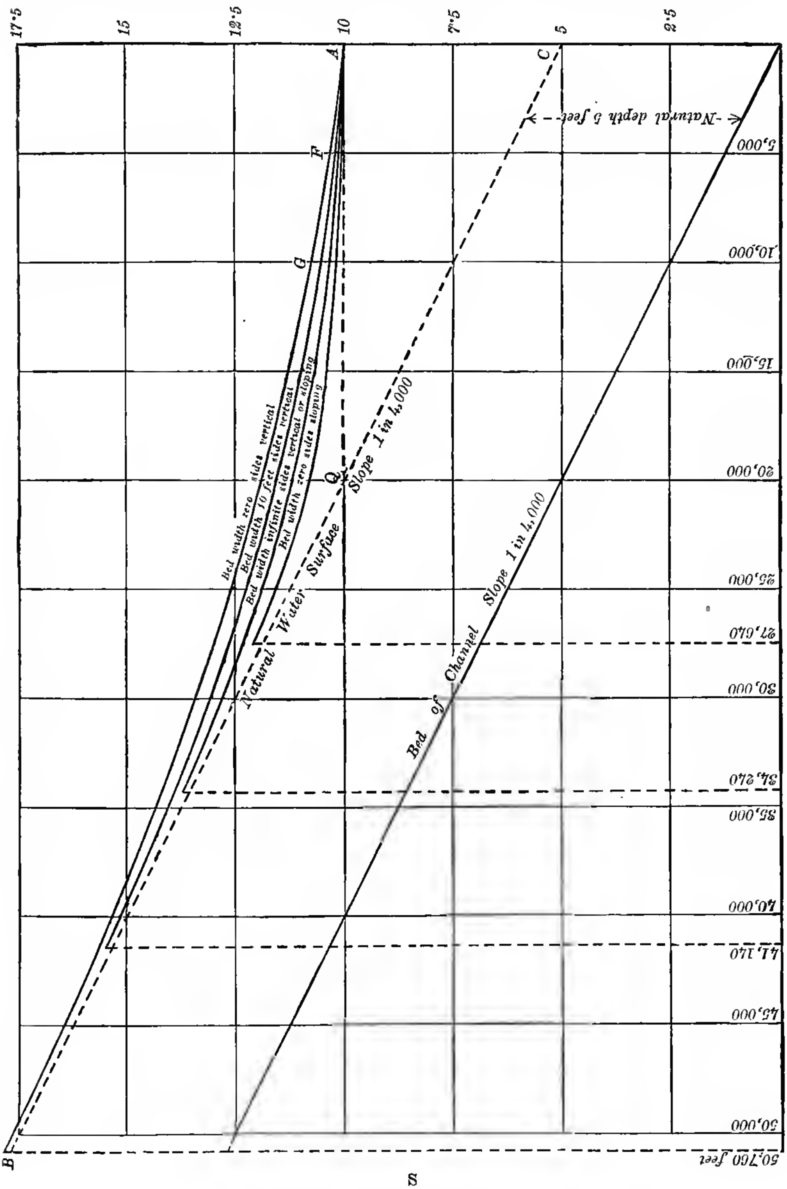


Fig. 139.

S

tions of the case, are of very limited use. For channels with vertical sides they are not accurate, for others not even fairly accurate.

Fig. 139 shows four curves worked out length by length by equation 74 (p. 264), for streams 5 feet deep with a slope of 1 in 4000, the co-efficient C being about 60 when the depth is 5 feet. For other depths the co-efficient is suitably increased. The curves all tend to become straight lines as the depth increases. This is owing to the minuteness of the surface-slope at great depths. The fall in GF has a great relative difference to the fall in FA, but both are so small that the divergence of the curve from a straight line is sometimes imperceptible. The curves are drawn up to a depth of 10 feet in one direction and 5·125 feet[1] in the other. Below this depth the curve again tends to become straight. The three uppermost curves are for channels of rectangular section. The uppermost curve represents the extreme limit possible, the bed being assumed of width zero, or, what is the same thing, assumed to be quite smooth, the sides being only taken into account in calculating R, which is therefore constant. In the second curve R increases from 2·50 feet to 3·33 feet. The third curve is for a channel of infinite width, but it is not the imaginary curve mentioned above, because the co-efficient C has been increased as D increases, instead of being constant. As D increases from 5 to 10 feet R also increases from 5 to 10 feet. In channels with sloping sides increase of depth is accompanied by a rapid increase of section and of R and C. The profiles curve more rapidly, and the points where the curves become straight are sooner reached. The lowest curve is for a triangular section (bed-width zero), and represents the extreme limit possible. For greater bed-widths the effect of the side-slopes becomes less and vanishes when the bed-width is infinite. The third curve, therefore, represents the other limit in this case. The surface-slopes at A are, for the four curves, $\frac{1}{16{,}085}$, $\frac{1}{24{,}481}$, $\frac{1}{46{,}233}$, and $\frac{1}{182{,}559}$, the last being only $\frac{1}{46}$th of the slope at B.

The total length of the curve—up to the point where $D = 1{\cdot}025D'$—is 2·538, 2·057, 1·732, or 1·382 times the length of the horizontal line AQ. The heading up at Q is ·375, ·313, ·234, or ·164 of the heading up at A.

It will be seen directly that as long as the proportions of the channel are maintained—even though its roughness or gradient may alter—the curves, including the particular ratios just mentioned, remain in most cases essentially the same. For a large number of cases it will suffice merely to take the depths by scale from one

[1] That is, to $1{\cdot}025\,D'$.

of the curves of Fig. 139—or any part of it—or any intermediate curve that may be estimated to suit the case. The vertical and horizontal scales of the diagram can be altered without altering the actual diagram.

It will be useful to consider these curves further. Cross sections of the streams corresponding to the four curves of Fig. 139 are shown in Fig. 139A. The increase of C^2R as D increases from

Section Ratio (Table LI.).	Reference to Fig. 139.	Increase in C^2R (per cent.).
...	1st curve	...
2	2nd curve	50
4	...	83
Infinity	3rd curve	175
3	...	...
·75	...	...
zero	4th curve	175

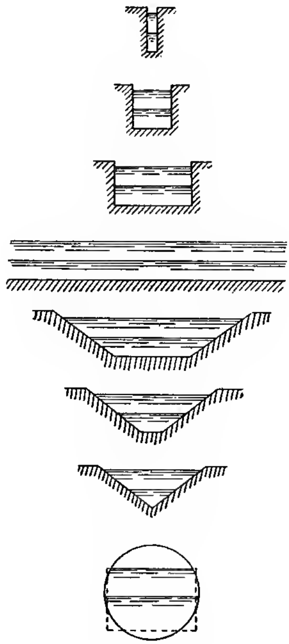


Fig. 139A.

D' to $2D'$ is also shown. In equation 74 let h_v, which is generally very small, be neglected. Then

$$L = \frac{C^2R(D_2 - D_1)}{C^2RS' - V^2} \text{ nearly} \quad . \; . \; (80\text{B})$$

Consider channels with vertical sides. As D increases from D' to $2D'$, V^2 is reduced by 75 per cent. The numerator and the first term in the denominator of the above fraction both increase at the same rate. When D only slightly exceeds D', V^2 is only slightly less than C^2RS', and the denominator of the fraction is far less than C^2RS'. When D is about $2D'$, V^2 is small and the denominator greatly increased. Thus L, for a given value of D_2-D_1, decreases as D' increases and tends to become constant. The greater the bed width of the channel the greater the rate of increase of C^2R, the less the relative value of V^2 and the less the value of L. This is especially the case when D is great. Considering, say, the second and third curves, the lower one has everywhere the lesser value of L, but the difference is greatest when D is greatest. The two curves are essentially different.

The equation obtained by integration and referred to above is :—

$$L=\frac{D_2-D_1}{S'}+D'\left(\frac{1}{S'}-\frac{C^2}{g}\right)\left\{\phi\left(\frac{D_1}{D'}\right)-\phi\left(\frac{D_2}{D'}\right)\right\}$$

The function ϕ—called the backwater function—is complicated,[1] but values of it are given in tables for various values of $\frac{D'}{D}$. For the usual flat slopes $\frac{C^2}{g}$ is only a small fraction of $\frac{1}{S'}$ so that L depends very little on C. It depends almost entirely on (D_2-D_1) and obviously cannot be correct for the various ratios of width to depth. The value of L obtained by using it may be wrong, even though the value taken for C^2 may be selected so as to suit the stream in question.

For a channel of triangular section R increases at the same rate as in the case represented by the third curve, being doubled when D' is doubled, but A is then quadrupled and V^2 is reduced by about 94 per cent. The reduction of L for great depths is more marked. In using the backwater function tables for channels with sloping sides D is taken as the sectional area divided by the surface width, but even in this case the results are liable to be quite wrong.

In cases where scaling from the diagram is not sufficiently precise the procedure may be as follows. From equation 74 (p. 264),

$$\frac{1}{L}=\frac{V^2}{C^2R(D_1-D_2+h_v)}-\frac{S'}{D_1-D_2+h_v} \quad . \; . \; . \; (81).$$

Let $x'=\frac{D_1-D_2}{S'}$, then x' is the length in which the bed-level changes by (D_1-D_2) feet, and L is the length in which the depth changes by (D_1-D_2) feet. If the ratio $\frac{x'}{L}$ is known L can be easily found. This ratio, for each of the above curves (except the uppermost, which is not needed) and for some intermediate cases, is given approximately in table li. for a range of depth extending up to

[1] It is $\phi\left(\frac{D}{D'}\right)=\phi(x)=\frac{1}{6}\log_e\frac{x^2+x+1}{(x-1)^2}-\frac{1}{\sqrt{3}}\,\text{arc. cot.}\,\frac{2x+1}{\sqrt{3}}$.

$2D'$, the value of $(D_1 - D_2)$ being usually $\frac{D'}{10}$, which gives reaches sufficiently short to enable equation 74 or 81 to apply without any considerable error. The approximate ratios $\frac{x'}{L}$ are easily found by disregarding h_v. Then, putting $C^2RS' = V'^2$, from equation 81,

$$\frac{x'}{L} = \frac{D_1 - D_2}{S'L} = \frac{V^2}{V'^2} - 1 \quad . \quad . \quad . \quad (82).$$

This quantity, since $D_2 > D_1$, is negative, and in table li. the quantity $1 - \frac{V^2}{V'^2}$ is shown instead.

Now the ratios $\frac{x'}{L}$ in table li. apply, not only to the cases from which they were deduced, but to a very large proportion of other cases. Let the size, roughness, or bed-slope of the stream alter in any manner, the proportions of the stream being maintained, and the proportionate change in C with change of R being also maintained, and let $\frac{D_1 - D_2}{D'}$ be as before, then $\frac{V^2}{V'^2}$ and $\frac{x'}{L}$ are as before. Thus the ratios in table li. can be used, with suitable interpolations, for any channel whose section is rectangular or trapezoidal. For a curvilinear or irregular section the section most resembling it can be adopted.[1]

Still greater exactness can be obtained as follows:—

Denoting by C_1 the value of C for the natural depth D', and C_2 the value for the headed-up depth $2D'$, column 14 of table li. shows the ratios $\frac{C_2}{C_1}$ or M, which actually occurred in the cases worked out. These ratios are fair averages, being such as occur with streams 5 feet to 10 feet deep with N about ·0275, but for other cases the ratio may be different. For a very smooth deep stream it will be less, and for a rough shallow stream more. For values of R (in the reach of natural flow) ranging from 2 feet to 8 feet, and N ranging from ·017 to ·030, the value of M (Kutter and Bazin) may possibly vary as shown in columns 15 and 16. For any given stream it will be difficult to say what the value is, and the extreme values shown are not likely to occur. Suppose that, for the second case shown in table li., it is believed that M' is 1·16. Then $\frac{M'}{M} = \frac{1 \cdot 16}{1 \cdot 10} = 1 \cdot 055$ and $\frac{M'^2}{M^2} = 1 \cdot 11$ nearly. Corrections can be applied as follows:—

	Column of table li.:	3,	4,	5,	. .	11,	12,	13
Corrections	In C^2R or V'^2 (+) say,	$\frac{1}{2}$,	1,	2,	. . .	9,	10,	11 per cent.
	In $V^2 \div V'^2$ (−) say,	$\frac{1}{2}$,	1,	2,	. . .	8,	9,	10 per cent[2]
	In $\frac{x'}{L}$ (+) say,	$4\frac{1}{2}$,	4,	4,	. . .	$2\frac{1}{2}$,	2,	2 per cent.

The correction to be applied to $\frac{x'}{L}$ is + or − according as $\frac{M'}{M}$ is $> 1 \cdot 0$ or $< 1 \cdot 0$.

[1] For recent tests of tables li. and lii. see Notes at end of chapter.

[2] Since $100 \div 1 \cdot 11 = 90$ nearly.

For trapezoidal channels table li. gives the ratio $\frac{A_b}{A_s}$, but the channels concerned had side-slopes of 4 to 3. For other side-slopes the increase of R, even with the same value of $\frac{A_b}{A_s}$, may differ somewhat, but the difference is likely to be considerable only for a deep narrow channel. In any case a correction can be made, as above, by considering the change in $\frac{C_2^2 R_2}{C_1^2 R_1}$ instead of in $\frac{C_2^2}{C_1^2}$. The actual values of R_1 and R_2 were as follows:—

Section ratio	= Infinity	3	·75	0·0
R_1	= 5·0	3·64	2·69	2·0
R_2	= 10·0	6·25	4·78	4·0
$\frac{R_2}{R_1}$	= 2·0	1·72	1·78	2·0

Regarding the hitherto neglected quantity h_v, the following table shows such values of it as have been worked out for the above cases. Except with high velocities h_v is small compared to $(D_1 - D_2)$. For a smaller channel $(D_1 - D_2)$ will be less, but probably V and h_v will also be less. By interpolating and noting that h_v is as V^2 the values of h_v for any case can be approximately obtained and $\frac{x'}{L}$ corrected by multiplying it by $\frac{D_1 - D_2}{D_1 - D_2 + h_v}$, which, since $D_2 > D_1$, is greater than unity, so that the correction increases $\frac{x'}{L}$.

Values of h_v.

Section Ratio (see table li.).		Velocity where depth is 5 feet.	Depths of Water: 5·125 to 5·25	5·25 to 5·5	5.5 to 6	6 to 6·5	6·5 to 7	7 to 7·5	7·5 to 8	8 to 8·5	8·5 to 9	9 to 9·5	9·5 to 10
			Values of $D_1 - D_2$: ·125	·25	·5	·5	·5	·5	·5	·5	·5	·5	·5
Rectangular.	2	6·0	·025	·046	·074	·058	·046	·036	·030	·026	·021	·018	·015
	4	1·73	...	...	·006	·005	...	...	·0025	...	...	·0014	·0013
	Infinity	2·12	·003	·006	·009	·007	...	·0046	...	...	...	...	·002
Trapezoidal.	3	1·81	...	...	·008	·006	...	...	...	·0023	...	...	...
	·75	1·56	...	...	·007	...	...	...	...	·0015	...	·001	...
	0·0	2·68	...	·013	·023	·015	·018	·007	·005	·004	...	...	·0016

Ordinarily the corrections have little effect, because D changes less rapidly than $\frac{x'}{L}$. Suppose the ratio $\frac{x'}{L}$ used is wrong by 4 per cent., then instead of giving the point where D is, say, 1·30, it gives the point where D is 1·28 or 1·32.

The profile can be easily extended with accuracy to a point where the depth is greater than $2D'$ by simply calculating the surface-slopes at the two ends of the extension and drawing two straight lines or even one.

Table lii. shows some co-efficients $\frac{x''}{L}$ for cases of drawing-down extending to half the natural depth. As with the curves of heading-up the greatest change of slope and the shortest curve occurs with a channel of triangular section. Fig. 140 shows one of the

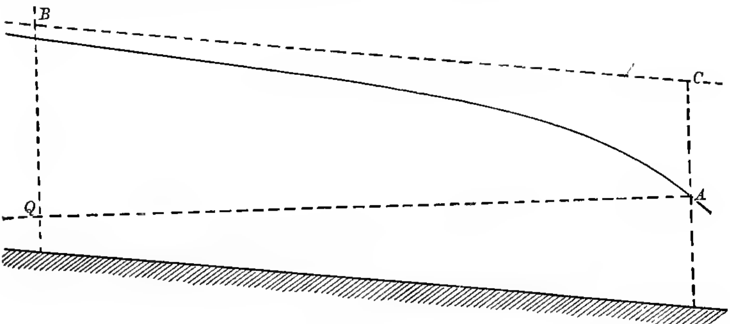


FIG. 140.

curves. The channels are the same as before, but the natural depth D' is now 10 feet, so that column 1 is not as before, and $D_1 - D_2$ is $\frac{D'}{20}$.

C_1 now refers to the depth D' and C_2 to the depth $\frac{D'}{2}$. The correction to be applied to $\frac{x''}{L}$ for change in M is, as before, + or − according as $\frac{M'}{M}$ is $> 1·0$ or $< 1·0$, but it is greater than before in relative amount. The values of $\frac{R_1}{R_2}$ for the trapezoidal channels are the same as the values of $\frac{R_2}{R_1}$ given above. The correction for h_v is the same as before, and, as before, has the effect of increasing $\frac{x''}{L}$.

Where D is not much less than D' the surface-curve is very similar to that of heading-up, with similar proportionate depths; but as D decreases the resemblance ceases, and the curvature increases rapidly, a tangent to the curve tending to eventually become vertical instead of horizontal as in heading-up.

The ratios in tables li. and lii. have been arranged in the form given so as to admit of corrections being applied, or at least to show how the corrections affect them. Otherwise it would be more convenient to show $\frac{L}{x}$ instead of $\frac{x}{L}$. It is, however, easy to convert the figures. If they are converted and L is great it can be found once for all by adding up the various values of $\frac{L}{x}$ and multiplying by x.

14. Calculations of Discharges and Water-levels.—When the flow in a reach is not variable throughout, the discharge can be found from the depth—or *vice versâ*—in its upper portion, and thus V is known. Then, the depth at the lower end, or at any point in the variable length, being also known, the surface-curve can be found by the method of the preceding article.

When the flow is variable throughout a reach, such as AK (Figs. 122 and 123, p. 255), supposing a breach in uniformity to occur at K, an approximate discharge can be found by the formula for uniform flow, the slope being KA and the depth being greater or less than the mean of the two depths at K and A, according as draw or heading-up exists. The reach can then be divided into a few lengths, or left undivided (according as the relative difference in the two depths at K and A is great or small), and a nearer approximation made by using equation 74. If the depths at K and A are very different the channel can be assumed to extend up to B and table li. or lii. used. In any case the correct discharge is obtained when, the water-level at one end being assumed, that at the other end comes out correct.

Whether or not the flow is variable throughout the reach, if the discharge is so great as to affect the original water-level at the head of the reach, allowance must be made for this in assuming the water-level at B or K.

A case occurred [1] in which a cut, BA, with a level bed (Fig. 135, p. 266) connected two rivers. It was desired to ascertain how much water would flow along the cut. The writer of the article worked out the discharge from first principles by the aid of the calculus, the working occupying several pages. This case, as well as that shown in Fig. 136, can be dealt with as above, except that, D' being infinite, tables li. and lii. cannot be used, and that for the level bed equation 79 (which is simpler) is to be used instead of 74.

To find approximately the depth AN (Fig. 135) for which the

[1] *Minutes of Proceedings, Institution of Civil Engineers*, vol. liii.

discharge will be a maximum, BM being given, let $BM=D$ and $NA=y$. The section CQ is nearly as $\frac{D+y}{2}$, $\sqrt{R}$ as $\sqrt{\frac{D+y}{2}}$, and $\sqrt{S}$ as $\sqrt{\frac{D-y}{L}}$. Then assuming C constant, Q is nearly as $(D+y)(D^2-y^2)^{\frac{1}{2}}$;

$$\frac{dQ}{dy}=\text{constant}\times\{(D^2-y)^{\frac{1}{2}}-y(D+y)(D^2-y^2)^{\frac{1}{2}}\};$$
$$=\text{constant}\times(D^2-y^2-Dy-y^2).$$

When the expression in brackets is zero $y+\frac{D}{4}=\pm\frac{3D}{4}$.

The discharge is a maximum when $y=\frac{D}{2}$ and a minimum when $y=D$. The discharge, however, varies little for a considerable variation in y. In the case just referred to, when D was 8 feet, the discharges found were, C being constant,

$y=$ 1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.
$Q=$ 249	253	255	259	240	229.

Similar interesting problems occur on Inundation Canals, though, owing to the temporary nature of the conditions, approximate solutions are sufficient. When the head-reach of a canal is silted and the time is approaching when the canal, owing to the falling of the river, will go dry, a reserve head-channel is often opened. Sometimes the first one is also left open. Whether it should be left open or not depends on what extra supply it will give (when the water-level at the junction is raised by the opening of the reserve head) and on whether the slope in it will be so flat as to cause it to silt excessively. If only one head is to be open it is sometimes better to keep the reserve head closed, as the slope along it may be flat owing to the conditions in the shifting river.

On the Choa branch of the Sirhind Canal the water, four miles from the head, was headed up in order to work a mill, and the variable flow extended up to the head, thus vitiating the discharge table which depended on the reading of the head-gauge. The use of the table was abandoned, but it would be possible to correct it on the above principles, a gauge above the mill being also read. The case of a silted canal head (art. 8) is different because the bed is constantly changing.

Section IV.—Variable Flow in General

15. Flow in a Variable Channel.—Sections ii. and iii. of this chapter treat of uniform channels, but though the propositions are more easily stated and proved for uniform channels, they apply with certain modifications, which will readily suggest themselves, to variable channels. In uniform channels 'natural flow' and 'uniform flow' both have the same meaning. In a variable channel, if the water surfaces corresponding to various discharges are termed the natural surfaces, and if 'natural flow' is substituted for 'uniform flow,' nearly the whole of section ii. applies. For instance, if a weir is made, or a branch opened, the flow downstream of the alteration is still natural. The causes of variable flow described in article 5 may be causes of heading-up or drawing-down, or they may counteract each other, leaving the flow natural.

Generally a variable channel is in actual flow, so that the water-level, for at least one discharge, can be observed. One problem is to find the change of water-level which will be produced by a change in the channel. The only way of finding the surface profile exactly is to divide the channel into short lengths, in each of which the section is either uniform or is varying in one direction, and to use equation 74 (p. 264), which then applies. If the channel is so variable as to consist of a number of pools and rapids, the effect of a change of level at any point will often extend back only to the next rapid.

The surface-slope at any point is always given by equation 76 (p. 265), that is, roughly, by the equation $V=C\sqrt{RS}$, where S is the surface-slope. At any selected point let B be the width of the water-surface and d the mean depth. Then roughly

$$Q=AV=BdC\sqrt{RS}, \text{ or } S=\frac{Q^2}{B^2d^2C^2R}$$

Since $Bd=A$, therefore S changes in the opposite manner to A. Fig. 141 represents a case in which B is constant. Here d, R,

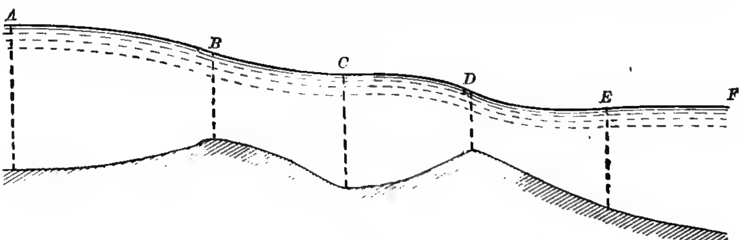


FIG. 141.

and C all change in the same manner, and the changes in S are very great. The vertical lines mark the points where it is a maximum or minimum. The convex and concave surface-curves touch one another at these points. The changes in S follow those in the bed, but are less pronounced. If, instead of d, B is supposed to vary, the profile is similar, but the changes in S less pronounced. If Fig. 141 is supposed to be a plan of such a channel, instead of a longitudinal section, the surface will still be like AF. If the changes of width and depth both occur together, and are of the same kind, the changes in S are greater.

If from any cause heading-up or drawing-down occurs at F the surface will undulate somewhat as before, approaching the natural surface towards A. The greater the depth of water in a channel the less the effect of inequalities in the bed. A stream which, at high water, has a fairly uniform surface-slope, may at low water form a succession of pools and rapids.

It has been stated that in a channel of varying width the discharge depends only on the least width, and that in clearing silt all clearance beyond the minimum width is useless. These statements are quite incorrect.

A stream may be so irregular in plan and section that the direction of the current is not parallel to what may seem to be the axis of the channel and the water-surface far from level across. The irregularities, if examined, will be found to be developments of those discussed under curves, obstructions, etc. Very often the excessive irregularity occurs only at low water.

16. Uniform and Variable Flow.—Whether variable flow takes place in a uniform or in a variable channel there are many degrees of variability. When the variability is very slight all the results found for uniform flow obviously apply, and the same is true, except as regards formulæ and exact calculations, when the variability is great. It will be clear, on consideration, that the discussions of chapter vi. all apply, even if two successive sections are not quite equal or similar.

In a variable stream a short length l can generally be found in which the flow is uniform. If observations are made in such a length for the purpose of finding C, the formula for uniform flow applies if S is the local slope. If the fall in l is very small the slope observations are often extended outside it. This was done in some of the Roorkee experiments on earthen channels, where the stream, though of uniform width, varied much in depth. The results seemed to disagree with Kutter's co-efficients, but when allowance was made for the variable flow they agreed quite well. No doubt similar error has occurred in many experiments. The proper method in such a case would be to observe V and R over the same length as that for which S is observed.

The surface-slopes at opposite banks of a stream are not generally equal unless it is quite uniform and straight.

17. **Rivers.**—A river, especially at low water, may be a series of separate streams with numerous junctions and bifurcations. The water-level in a side-channel *CAE* (Fig. 142) may afford only a very poor indication of the general water-level in the river. Suppose that with a good supply the water-level at *A* is the same as that at *B*. If there is silt in the channel *CA*—the silt being deepest at *C*—a moderate decrease of the river discharge may cause a great decrease in the discharge of *CA*, or even a total cessation of discharge. This causes great difficulties in the matter of gauge-readings in some Indian rivers. Suppose a gauge to have been originally at *B*. If erosion of the bank sets in the gauge has to be moved, and sometimes it is difficult to find another place (free from practical difficulties in the matter of reading the gauge and despatch of readings), except at such a place as *A* in a side channel. In floods, especially when the sandbanks between the channels are submerged, there is a general tendency for the water-surface to become level across, but it by no means follows that it becomes so. When the deep stream is at one side of the river channel the flood-level is nearly always higher on that side than at the opposite side.

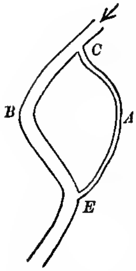


Fig. 142.

Since a small cross-section tends to cause scour and a large one silting, it follows that every stream tends to become uniform in section. The remarks made in articles 1, 2, and 8 also show that it tends to destroy obstructions, to assume a constant slope, and to become curved in such a way that its velocity will suit the soil through which it flows. If a river always discharged a constant volume its regimen would probably be permanent. It is the fluctuations in the discharge that cause disturbance.

Notes to Chapter VII

Momentum (arts. 1, 2, 3).—The effect of the momentum of flowing water is apt to be exaggerated. When a stream enters a tank or lake its current is quickly destroyed. When a large river enters the sea its effect on the colour or saltness of the water may be perceptible for a great distance, but this is because the sea level at the mouth of the river becomes very slightly raised so that currents are caused. These extend not only straight out to sea, but to right and left.

In the case of a sharp bend in a large river statements are sometimes made to the effect that the 'full force of the stream' has to be contended with. It seems to be implied that the momentum of the great mass of water is the danger. The scour along the concave bank is due to velocity, not to momentum. Sometimes it is implied that there is a danger of the river taking a straight course. This again depends on scour and velocity and is a rare occurrence, except as regards changes occurring within the sandy channel of a broad river.

Equation for Variable Flow (chap. ii. art. 10, and chap. iv. art. 15).—In a cross-section of a stream the mean of the squares of all the velocities exceeds the square of the mean. In the case of the numbers 2, 3, 4, the one quantity is 9·67 and the other 9·0. The proper percentage to be added to $\frac{V_1^2 - V_2^2}{2g}$ can only be decided by observation at the place, but can probably in the case of a contracted channel[1] be taken at 11 or $\frac{1}{9}$th.

Tests of Tables LI. and LII.—Curves of heading up and draw-down for two concrete conduits, one rectangular and one circular, have been worked out by Jameson.[2] For a rectangular section 7·08 feet wide, with $D' = 2{\cdot}875$ feet (section ratio 2·46), the lengths in which D increased from 4 feet to 4·5 feet and from 4·5 feet to 5 feet were 2370 feet and 2087 feet respectively. The figures arrived at by using table li. are 2409 feet and 2157 feet. The difference is no doubt due chiefly to the effect of h_v, which in the conduit was quite appreciable.

The section of the second channel is shown in Fig. 139A, the diameter being 7·3 feet, and the natural depth 3·4 feet, with a heading up of 2·08 feet. The curve of heading up is practically parallel to that for the rectangular channel. This was to be expected, since the sides are nearly vertical and the relative increase in sectional area and hydraulic radius nearly as before. In using table li. for such a channel the section would be assumed to be as shown by the broken lines.

In the case of the conduit of rectangular section above-mentioned the lengths in which D—in a case of draw-down—decreased from 2·5 feet to 2 feet and from 2 feet to 1·5 feet were 1474 feet and 345 feet respectively. By table lii. the lengths are 1735 feet and 513 feet. The difference is again due to the effect of h_v, this

[1] *Calculation of Flow in Open Channels.* Houk. See chap. iv. art. 15.

[2] Paper read at Inst. of Water Engineers, 5th December 1919.

quantity—considering in each case the whole length, and not dividing it up—amounting to ·089 and ·191, while $(D_1 - D_2)$ is in each case ·50. When D was 1·5 feet V was 5·3 feet per second. With such high ratios of V to D the correction for h_v is considerable.

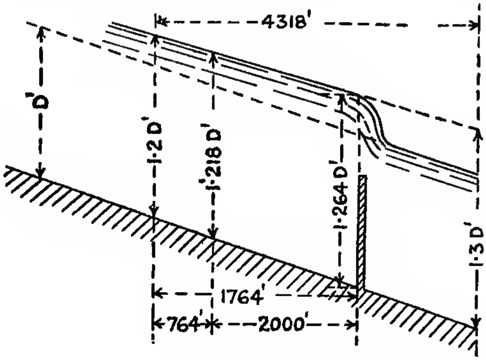


FIGURE FOR EXAMPLE 1.

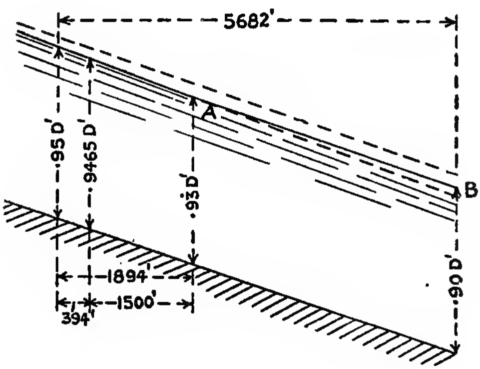


FIGURE FOR EXAMPLE 2.

EXAMPLES

Example 1.—In the channel considered in example 3 of chapter vi. a heading-up of 1·25 ft. is caused by a weir. What heading-up is caused 2000 feet upstream of the weir?

Table xlv. shows $A = 402{\cdot}6$ sq. ft. Also $A_b = 80 \times 4{\cdot}75 = 380$ sq. ft. $\therefore A_s = 22{\cdot}6$ sq. ft. and $\frac{A_b}{A_s} = 17$ nearly, so that $\frac{x'}{L}$ lies between the values for the first and second cases in the second part of table li., and somewhat nearer to the first than the second.

Since $S' = \frac{1}{5000}$ and $D_1 - D_2 = \frac{D'}{10} = \cdot 475$ ft. $\therefore$ $x' = \frac{D_1 - D_2}{S'} = \cdot 475 \times 5000 = 2375$ ft.

The headed-up depth at the weir is 6 ft. $= 4\cdot 75 \times 1\cdot 264$. From table li. $\frac{x'}{L}$ is about $\cdot 550$ when D_1 is $1\cdot 2D'$ and D_2 is $1\cdot 3D'$. Therefore $L = \frac{x'}{\cdot 550} = \frac{2375}{\cdot 550} = 4318$ ft. The distance of the weir downstream from the point where the depth is $1\cdot 20D'$ is $\frac{1\cdot 264 - 1\cdot 200}{1\cdot 30 - 1\cdot 20} \times 4318 = 2764$ ft. The point 2000 ft. upstream of the weir is thus 764 ft. from the above point, and the change of depth in this length is $(1\cdot 30 - 1\cdot 20)D' \times \frac{764}{4318} = \cdot 018D'$, so that the heading-up is $(1\cdot 218 - 1\cdot 00)D'$, or $\cdot 218 \times \cdot 475$ ft., or $1\cdot 04$ ft. Corrections if applied to this case might alter the result by $\cdot 01$ ft.

Example 2.—From the stream considered in the first trial in example 2 of chapter vi. a branch is taken off and discharges 120 c. ft. per second. What lowering of the water-level is caused 1500 ft. upstream of the branch ?

Table xlv. shows $A = 356\cdot 3$. Also $A_b = 40 \times 7\cdot 5 = 300$ sq. ft. $\therefore$ $A_s = 56\cdot 3$ sq. ft. and $\frac{A_b}{A_s} = 5\cdot 32$, so that $\frac{x''}{L}$ lies between the values in the first two lines of the second part of table lii. The discharge below the bifurcation is 967 c. ft., and this is given by a depth of 7 ft., so that the lowering is $\cdot 5$ ft.

Since $S'' = \frac{1}{5000}$ and $D_1 - D_2 = \frac{D'}{20} = \cdot 375$ ft. $\therefore$ $x'' = \frac{D_1 - D_2}{S''} = \cdot 375 \times 5000 = 1875$ ft. The drawn-down depth at the bifurcation is 7 ft. $= 7\cdot 5 \times \cdot 9\dot{3}$ ft. From table lii. $\frac{x''}{L}$ is about $\cdot 33$, when D_1 is $\cdot 95D'$ and D_2 is $\cdot 90D'$. Therefore $L = \frac{x''}{\cdot 33} = \frac{1875}{\cdot 33} = 5682$ ft. The distance of the bifurcation downstream from the point where the depth is $\cdot 95D'$ is $\frac{\cdot 95 - \cdot 9\dot{3}}{\cdot 95 - \cdot 90} \times 5682 = 1894$ ft. The point 1500 ft. upstream of the branch is thus 394 ft. from the above point, and the change of depth in this length is $(\cdot 95 - \cdot 90)D' \times \frac{394}{5682} = \cdot 00347D'$, so that the drawing-down is $D' - (\cdot 95 - \cdot 0035)D'$ or $\cdot 0535 \times 7\cdot 5 = \cdot 401$ ft.

Table LI.—Ratios for calculating Profile of Surface when headed up. (Art. 13.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Section Ratio.	Ratios. $\frac{V^2}{V'^2}$ and $1-\frac{V^2}{V'^2}$	Depth Ratios Upper figures show $\frac{D_1}{D'}$, lower figures $\frac{D_2}{D'}$.											Values of M or $\frac{C_2}{C_1}$.		
		1·025 to 1·05	1·05 to 1·10	1·10 to 1·20	1·20 to 1·30	1·30 to 1·40	1·40 to 1·50	1·50 to 1·60	1·60 to 1·70	1·70 to 1·80	1·80 to 1·90	1·90 to 2·0	Actual which occurred.	Extreme Values. Maximum.	Extreme Values. Minimum.
Rectangular Sections. Ratio of Width to Depth as in column 1.															
2	$V^2 \div V'^2$	·903	·820	·682	·548	·448	·371	·313	·267	·229	·199	·175	1·07	1·12	1·02
	$\frac{x'}{L}$ or $1-(V^2 \div V'^2)$	·097	·180	·318	·452	·552	·629	·687	·733	·771	·801	·825			
4	$V^2 \div V'^2$	·892	·804	·659	·516	·416	·336	·280	·234	·201	·171	·149	1·10	1·16	1·03
	$\frac{x'}{L}$ or $1-(V^2 \div V'^2)$	·108	·196	·341	·484	·584	·664	·720	·766	·799	·829	·851			
Infinity	$V^2 \div V'^2$	·880	·777	·614	·458	·351	·274	·218	·175	·143	·118	·099	1·17	1·28	1·05
	$\frac{x'}{L}$ or $1-(V^2 \div V^2)$	·120	·223	·386	·542	·649	·726	·782	·825	·857	·882	·901			
Trapezoidal Sections. Ratio $\frac{A_b}{A_s} = \frac{\text{area of section over bed}}{\text{area over side-slopes}}$, as in column 1.															
Infinity	(The figures are the same as for the preceding case.)												1·17	1·28	1·05
3	$V^2 \div V'^2$	·864	·760	·600	·440	·334	·257	·198	·154	·122	·098	·080	1·13	1·21	1·04
	$\frac{x'}{L}$ or $1-(V^2 \div V'^2)$	·136	·240	·400	·560	·666	·743	·802	·846	·878	·902	·920			
·75	$V^2 \div V'^2$	·847	·780	·543	·870	·269	·194	·146	·109	·082	·064	·052	1·13	1·21	1·04
	$\frac{x'}{L}$ or $1-(V^2 \div V'^2)$	·153	·270	·457	·630	·731	·806	·854	·891	·918	·936	·948			
0·0	$V^2 \div V'^2$	·815	·672	·420	·292	·192	·130	·090	·064	·046	·034	·026	1·18	1·28	1·05
	$\frac{x'}{L}$ or $1-(V^2 \div V'^2)$	·185	·328	·580	·708	·808	·870	·910	·936	·954	·966	·974			

Table LII.—Ratios for calculating Profile of Surface when drawn down.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Section Ratio.	Ratios. $\frac{V^2}{V'^2}$ and $\frac{V^2}{V'^2}-1$	Depth Ratios. Upper figures show $\frac{D_1}{D'}$, lower figures $\frac{D_2}{D'}$.									Values of M or $\frac{C_2}{C_1}$.		
		·95 to ·90	·90 to ·85	·85 to ·80	·80 to ·75	·75 to ·70	·70 to ·65	·65 to ·60	·60 to ·55	·55 to ·50	Actual which occurred.	Extreme Values. Maximum.	Extreme Values. Minimum.
Rectangular Sections. Ratio of Width to Depth as in column 1.													
1	$V^2 \div V'^2$	1·21	1·39	1·62	1·90	2·25	2·72	3·33	4·14	5·31	·935	·89	·98
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·21	·39	·62	·90	1·25	1·72	2·33	3·14	4·31			
2	$V^2 \div V'^2$	1·24	1·45	1·69	2·02	2·42	3·00	3·72	4·76	6·24	·909	·86	·97
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·24	·45	·69	1·02	1·42	2·00	2·72	3·76	5·24			
Infinity	$V^2 \div V'^2$	1·31	1·59	1·94	2·42	3·04	3·89	5·07	6·80	9·35	·85	·78	·95
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·31	·59	·94	1·42	2·04	2·89	4·07	5·80	8·35			
Trapezoidal Sections. Ratio $\frac{A_b}{A_s} = \frac{\text{area of section over bed}}{\text{area over side-slopes}}$, as in column 1.													
Infinity	(The figures are the same as for the preceding case.)										·85	·78	·95
1·5	$V^2 \div V'^2$	1·34	1·67	2·10	2·70	3·50	4·55	6·00	8·18	11·35	·88	·83	·96
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·34	·67	1·10	1·70	2·50	3·55	5·00	7·18	10·35			
·375	$V^2 \div V'^2$	1·35	1·73	2·30	3·08	4·12	5·69	7·84	11·50	17·17	·88	·83	·96
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·35	·73	1·30	2·08	3·12	4·69	6·84	10·50	16·17			
0·0	$V^2 \div V'^2$	1·52	2·06	2·84	3·99	5·77	8·49	12·94	18·62	31·32	·84	·78	·95
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·52	1·06	1·84	2·99	4·77	7·49	11·94	17·62	30·32			

CHAPTER VIII

HYDRAULIC OBSERVATIONS

[For general remarks on Hydraulic Observations, see chap. ii. art. 25]

SECTION I.—GENERAL

1. **Velocities.**—When the velocity is observed at one or more points in the cross-section of a stream, the process is termed 'Point Measurement.' When the mean velocity on a line in the plane of the cross-section is found directly, it is known as an 'Integrated Measurement.' Velocity-measuring instruments are of two classes, namely, 'Floats' and 'Fixed Instruments.' Fixed Instruments give the velocities in one cross-section of a stream. Floats give the average velocity in the 'run' or length over which they are timed, and not that at one cross-section. Floats are used only in open streams, but fixed instruments sometimes in pipes.

With most instruments time observations are necessary. The best instrument for this is a chronometer beating half-seconds, similar to those used at sea, or a stop-watch which can be read to quarter-seconds. The next best is a common pendulum swinging in half-seconds, and after that an ordinary watch. The error in timing with a chronometer is not likely to exceed half a second, with an ordinary watch it may be one or even two seconds. Some stop-watches and watches not only do not keep proper time, but are not regular in their speed. If any such defect is suspected the instrument should be tested. The time over which an observation extends should be such that any error in timing will be relatively small. In order to eliminate the 'personal equation' of the observer similar observations at the beginning and end of the time should be performed by the same individual, or if performed by two they should frequently change places.

Floats include surface-floats, sub-surface floats, and rod-floats. The first two are used for point measurement, the last for integrated measurements on vertical lines. A float travels with the stream, and so interferes little with the natural motion of the water. Its velocity is supposed to be the same as that of the water which it displaces.

Fixed Instruments are divided into Current Meters and Pressure Instruments. In the former the velocity of the stream is inferred from that of a revolving screw, in the latter from indications caused directly by the pressure of the water.[1] Velocities cannot be obtained by Fixed Instruments until they have been 'Rated,' that is, until it has been ascertained that certain indications of the instrument correspond to certain velocities. Fixed instruments interfere with the natural motion of the stream, but this need not cause error. The disturbance is almost entirely downstream of an obstruction (chap. ii. art. 21), and if those parts of the instrument which are intended to receive the effect of the current are kept well upstream, no difficulty arises, except perhaps in very small streams. If a boat is used the bow can be kept pointing upstream and the instrument upstream of the bow, a platform being made to project over the bow. Even if the boat or instrument is so large (which is not likely) relatively to the stream as to cause a general heading-up, this will not prevent a correct measurement of the discharge, though it may affect the surface-slope. In order that disturbance may not be caused by moorings the boat should (unless it is a steam-launch which can maintain its position) be held by shore-lines. If attached by its bow to a pulley running on a transverse rope, it can quickly be brought, by using the rudder, to any required point. Another transverse rope serves to keep the boat steady and, if divided by marks, shows its position. In a wide stream containing shallows the ropes may rest on trestles placed at the shallows. Where moorings must be used it is best to moor two boats side by side, as far apart as practicable, and to work from a platform between them, keeping the instrument well upstream.

The choice of an instrument for velocity measurement depends on various considerations. Floats require a regular stream, but fixed instruments can be used in any stream. In comparing the Current-Meter, or Pitot's Tube with Floats, regard must be had to the design and quality of the instruments available, and to the manner in which they were rated. Sub-surface floats are unsuit-

[1] Further information concerning Fixed Instruments is given in Sections IV. and V., but the varieties and details are very numerous and cannot all be discussed. There are many papers on these instruments in the *Minutes of Proceedings of the Institution of Civil Engineers* and *Transactions of the American Society of Civil Engineers*.

able when the stream is rapid or when there are weeds growing in it, fixed instruments unsuitable when the velocity is very low. For surface velocities alone surface-floats are, in regular streams, the best instruments unless there is considerable wind. For integrated measurements the rod-float is as good as any instrument, provided the bed is even enough to allow of a rod of the proper length, or nearly the proper length, being used.

The above considerations refer to accuracy only. As regards the time occupied and the number of observers required, fixed instruments generally have the advantage. In a discharge measurement of a large river current-meter integration measurements can be made while the soundings across the channel are being taken. On the other hand, the time occupied in rating the fixed instruments, their initial cost, and their liability to damage or loss; especially in out-of-the-way places, may be very important factors. If a stream is too wide to be reached at all points without a boat, has no suitable bridge, but is still narrow enough for the floats to be thrown in from the sides, and if no soundings are required, float observations may take less time than others.

2. **Discharges.**—The discharge of any small volume of water is best found not by measuring the velocity, but by letting the water pass into a tank and measuring the volume added in a given time. In this method nothing, or next to nothing, is left to assumption. Whenever leakage, absorption, or evaporation occur, allowance must be made for them. For very small discharges the water can be weighed. The methods adopted for high discharges are as follows.

The discharge of an open stream is usually found by observing the depths and mean velocities on a number of verticals. Let *ABC* (Fig. 143) be the mean velocity curve, and *ADEFC* a curve

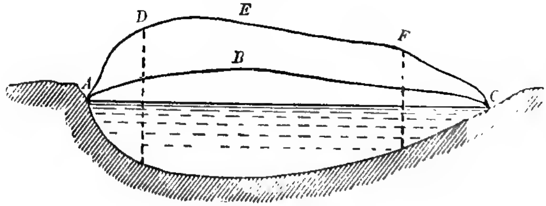


FIG. 143.

whose ordinates are found by multiplying the depth on each vertical by the corresponding velocity. Then *ADEFC* is the discharge curve, and its area is the discharge. If floats are used the velocities obtained are the averages in the run, and the depths must also be averages in the run. The more numerous the verticals the more accurate the result. For ordinary work ten is a fair number; for very accurate work, twenty. In the segments *AD*, *FC*, near the sides the verticals should be nearer together than elsewhere, because the ordinates change rapidly. The equal spacing of the verticals in each segment is not essential, but it simplifies the calculation, as it is only necessary to add together all the ordinates in a segment—deducting half the first and last—and multiply the sum by the distance between the ordinates. The discharges of all the segments added together gives that of the stream. If the number of equal spaces in a segment is even Simpson's rule can be used, but ordinarily the results of formulæ such as this differ very little from those of the simpler rule.

Sometimes the spacing in a segment cannot be equal. If there is in the cross-section any marked angle, whether salient or re-entering, a measurement should be made there. Sometimes when floats are used in rivers the velocities must be observed where the floats happen to run. In such cases the depths at these exact points need not be measured, but may be inferred from those observed at fixed intervals or found by plotting the section.

If the mean velocity on a vertical is obtained by multiplying the observed surface velocity by the co-efficient β (chap. vi. art. 9), and if β is the same for all verticals, the discharge may be calculated as if the surface velocities were the means on verticals and the whole discharge multiplied by β.

Discharge observations in an open stream are greatly facilitated by the construction of a 'Flume.' A short length of the channel is constructed of masonry or timber. The sides may be sloping but are preferably vertical. In the absence of silt deposit the section of the stream is known from the water-level, and if rod-floats are used they are all of one length. Flumes may, however, prevent proper surface-slope observations (chap. vii. art. 5). Discharges can be obtained with more or less exactness by the observation of U or U_s and the use of α or δ (chap. vi. art. 10), but a flume may be unsuitable for this (chap. ii. art. 21) if there is any abrupt change at its upstream end.

When the velocities in the whole cross-section of any open stream cannot be observed, and even the approximate method just mentioned is impracticable—as, for instance, in the case of a flood—

the velocity is calculated from the surface slope and cross-section. At the time of the flood, stakes should be driven in at the water level, or other marks made. If this is not done flood marks on trees or other objects should be observed in as great a number as possible and discrepancies averaged. Flood discharges can also be calculated from the water levels at bridge openings or contracted portions of channel (chap. iv. art. 15).

Whenever discharges of open streams are observed it is highly desirable to observe the surface slope and so ascertain C if for no other purpose than that of adding to existing information as to co-efficients and values of N. But such observations cannot usefully be made in any perfunctory manner. The greatest care is required. In an earthen channel there is often the chance of the sectional area varying within the slope length. The errors which occurred in the Roorkee Hydraulic Experiments have been mentioned (chap. vii. arts. 5 and 16). Preliminary longitudinal soundings should if possible be taken over the whole slope length or V should be observed at several places within the slope length. If the channel is decidedly irregular, as in the case of many rivers, several cross-sections should be taken within the slope length and the mean value of V computed. Neglect of such precautions as the above has led to remarkably erratic values of N being reported. (See also art. 7).

The discharge, Q, of a small body of water can be ascertained by "chemical gauging." A small and steady supply, q, of a soluble material, say salt, is introduced into the stream. At a point further downstream where thorough mixture has taken place samples of the water are taken. A cubic foot, 62·4 lbs., is found to contain a certain weight w of the chemical. The ratio of Q to q is the same as that of 62·4 to w. Parker[1] mentions some practical difficulties which may occur. He considers that, in order to obtain a steady supply, a concentrated solution of the chemical must be made and discharged through an orifice, but that, owing to impurities and evaporation, the discharge will not be uniform unless the co-efficient for the orifice is ascertained for each fresh batch of the chemical.

The discharge of a large pipe can be found by observing the velocities by means of the Pitot tube (art. 14). The co-efficients for orifices and weirs in thin walls being well determined, these are frequently used as instruments for measuring the discharges of small open channels or large pipes; or weirs or orifices of kinds other than thin-wall. The same is the case with the Venturi

[1] *The Control of Water.*

water meter (art. 16). In all the above cases the chief assumption made is the value of the co-efficients—generally well known—appertaining to the instruments or devices used.

When a discharge table has been prepared for any site or aperture the discharge can be found by simply observing the water-level[1] or head or—in the case of a pipe—the hydraulic gradient. The discharge of a pipe may be altered by incrustation or vegetable growths, and that of a channel by changes occurring, not only at the site but downstream of it. Frequent measurement of the discharge may be necessary in order to correct the table. In such cases the sectional areas and velocities should be tabulated so that causes of error may be the more readily traced.

3. Soundings.—Soundings are generally taken to obtain a cross-section of a stream, but longitudinal sections may be required in order to find the most regular site, or in connection with float observations. In water not more than about 15 feet deep soundings are best taken with a rod, which may carry a flat shoe to prevent its being driven into the bed. In greater depths a weighted line is used.

Unless the velocity is very low it is best to observe soundings from a boat drifting downstream. The current then exerts little force on the rod or line, which can thus be kept vertical. It can be held so as to clear the bed by a small amount, and lowered at the proper moment. This plan is particularly suitable for obtaining the mean cross-section in the run when floats are used. As the boat drifts the bottom is frequently touched with the rod or line, and the readings booked and averaged. Any local shallow likely to interfere with the use of rod-floats is also thus detected. When shore-lines can be used the boat can be worked and the widths measured as described in article 1. In wide rivers lines of flags or 'range-poles' are used instead of ropes. An observer on the boat or on shore can note the moment when the boat crosses the line, and give a signal for the soundings to be taken. To determine the distance of the boat from the bank an observer in the boat reads an angle with a sextant, or an observer on shore reads it with a theodolite, following the boat with his instrument and keeping the cross wires on some part of it. When the line is reached the motion of the instrument is stopped and the angle read off.

4. Miscellaneous.—The diameters of pipes, while water was flowing, were measured by Williams, Hubbell, and Fenkell by

[1] This may be done by a self-recording gauge (art. 5).

means of a rod with a hook inserted through a stuffing-box. For obtaining the mean diameter in a length of pipe one method is to fill it with water, which is afterwards measured or weighed. If the joints are not closely filled in some error may be caused, and Smith in some experiments filled each separate piece of pipe before it was laid, and weighed the water it contained.

For ascertaining c_v and c_c for orifices special arrangements are required. The velocity of the jet is found by observing its range on a horizontal plane. A ring or movable orifice of nearly the size of the section of the jet may be placed so that the jet passes through it, the flow stopped, and the necessary distances measured. The actual velocity can then be found from equation 29 or 30 (p. 52), and, the actual head being measured, c_v is easily found.[1]

When observations of any kind are made a suitable form should be prepared and filled in. It should have spaces set apart for recording the date, time, gauge-reading, and (at least when floats are used) the direction and force of the wind.

When extreme accuracy is required, as in the case of important experiments, many precautions have to be taken. In small orifices the edges have to be got up with very great accuracy. Excellent work of this kind was done by Bilton (chap. iii. art. 8). With a weir great care is necessary in observing the head. Nearly all detailed accounts of hydraulic experiments, such as those referred to in this work, contain instructive details as to methods adopted.

Before undertaking any important experiments those concerned should carefully study in every detail the instruments and methods to be adopted and obtain preliminary practice with them.

On the question how far it is correct to disregard any co-efficient or experimental result which seems to be abnormal, it is to be noted that all observations made by any one person with equal care and under similar conditions are entitled to equal weight. If one experiment in a set gives a result greatly differing from the rest, it is often rejected by the observer himself, the inference being that there was a mistake, say in timing. This implies a reduced degree of care in that observation. Whether the difference is great enough to warrant rejection is a matter for the judgment of the observer. When it comes to an author accepting or rejecting the result of an observation at which he was not present, the difficulty is far greater because he does not know all the facts. In some cases experiments have been rejected

[1] But see chap. iii. art. 9.

without any reason being given, but apparently on the sole ground that the results disagree with those of some other experiments.

Section II.—Water-levels and Pressure Heads

5. **Gauges.**—For observing the water-level of an open stream the simplest kind of gauge is a vertical scale fixed in the stream and graduated to tenths of a foot. It may be of enamelled iron, screwed to a wooden post which is driven into the bed or spiked to a masonry work. The zero may conveniently be at the bed-level, so that the reading gives the depth of water. The actual gauge may extend only down to low-water level. If a gauge is exposed to the current it may be damaged by floating bodies, and it is difficult to read it accurately, owing to the piling-up of the water against the upstream face and the formation of a hollow downstream. These irregularities can be greatly reduced by sharpening the upstream and downstream faces of the post or the upstream face only.[1] Greater accuracy can be obtained by placing the gauge in a recess in the bank, but not where it is exposed to the effects of irregularities in the channel (chap. vii. art. 2), and by watching the fluctuations of the water-level, noting the highest and lowest readings within a period of about half a minute, and taking their mean; but very great accuracy by direct reading of a fixed gauge is difficult, because of the adhesion of the water to the gauge, and the differences in level of the point observed and the eye of the observer.

With floating gauges these difficulties are almost got rid of. The graduated rod is attached at its lower end to a float which rises and falls with the water-level. The rod travels vertically between guides, and it is read by means of a fixed pointer on a level with the eye of the observer. The float and rod should be of metal, so that they may not alter in weight by absorbing moisture; the float perfectly water-tight and its top conical, so that it may not

[1] Ward's Gauge, well known in India, consists of two vertical planks joined so as to form an angle upstream. The gauge is placed between the planks on the downstream side.

In a type of gauge used in tidal waters a pipe containing air extends down below the water. As the tide rises the air is compressed. The recording apparatus is actuated by a float resting on mercury in one leg of a U-tube, the other leg being in communication with the pipe. The record can be made at a considerable distance away (*Min. Proc. Inst. C.E.*, vol. cxcv.).

form a resting-place for solid matter. The gauge should occasionally be tested by comparison with a fixed gauge or bench-mark. For a given weight of float and rod the smaller the horizontal section of the float at the water-surface the more sensitive the gauge will be.

To reduce the oscillations of the surface a gauge, whether fixed or floating, may be placed in a masonry well communicating with the stream by a narrow vertical slit. It is not certain that the average water-level in the well is exactly the same as in the stream, but the difference can only be minute. The larger the well the better the light, and the less the oscillation of the water. The advantage of a slit as compared with a number of holes is that it can always be seen whether the communication is open, but in order to avoid the necessity for frequent inspection the oscillation of the water in the well should not be entirely destroyed. In observations made downstream of the head-gates of irrigation distributaries in India the oscillations were very violent—amounting to ·60 foot—but they were reduced to ·03 foot in the well by slits ·005 foot wide.[1]

Where a gauge does not exist the water-level can be measured from the edge of a wall or other fixed point, either above or below the surface. Owing to the oscillation of the water the end of the measuring-rod cannot be held exactly at the mean water-level. It should be held against the fixed point, and the mean reading taken as explained above. A self-registering gauge can be made by means of a paper band travelling horizontally and moved by clock-work and a pencil moving vertically and actuated by a float. The pencil draws a diagram[2] showing the gauge-readings. The water-level in a tank may be shown by a graduated glass tube fixed outside the tank and communicating with it.

The level of still water can be observed with extraordinary accuracy by Boyden's Hook-Gauge, which consists of a graduated rod with a hook at its lower end. The rod slides in a frame carrying a fixed vernier, and is worked by a slow-motion screw. If the hook is submerged, the frame fixed, and the rod moved upwards, the point of the hook, before emerging, causes a small capillary elevation of the surface. The rod is then depressed till the elevation is just visible. By this means the water-level can be read to the thousandth of a foot, and even to one five-thousandth in still water, by a skilled observer in certain lights. The hook-gauge is not of much use in streams because of the surface oscillation. It is most used in still water upstream of weirs.

[1] Gourley and Crimp used two 9-inch stoneware pipes placed on end, one above the other.

[2] Also see Notes at end of chapter.

To destroy oscillation and ripples, a box having holes in it may be placed in the water and the readings taken in the box. When observing with a hook-gauge in water not perfectly still the point of the hook should be set so as to be visible half the time. A pointed plumb-bob hung over the water from a closely graduated steel tape is sometimes used, and by it the surface-level can be observed to within ·005 foot. The adjustment of the level of the zero of the gauge above a weir may be effected by a levelling instrument. If effected from the level of the water when just beginning to flow over the crest capillary action may cause some error.

6. **Piezometers.**—The name 'Piezometer,' used chiefly for the pressure column of a pipe, is also used to include a gauge-well and its accompanying arrangements. In all such cases the surface, where the opening is, should be parallel to the direction of flow and flush with the general boundary of the stream, and the opening should be at right angles.[1] If it is oblique the water-level in the piezometer will be raised or depressed according as the opening points upstream or downstream. The well or pressure tube can be connected with any convenient point by flexible hose terminating in fixed glass graduated tubes. With high pressures the piezometers may be connected with columns of mercury, which may be surrounded by a water-jacket to keep the temperature nearly constant. Common pressure gauges are not accurate enough.

In the piezometers of pipes air is somewhat liable to accumulate and cause erroneous readings. When the presence of air is suspected the tubes should be allowed to flow freely for a few minutes. If flexible they can be shaken, and if stiff rapped with a hammer. Very small tubes are liable to obstruction by leaves or deposits and should be avoided, as also should glass gauge-tubes small enough to be affected by capillarity. The orifices should be drilled and made carefully flush. Instead of one orifice there may be four, 90° apart, in one cross-section of a pipe, all opening into an annular space from which the piezometer tube opens. It is not certain that this gives greater exactness, but with a single opening from the top of the pipe the accumulation of air is probably greatest. The air probably travels along the pipe at the top.

Pulsations with fluctuation of the water level may occur in piezometers and should be dealt with as described in art. 5.

[1] The sectional area of the pipe at the point of attachment should be the same as the mean area in the length over which the slope is measured.

The arrangements at the weirs where the most important observations (chap. iv. art. 1) have been made were as below. In all cases the surface containing the orifice was parallel to the axis of the stream.

Bazin.—An opening near the bed 4 inches square communicating with a well.

Francis.—A small box[1] with 1-inch holes in the bottom.

Fteley and Stearns.—For the 19-foot weir there was an opening ·04 foot in diameter and ·4 feet lower than the crest of the weir. From the opening a rubber pipe led to a pail below the weir.

For the 5-foot weir there was a board parallel to the side of the channel and 1·5 feet from it. The pipe leading to the pail started from an auger-hole in the board ·9 feet above the bed of the channel.

To find the heads on weirs piezometers connected with perforated tubes placed horizontally in the channel have been used in America, but they appear to give unreliable results, even when the holes open vertically. In experiments made at Cornell University[2] the 'middle piezometer' was a transverse 1-inch pipe, laid 8 inches above the bed and 10 feet upstream of the weir. The 'upper piezometer' was similar, but 15 feet further upstream. A 'flush piezometer' was also 'set in the bottom of the flume,' 6 inches upstream of the upper piezometer. The readings of these two differed on one occasion by ·3 foot. The readings of the upper and the middle also differed. It is believed that the opening from the rounded surface of the pipe, instead of from a plane surface, causes error, and that the error is one of defect. A 'longitudinal piezometer' was formed by certain perforated pipes. With high heads—a little over 3 feet—the longitudinal piezometer read ·099 foot higher than the upper piezometer. With a head of about ·17 foot there was no difference between the two. Experiments made by Williams[3] also show that the readings obtained with a transverse pipe with holes opening downwards, do not agree with those obtained by a simple opening in the side of the channel, being higher with low supplies and lower with higher supplies. It seems clear that all perforated pipe arrangements are to be avoided until their action is better understood.

7. **Surface-slope.**—Probably the best method of observing the slope in a short length of open stream is to dig two ditches from the extremities of the slope length, both leading into a well divided into two by a thin partition. The difference between the water-levels on the two sides of the partition is the local surface-fall. It can be very accurately measured, especially if the ditches

[1] The box projected somewhat into the stream, and this was not free from objection, as it caused an abrupt change.

[2] *Transactions of the American Society of Civil Engineers*, vol. xliv.

[3] *Ibid.* vol. xliv.

are treated as gauge-wells, that is, open into the stream by narrow slits. Slight leakage in the partition is probably of no consequence as long as it gives rise to no perceptible current in the ditch. The slope should, unless the stream is perfectly uniform and straight, be observed at both banks and the mean taken (chap. vii. art. 16).

For measuring the loss of pressure head in a short length of pipe or channel a differential gauge consisting of two parallel glass tubes with a scale fixed between them is commonly used. The two tubes are connected at the top where there is a cock, and their lower ends are connected by hose pipes with the two points in the pipe or channel. Capillarity does not vitiate the results because it is the difference that is taken. If the tubes are partly filled with water and the space above the water is occupied by air the difference in heights of the water columns gives the difference in head. When this difference would be too small to be accurately observed, paraffin—specific gravity, say, ·80—can be substituted for air. It is then as if the specific gravity of the water in the tube was equal to the difference between the specific gravities of water and paraffin. The difference in the heights of the two water columns is five times, more accurately 5·3 times, what it was. Also see art. 14.

In whatever way slope is observed the openings of any pair of gauge-wells, ditches, or piezometers must be exactly similar, and the observations should be repeated at intervals as long as the velocity observations last.

Section III.—Floats

8. Floats in general.—The size of a float used for point measurement is limited by the consideration that the mean velocity of the stream within the 'direct area' of the float (the area of its projection on a cross-section of the stream) must be practically equal to that at the point where the velocity is sought. The depth of the submerged part of a surface-float may be about one-twentieth of the depth of water, and the depth of a sub-surface float one-tenth, or, at the point of maximum velocity, one-twentieth of the depth of water. The width of a float of any kind may be about one-twentieth of the width of the stream, except for use near the bank, when it may be about one-tenth of the distance from the bank to the line of the float. The length is similarly limited because the float may revolve. The exposed part of a surface-float should be small compared to the submerged part. For deep water a good surface-float is made by a bottle submerged all but the neck, or a log deeply submerged; for shallow water by a disc almost totally submerged and carrying a small cylinder or knob. With all kinds of floats the exposed part should be of such a colour that it can easily be seen.

The 'lines' or boundaries of the run are marked by ropes stretched across the stream at right angles, or, if the width is great, by lines of flags. Observers signal each float as it crosses the lines, and another observer notes the times. When ropes are used the float-courses can be marked by 'pendants' of cloth or brass. Usually about three floats are signalled in rapid succession at the first line and then at the second. If on reaching the second line they have changed order, this affects the individual times recorded, but not the mean. With a stop-watch the time-observer may also be the float-observer. He can start and stop the watch while noting the float. But in this case each float must complete its course before another can be timed. With a slow current the time observer may also start the floats, and he may even use an ordinary watch. In a wide river the course of a float can be observed by an angular instrument (see art. 3).

A float required to travel in any course usually deviates from it. The deviation increases the distance over which it travels, but this is of no consequence because the object is to obtain the forward velocity (chap. i. art. 3). The deviation is of consequence only when the velocities in adjacent parts of the stream differ much from one another, that is, near banks or shallows. In such cases the 'run' of the float can be shortened, the deviation noted, and the mean course adopted. When ropes are used the approximate deviation can be seen by the float-starter by means of the pendants, especially when the rope is at a low level.

The length over which a float travels, upstream of the run, in order that it may acquire the velocity of the water, is called the 'dead run.' The float may be taken out into the stream, or thrown in from the bank, or placed in it from a bridge or boat. Throwing-in is often practicable with surface-floats, and sometimes with rods. A low-level single-span bridge is the most suitable arrangement, but if there are piers or abutments which interfere with the stream they disturb the flow, and a site downstream of them is unsuitable for velocity measurements, at least with floats (chap. ii. art. 21). Even a boat causes disturbance downstream. Two small boats or pontoons carrying a platform are better than a large boat.

The length of run to be adopted depends on the velocity and uniformity of the stream, the accuracy of the timing, and the distance of the float-course from the bank, this last consideration having reference to deviation. Ordinarily the length may be so fixed that the probable maximum error in timing will be only a small percentage of the time occupied. The length may, however, have to be reduced if the stream is not regular, especially if rods are used. Reduction of the length in order to avoid excessive deviation is most likely to be necessary for observations near the bank, especially with surface-floats. The surface-currents near the bank set towards the centre of the stream (chap. vi. art. 7), so that the tendency to deviation is greater, while the admissible deviation is less. Most observations are made at a distance from the bank, and the rejections for excessive deviation need not generally be numerous. A moderate number of rejections, owing to a long run, does not cause much loss of time, because in order to obtain a particular degree of approximation to the average velocity of the stream the number of floats recorded must be inversely proportional to the length of the run.

9. Sub-surface Floats.—A float used for measuring the velocity at a given depth below the surface is called a 'double-float.' A submerged 'lower float' somewhat heavier than water, is suspended by a thin 'cord' from a 'buoy' which moves on the surface. In the ordinary kind of double-float the buoy is made small, and the velocity of the instrument is assumed to be that of the stream at a depth represented by the length of the cord, but it is really different because of the current pressures on the buoy and cord, and the 'lift' of the float due to these pressures. There is also 'instability' of the lower float, caused chiefly by the eddies which rise from the bed. Any lateral deviation of the lower float adds to the lift, but otherwise is not of consequence, except near the banks. The resultant effect of all the faults is a distortion of the velocity curves obtained. When the maximum velocity is at the surface (Fig. 112, p. 184) the buoy and cord accelerate the lower float, and the lift brings it into a part of the stream where the velocity exceeds that at the assumed depth. Hence the velocity obtained is always too great, and the 'observation curve,' which is shown dotted, lies outside the true curve. When the maximum velocity is below the surface the curve is distorted as in Fig. 113.

A double-float is best suited to a slow current. The higher the velocity of the stream the greater the differences among the velocities at different levels and the greater the lift of the lower float; the greater also the strength of the eddies and the instability.

The defects of the double-float cannot be removed, but they can be much reduced by attention to the design. In order that the lower float may be as free as possible from instability, its shape should be such as to afford little hold to upward eddies. In order that it may be little affected by the current pressures on the buoy and cord, it should afford a good hold to the horizontal current. It should therefore consist of vertical plates, say of two cutting each other at right angles, with smooth surfaces, and lower edges sharpened. The upper edges should not be sharpened. Any downward current will then act as a corrective to instability. If the float tilts much its efficiency is reduced, but tilting can be prevented by avoiding a high ratio of width to height, and by making the upper and lower parts respectively of light and heavy materials, say wood and lead. If the thickness of the plates is uniform the resistance to tilting is a maximum when the heights of the heavy and light portions are inversely as the square roots of the specific gravities of the materials. It is an improvement to remove the central portions of the plates and to substitute for them a hollow vertical cylinder, in the middle of which the cord is attached by a swivel. This causes the pull of the cord, however the float revolves on its vertical axis, to be applied at the point where the average horizontal current pressure acts. The cord should be of the finest wire, and the buoy of light material, say hollow metal, smooth and spindle-shaped, the cord being attached towards one end, so as to make the float point in the direction of the resistance.

Given the velocity of the stream the force tending to cause instability of the lower float depends on its superficial area. Its stability depends on the ratio of its weight to its superficial area, that is, on the thickness of the plates. For all floats of the same shape and materials there is a certain thickness of plate which is the least consistent with stability, and a float should be composed of plates of this thickness, in order that the thickness of the cord and volume of buoy may be small. This thickness cannot be determined theoretically, but is a matter of judgment and experience. Of any two similar double-floats, that which has the larger lower float is the more efficient. If the direct areas of the lower floats are as 4 and 1, their weights and the submerged volumes of the buoys are as 4 and 1. But the direct areas of the buoys, if their shapes are similar, are as $4^{\frac{2}{3}}$ and 1 or nearly as 2·5 and 1. The thicknesses and direct areas of the cords are also theoretically as 2 and 1. In both cases the larger instrument has greatly the advantage, and practically, if the lower float is small, it is physically impossible to make the cord thin enough. The dimensions are limited by the considerations set forth above. The larger the stream the greater the admissible size of float.

The following statement shows that the double-floats which have been actually used in important experiments have been of bad design :—

Channel.	Observer.	Greatest Depth of Water.	Description of Lower Float.	Ratio of Direct Areas at Maximum Depth.		
				Lower Float.	Cord.	Buoy.
		Feet.				
Mississippi.	Humphreys and Abbott.	110	Keg with top and bottom removed.	1·0	1·75	·03
Irrawaddy.	Gordon.	70	Block of wood loaded with clay.	1·0	·73	·06
Ganges Canal.	Cunningham.	11	Ball (3 inches and $1\frac{5}{8}$ inch).	1·0	·48 ·72	·10

It is obvious that when the lower float was near the bed—or supposed to be near it—the observed velocities must, owing to the very great relative current-actions on the cord, and probably also to instability, have been so much in excess of the truth as to render them mere approximations, the general values found for bed velocities being perhaps about halfway between the real bed velocity and the mean velocity from the surface to the bed. The vertical velocity curves obtained with the above instruments often show marked peculiarities in form, the velocity sometimes seeming to remain constant or even increase as the bed is approached.

In the 'twin-float' the submerged part of the buoy or 'upper float' is of the same size, shape, and roughness as the lower float, and the velocity of the instrument is assumed to be a mean between the stream velocities at the surface and at the level of the lower float. The surface velocity is observed separately and eliminated. This causes additional trouble. The best form and size for the lower float are arrived at in the same manner as in the ordinary double-float. The difficulties arising from tilting and instability can be overcome by making the lower float heavy and the upper one light. The current pressure on the cord is less than with the ordinary double-float, but its inclination greater. The instrument has been very little used.

Cunningham has proposed a triple float for measuring the mean velocity on a vertical when the depth is too great for rod-floats, or the bed too uneven. It has a small buoy and two large submerged floats at depths of ·21 and ·79 respectively of the full depth, the upper of the two being light and the lower heavy. The instrument is supposed to give the mean of the velocities at these two depths, and this is nearly equal to the mean on the whole vertical. The figures ·21 and ·79 were arrived at theoretically by Cunningham, and they are the best for general use, the depth of the line of maximum velocity being supposed to be unknown. It would be preferable to use a multiple float with several equidistant submerged floats, the lower ones heavy and the upper ones light, the distance of the lowest from the bed and of the highest from the surface being half the distance between two adjoining floats. All these floats are best suited to slow currents.

10. Rod-floats.—A rod-float is a cylinder or prism ballasted so that in still water it floats upright. In flowing water it tilts because of the differences in the velocities of the stream. By using a rod reaching nearly to the bed the mean velocity on the vertical is obtained. Owing to the irregular movements of the water both the submerged length and the tilt of the rod vary slightly. The clearance below the bottom of the rod must be sufficient to prevent the bed being touched. The great advantage of a rod as compared with a multiple float is that there is no uncertainty as regards lift and instability.

Rods are usually made of wood or tin and weighted with lead. A wooden rod is liable to alter in weight from absorption of water, and it may then become too deeply submerged or sink. A cap containing shot fitted to the lower end of the rod gives a ready means of adjustment. In a rapid stream a wooden rod may have an excessive tilt, and a tin rod is better. It is lighter and can carr more ballast. It is, however, liable to damage and to spring a leak. A rod may sometimes sink, owing to its grounding and being turned over by the current. In a rapid stream a wooden rod may be turned over even without grounding. Wooden rods can be more easily made square than of other sections. In any case the section and degree of roughness must be uniform throughout.

For a rod 1 foot long, 1 inch; and for one 10 feet long, $2\frac{1}{2}$ inches are suitable diameters. Rods are often made up in sets, the lengths increasing by half-feet, or for small depths by quarter-feet, but this does not give sufficient exactitude, and it often leads to the use of rods much too short. Owing to the unevenness of the bed a rod of the proper theoretical length is usually too long, and the next length is perhaps 10 or 15 per cent. shorter. A set of short adjusting pieces to screw on to the tops of the rods should be provided. Rods for use in very deep water are sometimes made in lengths screwed together. It is convenient to have rods divided into feet, beginning from the bottom. If the tilt is likely to be great, allowance can be made for it in selecting the length to be used.

It has been said that a rod, owing to its not reaching down to the slowest part of the stream, must move with a velocity greater than the mean on the whole vertical. Cunningham has attempted to show theoretically that the length[1] of a rod must be ·945, ·927, or ·950 of the full depth of water according as the point of maximum velocity is at the surface, at one-third depth, or at half-depth. The proof rests on the assumption that the vertical velocity curve is a parabola. It has been shown (chap. vi. art. 9) that it is not a parabola, and that the velocity probably decreases very rapidly close to the bed, and for this last reason it is probable that a rod reaching close to the bed would move too slowly. The proper length of rod cannot be calculated theoretically in the present state of knowledge.

A large number of experiments with rod-floats were made by Francis. The discharges obtained by rods in a masonry flume of rectangular section with a depth of water of 6 feet to 10 feet were compared with the discharges obtained from a weir in a thin wall, and the following formula was deduced—

$$V = V_r\left(1{\cdot}012 - {\cdot}116\sqrt{\frac{D-d}{D}}\right),$$

when V is the mean velocity on the vertical, V_r the rod velocity, d the length[1] of the rod, and D the depth of the stream. According to this formula the correct length of rod, so that V and V_r may be equal, is $\cdot 99D$, and the errors due to shortness of rod are as follows:—

$\frac{d}{D}=$	·75	·80	·85	·90	·93	·95	·96	·97	·98	·99
$\frac{V}{V_r}=$	·954	·961	·968	·975	·981	·986	·989	·992	·996	1·00

[1] That is, submerged length.

The discharges obtained by the weir are believed to be very nearly correct, and the acceptance of the above figures is recommended. Accepting them, the proper length of a rod is ·99 of the full depth, and if the length is only ·93 of the full depth the velocity found is 2 per cent. in excess. In earthen channels a rod of the proper length can hardly ever be used, but allowance can be made for its shortness.

SECTION IV.—CURRENT-METERS

11. General Description.—The current-meter consists of a screw, resembling that of a ship, and mechanism for recording the number of its revolutions. Frequently this mechanism is on the same frame as the screw, and by means of a cord it can be put in and out of gear. The reading having been noted, the meter is placed in the water, the recording apparatus brought into gear, and, after a measured time, put out of gear and a fresh reading taken. The difference in the readings gives the number of revolutions, and this divided by the time gives the number of revolutions per second. This again, by the application of a suitable co-efficient, determined when the instrument is rated, can be converted into the velocity of the stream. The co-efficient depends on the 'slip of the screw,' and varies for each instrument and each velocity. With many meters the recording apparatus is above water, and there is electric communication between it and the screw. The meter can then be allowed to run for an indefinite time without raising to read. For each meter there is a minimum velocity below which the screw ceases to revolve. This may be as low as six feet per minute.

Sometimes a current-meter is carried on a vertical pivot and provided with a vane. The irregularity of the current causes the instrument to swing about, and so to register the total and not the 'forward' velocity. It is better to keep the instrument fixed with the axis parallel to that of the stream, but if the axis swings through a total angle of 20°—10° either way—the velocity registered is only ·75 per cent. in excess of the forward velocity, and if the total angle is 40°, 3 per cent. in excess.

A current-meter may be used in a small stream from the bank or from a bridge, but generally it is used from a boat. This has already been referred to (art. 1). The rod or chain to which the meter is attached should be graduated. If a rod is used, it may be sharpened or rounded on its upstream face, the downstream face being flat, and resting against a portion of the platform fixed at right angles to the centre line of the boat. The rod can be provided with a collar, which can be clamped on to it in such a position, that when it rests on the platform the meter is at the depth required. In water 53 feet deep Revy attached the meter to a horizontal iron bar, which was lowered by ropes fastened to its ends, and was kept in position by diagonal ropes. In shallow water an iron rod is sometimes fixed, on which the meter slides up and down, but this causes delay.

In some experiments the time in quarter-seconds, position of the meter, and number of revolutions of the screw have been automatically recorded on a band driven by clockwork. With a meter having electric communication with the bank a wire rope has been stretched across a wide stream, the meter carried on a frame slung from the rope, and the discharge of the stream thus observed. In other cases the observers travel in a cage slung from a wire rope. It is quite usual to have several meters working simultaneously at different depths. In integration it is not necessary for the descending and ascending velocities to be equal, and two or three up and down movements may be made without raising to read. It is a common practice, after taking an observation lasting a few minutes, to check it by a shorter one. To facilitate the computation of the meter velocity the times may be whole numbers of hundreds of seconds. A stop-watch may be started and stopped by the same movement, which puts the instrument in and out of gear.

The rate of a current meter is liable, at first, to increase slightly, owing to the bearings working smoother by use. It should be allowed to run for some time before being rated. Oil should not be used, as it is gradually removed by the water, and the rate may then alter. Every time a meter is used the screw should be spun round by hand to see that it is working smoothly. A gentle breeze should keep it revolving. A second instrument should be kept at hand for comparison. A short test of the rating should frequently be made. If tests made at two or three velocities all show small or proportionate changes of one kind similar corrections may be applied to other velocities, but if the changes are great or irregular the instrument should be rated afresh.

The speed of a current-meter is liable to be affected by weeds, leaves, etc., becoming entangled in the working parts. If any are found when the instrument is read the observation can be rejected, but some may become entangled and detached again without being seen. The effect must be to reduce the velocity, and any abnormally low result may be rejected. The rate of the instrument is also liable to be affected by silt and grit getting into the working parts and increasing the friction. The only rubbing surface which has a high velocity is the axis of the screw, and this is probably the part chiefly affected. In using a current-meter of the kind illustrated (Fig. 144) it was found on one occasion that it rapidly became stiff. The meter having been cleaned, the screw ran freely again, but again became stiff. The stream was six feet deep and

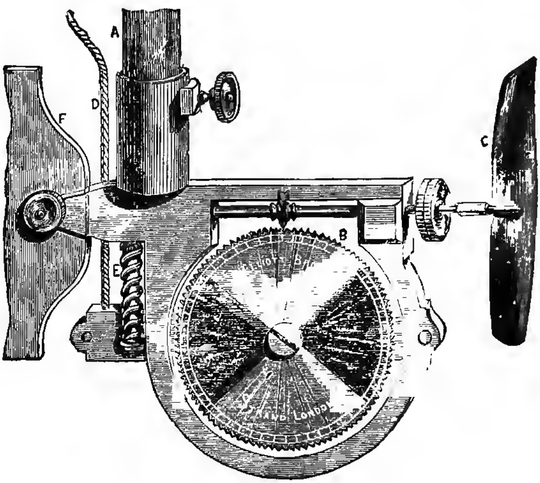


FIG. 144.

had a velocity of about seven feet per second. The water contained silt and probably fine sand, which gradually increased the friction. The clogging was most rapid in observations below mid-depth, and it is probable that there was more sand in that part of the stream.

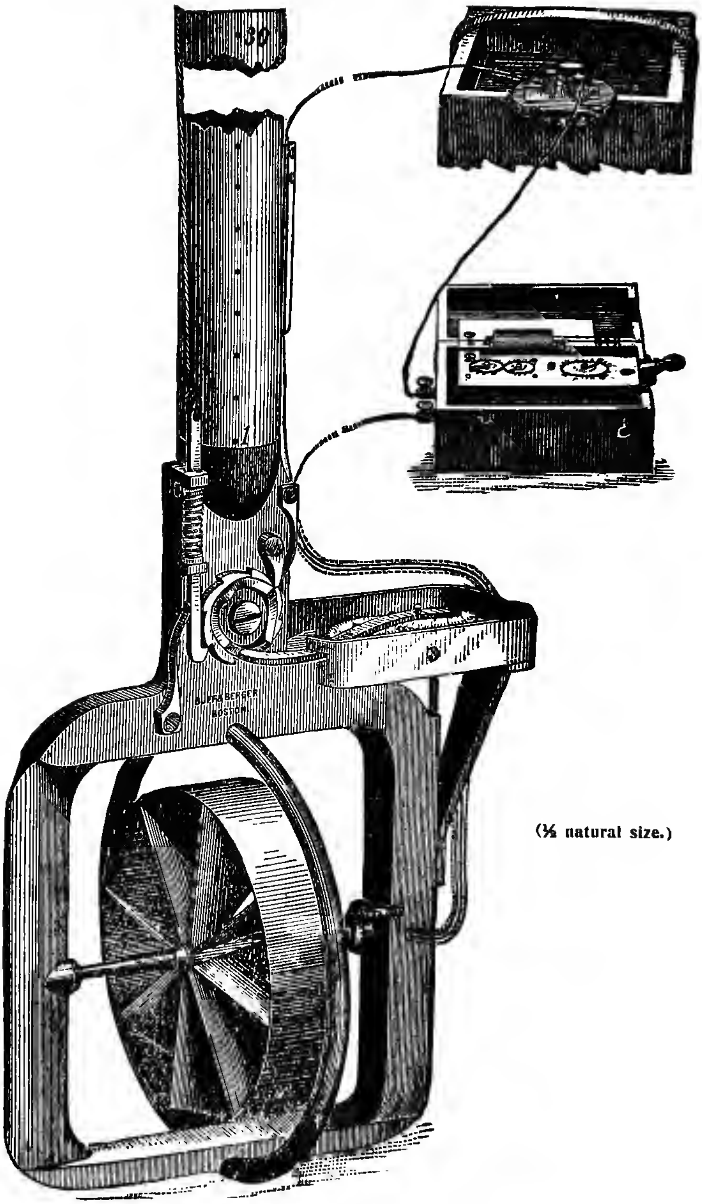
12. Varieties of Current-meters.—There are probably twenty kinds of current-meter. Each kind has its own special advantages or disadvantages. Fig. 144 shows a meter sold by Elliott Brothers, London. The instrument is attached by the clamping screw to a rod A. By pulling the cord D the wheel B is geared with the screw. A vane F can, if desired, be attached. A meter very similar to the above is made in the Canal workshops at Roorkee, India, but it is pivoted on the tube which carries the screw for clamping it to the rod.

In Revy's current-meter friction is reduced by a hollow boss on the axle of the screw of such a size that the weight of the whole is equal to that of the water displaced. The recording mechanism is enclosed in a box covered by a glass plate, filled with clear water, and communicating by a small hole with the water in the stream, so that the glass may not be broken by the pressure at great depths. A horizontal vane is added under the screw, so that it may revolve freely while the meter rests on the bed.

Moore's current-meter consists of a brass cylinder, 10½ inches long, provided with screw-blades. In front of the cylinder is an ogival head which is fixed to the frame. The cylinder, which is water-tight, revolves, and the reading apparatus is inside it, the reading being observed through a pane of glass. The instrument is hung from a cord or chain. This renders it easier to manipulate. To prevent its being forced far out of position, a weight is suspended to the frame, and it should be sufficient to prevent the instrument being temporarily displaced by the tightening of the gearing cord. The instrument has horizontal and vertical vanes and can swing in any direction.

In Harlacler's current-meter there is electric connection between the worm-wheel driven by the screw and a box above water. At every hundred revolutions of the screw the worm-wheel makes an electrical contact, and an electro-magnet in the box exposes and withdraws a coloured disc. The meter slides on a fixed wooden rod. A tube lying along the rod carries the electric wires, and serves to adjust the meter on the rod. In one variety the axle of the screw carries an eccentric which makes an electric contact every revolution, and thus enables individual revolutions to be noted.

Fig. 145 shows a current-meter sold by Buff and Berger, Boston, U.S.A. The object of the band encircling the screw is to protect the blades from accidental changes of form, which would cause a change in the rate of the instrument. A bar underneath the screw and a stout wire running round at a short distance outside it affords additional protection, and enables the instrument to be used close to the bed or side of a channel. There are two end bearings and a very light screw and axle, and the screw revolves with one-fourth of the velocity required to turn a similar one with the usual sleeve bearing. The friction is so small that the rate is not altered by silt or grit. The meter is fixed to a brass tube, which has a line along it to show the direction of the axis when the meter cannot be seen. The meter is sold with the recording apparatus either on the frame or with electric connection, as in the figure. Stearns used a meter of this type, and provided with two screws, either of which could be used. One



(½ natural size.)

FIG. 145.

had eight vanes and the other ten In the latter half the vanes had one pitch and the other half a different pitch. The eight-vane screw began to move with a velocity of ·104, and the ten-vane screw with a velocity of ·094, feet per second.

In the Haskell current meter (Fig. 145A) the screw is somewhat in the form of a cone with the apex upstream. This shape is intended to give it strength to resist damage from objects carried against it, and also to readily throw off weeds. Screws of two pitches are made. The one with the lower pitch—this appears to be the more generally used—is suitable for velocities from ·2 foot to 10 feet per second, the higher pitch for velocities from 1 foot to 16 feet per second. The bearings are of large surface and not liable to rapid wear and are under cover, so that grit cannot affect them. The velocity register is above water and has electric communication with the meter. Starting and stopping the watch makes and breaks the circuit.

The type meant for use in deep rivers (Fig. 145A) is suspended, swings on a vertical axis, and is provided with a torpedo-shaped lead weight. On the Irrawaddy it was found by Samuelson[1] that a weight of 80 lbs. was necessary. A type for use in small streams, and made in two sizes, is held on a graduated rod. It can be clamped, or can be left free to swing. A "set-back" velocity register is also supplied. This can be set back to zero after each observation. The Ritchie-Haskell "Direction Current Meter" indicates also the direction of the current, which in a tide-way may not always be the same as at the surface.

Another well-known current meter is Price's. Both this and Haskell's are made in the U.S.A.

In the cup pattern of current meter there is no screw. The wheel is provided with conical cups placed in a circle like the floats of a water-wheel. Each cup presents its open end to the stream and is driven downstream. It presents its conical end as it returns upstream.

Observations made by Groat[2] indicate that in perturbed water such as a tail race, the results given by a cup current meter may be 6 per cent. too high, those by a screw meter 1 per cent. too low, that in violently perturbed water the above differences may be 25 and 3 to 4 per cent. respectively, but that if the meters are allowed to run long enough the errors disappear.

[1] Note on the Irrawaddy River.

[2] *Proc. Am. Soc. C.E.*, 1912, vol. xxxviii.

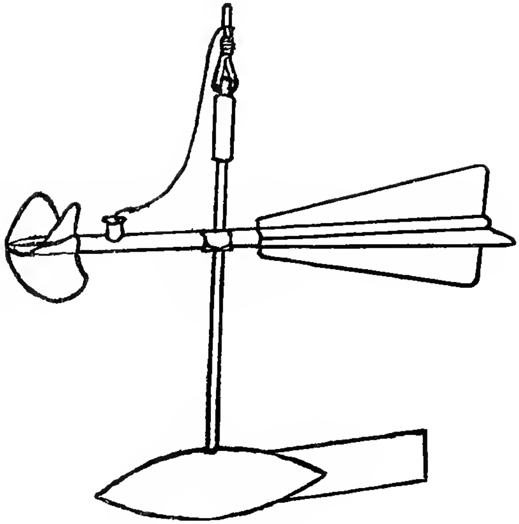


Fig. 145a.

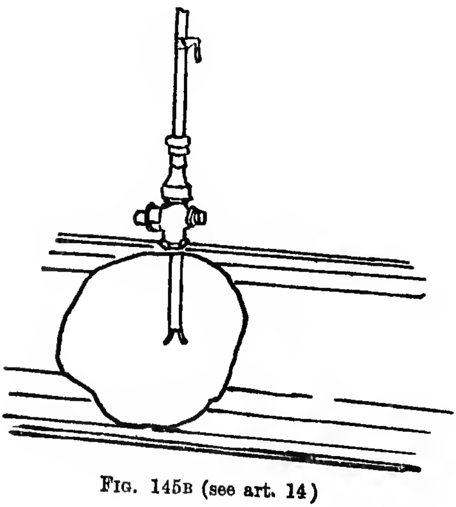


Fig. 145b (see art. 14)

A cup instrument must interfere considerably with the natural movement of the water. In an instrument of the screw type little more than the edges of the screw, when it is revolving, are presented to the current.

One kind of current-meter has no regular recording apparatus, but simply a device for making and breaking circuit and a sounder. The revolutions are counted by the clicks. A current-meter made by von Wagner gave its indications by sound, but the counting was effected by an arrangement like the seconds hand of a watch. At each stroke, or with high velocities at every fourth stroke, the observer pressed a button which caused the hand to move one division.

13. **Rating of Current-meters.**—The usual method of rating is to move the instrument through still water with a uniform velocity, and to repeat the process with other velocities covering a wide range. The instrument may be held at the bow of a boat, or attached to a car running on rails, or on a suspended wire. In case the water should not be quite still the runs should be taken alternately in reverse directions.

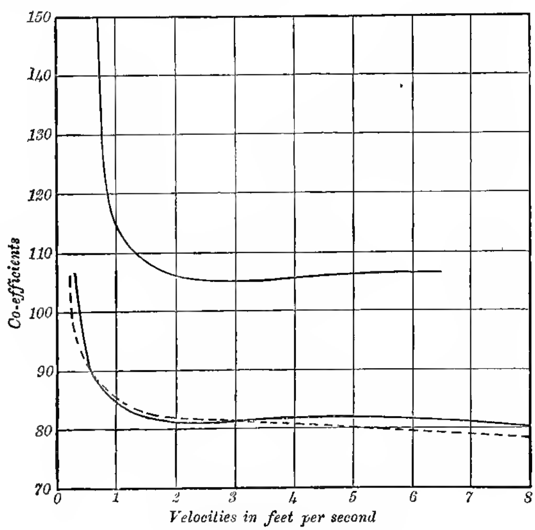


FIG. 146.

When rating a meter, the length of run being a fixed quantity, it is only necessary to record for each observation the time occupied and the difference of the meter readings. After several observations at nearly equal velocities the entries can be totalled and averaged.

The following table shows the values of the co-efficients of two current-meters, and Fig. 146 shows the curves obtained by plotting them. By means of the curves the co-efficients for intermediate velocities can be found. It will be noticed that the co-efficient changes rapidly for low velocities, while for high velocities it is nearly constant. For moderate velocities the co-efficients increase and then decrease again, causing a sag in the curve. The same thing occurred with other meters rated by Stearns. The cause is not known. The actual values of the co-efficients depend on the graduations of the reading dials.

Meter sold by Elliott Brothers, London. (Fig. 144.)			Stearns's Meter. (Fig. 145.)		
Velocities.		Co-efficient.	Velocities.		Co-efficient.
Actual.	Meter.		Actual.	Meter.	
·725	·484	1·50	·300	·280	1·070
1·16	1·05	1·11	·400	·406	·984
1·81	1·71	1·06	·500	·550	·909
2·97	2·84	1·05	·750	·862	·870
3·84	3·63	1·05	·950	1·13	·844
4·71	4·45	1·06	1·30	1·59	·823
5·85	5·52	1·06	1·60	1·97	·813
6·58	6·21	1·06	2·00	2·47	·809
			2·50	3·10	·806
			3·00	3·71	·809
			4·00	4·93	·813
			5·00	6·15	·813
			6·00	7·42	·809
			7·00	8·69	·806
			8·00	10·00	·800

Certain equations intended to show the law of the variation of the co-efficient have been arrived at theoretically. The most common is

$$v=aV+b,$$

where v is the actual velocity, V the meter velocity, and a and b are constant for any given instrument, their values being selected so as to make the agreement with the experimental co-efficients as close as possible. The above is the equation to a straight line, but the co-efficient is given by $c=\frac{v}{V}=a+\frac{b}{V}$, which is the equation to a curve. The values of the co-efficients for Stearns's meter calculated by this formula are shown dotted in Fig. 146. Another equation is

$$c^2=A+\frac{B}{V^2}.$$

Both equations give curves of the same general form, and becoming practically straight lines at high velocities. They can never agree exactly with curves having a sag, and as the constants cannot be arrived at until some experimental co-efficients have been found the equations are not of much practical value.

It has been shown by Stearns[1] that rating by ordinary towing through still water is not perfect. In a flowing stream the velocity and direction of the water constantly vary, but in rating this is not so. Stearns shows theoretically that the screw turns more rapidly when the velocity varies than when it is constant, that an ordinary screw probably turns more rapidly when the current strikes at an angle than when it is parallel to the axis; but that with his meter (Fig. 145) the band and parts of the frame intercept portions of the oblique currents, and so cause a decrease in the number of revolutions, the net result depending chiefly on the design of the instrument. He also moved the meter with mean velocities ranging up to 3·7 feet per second through still water, first with an irregularly varying velocity and then with its axis inclined to the direction of motion. He found that inclining the axis 8° and 11° had no appreciable effect, but that inclinations of 24° and 41° decreased the number of revolutions about 9 per cent.,[2] and that with irregular velocities the number of revolutions was increased, the increase varying from zero to 5 per cent., being generally greater for low velocities, and in one case reaching 13 per cent. when the mean velocity was only ·85 feet per second. This velocity was not a very low one when compared with that for which the screw ceased to revolve.

By measuring with the same current-meter the discharges in a masonry conduit, the depths varying from 1·5 to 4·5 feet, and the velocities from 1·7 to 2·9 feet per second, and comparing the results with others known to be practically correct, Stearns found that, with point measurement, the discharge given by the meter observations was practically correct, both in the ordinary condition of the stream and when the water was artificially disturbed, and that with integration the discharge was correct when the rate of integration was 5 per cent. of the velocity of the stream, but too small by 9 per cent. when the rate was 58 per cent. of the velocity. In the above experiments both the eight-bladed

[1] *Transactions of the American Society of Civil Engineers*, vol. xii.

[2] Other experiments have shown that inclinations of 25°, 35°, and 45° give a decrease in the number of revolutions of 8, 15, and 23 per cent. respectively.

and ten-bladed screws were used, the results being generally similar.

It seems clear that, with the instrument used, the increase in the velocity due to the variations in the velocity of the stream was counter-balanced by the decrease due to oblique currents, and that the instrument gave correct results with point measurements even when the water was disturbed; but with an instrument of different design, and especially one without a band, it seems probable that the results obtained by point measurement err in excess, that no additional error is introduced by a moderate inclination of the axis, or by slow integration, but that rapid integration causes error. These, however, are only probabilities. The real lesson to be derived from Stearns's investigations is that rating effected by steady motion in still water may be erroneous when applied to running streams, especially with rapid integration, and that additional tests should be adopted. To move a meter obliquely or with an irregular velocity would be troublesome, and would not produce the conditions existing in streams. It is best to place the meter in a running stream just below the surface, and to find the velocity by floats submerged to the same depth as the screw blades. If a sufficient range of velocities cannot be obtained the meter can be moved upstream or downstream with a known velocity. This plan can be combined with ordinary rating. The instrument can also be moved through still water while giving it a movement as in integration. A comparison of discharges obtained by the meter, with results known to be correct, affords a further test. An immense saving of labour is obviously effected by rating a number of meters together.

When it is necessary to rely on ordinary rating rapid integration should be avoided. The error, if any, will probably be less as the velocity is higher. For ordinary velocities the relative error is probably nearly constant, so that the results will be consistent with one another, and sometimes that is all that is required.

Section V.—Pressure Instruments

14. Pitot's Tube.—This instrument usually consists of two vertical glass tubes open at the ends placed side by side, one the 'pressure tube,' straight, and one the 'impact tube,' with its lower end bent at right angles and pointing upstream. The water-level in the pressure tube is nearly the same as that of the stream in which the instrument is immersed, but that in the impact tube is higher by a quantity which is equal to $K\frac{V^2}{2g}$, V being the velocity of the stream at the end of the tube, and K a co-efficient whose value has to be found by experiment.

The chief objections to this instrument were originally the fluctuation of the water-level in the tubes, owing to the irregularity of the velocity, and the difficulty in observing the height of a small column very close to the water-surface. Darcy in his gauge tube reduces the fluctuations by making the diameter of the orifice only 1·5 millimetres, that of the tube being one centimetre. The horizontal part of the tube tapers towards the point, and this minimises interference with the stream. The difficulty in reading is surmounted by means of a cock near the lower end of the instrument, which can be closed by pulling a cord. The instrument can then be raised and the reading taken. To give strength and to carry the cock, the lower parts of the tubes are of copper and are in one piece. For observations at small depths the heads of the water-columns are in the copper portion of the instrument, where they cannot be seen. To get over this difficulty the tops of the tubes are connected by a brass fixing and a stop-cock to a flexible tube terminating in a mouthpiece. By sucking the mouthpiece the air-pressure in the tubes is reduced, and both columns rise by the amount due to the difference between the atmospheric pressure and that in the tubes, but the difference in the levels of the two columns is unaltered. The upper cock being closed and the mouthpiece released, the reading can be taken. For reading the instrument a brass scale with verniers is fixed alongside the tubes. The instrument is attached to a vertical rod, to which it can be clamped at any height, and it can be turned in a horizontal plane, so that the horizontal part of the impact tube points upstream. To get rid of the effect of the fluctuations in the tube several readings, say three maximum and three minimum, can be taken in succession.

The Pitot tube has been improved by interposing a flexible hose between the nozzles and the gauge. The rod carrying the nozzles is thus more handy and the fluctuations of the water-column can be watched.

In the Detroit pipe experiments mentioned in chapter v. (art. 4) the tubes were inserted in the pipes through stuffing-boxes without interfering with the flow. The diameters of the orifices both impact and pressure were usually $\frac{1}{32}$ inch. When the impact tube was made to point at an angle with the axis of the stream the reading decreased. When the angle was a little over 45° negative readings occurred up to an angle of 180°, the greatest negative reading being for an angle of 90°.

In the Pitot tube the plane of the opening of the impact tube must be at right angles to the direction of flow. The exact form of the nozzle is of little consequence. In any case the water in the tube rises by a height almost exactly equal to $\frac{V^2}{2g}$. The chief difficulty is that the water level in the pressure tube is slightly different from that due to the pressure. It is usually lower—because subject to suction owing to the effect of the instrument on the current—and the co-efficient to be applied to the reading is then less than 1·0. It is usually ·80 to 1·0, but it may exceed 1·0.

The practical difficulty with a Pitot tube—as with a current meter—is the one of rating. Rating in still water may give results which are wrong by 5 or 10 per cent. The rating should include tests in a pipe or smooth channel, and the discharge should be measured in a tank. Tests made by holding the instrument with its orifice in the centre of a pipe are of course not reliable because the ratios of central to mean velocity (chap. v. art. 5) are not sufficiently well known and probably vary not only with D but with the roughness.

With the Pitot tube no time observations are required. The instrument is used chiefly in pipes—it can be inserted through a stuffing-box—and in small channels which are usually smooth. Parker (*Control of Water*) considers that it is unreliable in a large open stream. It is almost certainly unreliable in perturbed water. Probably it gives best results when the velocity is considerable. The stream should not be so small that the instrument seriously obstructs it.

In one pattern of the instrument the nozzles point one upstream and one downstream, the water in the former being raised and in the latter depressed. In the Pitometer, developed by Coles, the instrument has upstream and downstream nozzles, as above, and the two tubes are enclosed in a flat sheath (Fig. 145B, p. 314) and connected by flexible tubes—not shown in the figure—to a differential gauge.

There is also an electrically operated device in which the difference in the pressures in the two tubes is balanced by mercury in a special form of U-tube, and equal increments of discharge, in the pipe in which the tubes are inserted, are represented by equal divisions on a scale.[1] A photographic record of this can be kept.

15. Other Pressure Instruments.—

In Perrodil's Hydrodynamometer a vertical wire carries at its upper end a horizontal needle, and at its lower end a horizontal arm, to the end of which is fixed a vertical disc. The arm is connected with a graduated horizontal circle at the level of the needle. When the arm points downstream the needle points to zero on the circle. The needle is twisted round by hand till the arm is forced by the torsion of the wire to a position at right angles to the current. The pressure of the water on the disc is proportional to the square of its velocity, and it is proportional to and measured by the angle of torsion of the wire as given by the position of the needle. The disc oscillates owing to the unsteady motion of the stream, and the graduated circle oscillates with it, but the mean reading can be taken. The instrument has not been much used, but it is said to give good results and to register velocities as low as half an inch per second. It interferes somewhat with the free movement of any stream in which it is placed.

The Hydrometric Pendulum consists of a weight suspended from a string. The pressure of the current causes the string to become inclined to the vertical, and the angle of inclination can be read on a graduated arc. Except for observations near the surface the current pressure on the string must affect the reading. Bruning's Tachometer also has an arm and disc, but the pressure of the water, instead of being measured by the torsion of a wire, is measured by a weight carried on the arm of a lever. These two instruments have been little used, and it is not known how far their results can be relied on.

Section VI.—Pipes

16. The Venturi Meter.—The principle of this has been described in chap. v. art. 7. If D is the diameter of the pipe at the entrance to the meter, the lengths of the conical parts of the pipe are generally 2·5 D upstream and 7·5 D downstream, the angle of divergence in the latter portion being 5° 6′. The area A may be $4a$ to $18a$, but is usually $9a$. A has been as great as 60 square feet and as small as 2 square inches. The opening from the pipe into the pressure column may consist of one or more small orifices or there may be a gap and an annular chamber between the two portions of conical pipe. The Venturi meter if properly calibrated is an accurate and trustworthy instrument. It may be inaccurate with very low velocities unless these have been included in the calibration. With ordinary velocities c usually ranges in different instruments from ·96 to 1·00, increasing generally with the size of

[1] *Journal Am. Soc. Mech Eng.*, vol. xli. ; *Engineering Record*, vol. xlvii.

the meter, and for a given meter being nearly independent of the velocity.

In an investigation by Gibson[1] into peculiarities of the Venturi meter it is shown that when v is less than about ·5 foot per second c may be as low as ·75 or as high as 1·36, and that the instrument is not reliable for such low velocities unless it has been calibrated for them; that—since stream-line motion can occur in the converging cone at velocities much higher than in the main pipe—when v in the pipe is less than the critical velocity, stream-line motion may continue up to the throat and that, since in such a case the kinetic energy of the water is $\frac{V^2}{g}$ instead of $\frac{V^2}{2g}$, this may cause c to be as low as ·7. The effects of gaps of different widths were tested by experiments, and it was concluded that abnormally high co-efficients may occur owing to abnormal pressures in the throat column due to the accumulation of air—when the pressure is below atmospheric pressure—at the throat, but probably only when there is a gap at the throat and when the two measuring columns of the meter are independent. When negative pressures are anticipated a U-tube gauge should be used and not independent measuring columns.

17. Pipe Diaphragms.—The discharge of a pipe can be measured by means of an 'orifice in a thin wall,' the orifice being in a diaphragm (Fig. 90, p. 136). Holes are bored in the pipe upstream and downstream of the diaphragm, and pressure tubes are attached. The difference between the pressures in the two tubes can be ascertained by means of a U-tube. The pressure in the jet is no doubt a minimum at the most contracted section, and increases towards CD. This has been proved by observations by Gaskell[2] on a 4-inch pipe, the pressures being observed at various distances from the diaphragm. Further observations on pipes whose diameters were about 6 inches and 8 inches show that c_0 had very nearly the values given in chap. v. art. 6, provided the pressures were measured—both upstream and downstream—not more than 1·5 inches from the diaphragm. The pressure drop was not high compared to the pressure. Equation 70 (p. 141) applies to the case of a diaphragm if a is the area of the contracted stream and if c_v—say ·96 to ·98—is substituted for c.

It is convenient here to compare the various formulæ for orifices

[1] *Min. Proc. Inst. C.E.*, vol. cxcix.

[2] *Ibid.*, vol. cxcvii.

when there is velocity of approach. Let n (equation 8, p. 13) be taken as 1·0. In equation 23 (p. 48) let $A = ma = Ma'$. Then, since $c = c_v c_c$ and $a' = c_c a$,

$$Q = c_v c_c a\sqrt{2gH}\sqrt{\frac{1}{1-\frac{c_v^2}{M^2}}} = c_v c_c a\sqrt{2gH}\sqrt{\frac{M^2}{M^2-c_v^2}}$$

$$= c_v A\sqrt{2gH}\sqrt{\frac{1}{M^2-c_v^2}} \quad . \quad . \quad . \quad (82\text{A}).$$

In equation 70 (p. 141) let H be put for $H-h$ and c_v for c. There is no contraction. Then

$$Q = c_v A\sqrt{2gH}\sqrt{\frac{1}{m^2-1}} \quad . \quad . \quad . \quad (82\text{B}).$$

Since a is here the minimum area, m is the same as M in equation 82A. The two equations are for practical purposes identical. The slight difference is due to c_v being introduced at the beginning of the working leading up to equation 23. For pipe diaphragms Gaskell gives an approximate formula

$$Q = \cdot 60A\sqrt{2gH}\sqrt{\frac{1}{m^2-1}} \quad . \quad . \quad . \quad (82\text{C}).$$

In this case there is of course contraction. The results, calculated for orifices of various diameters (d), obtained from equations 82B and 82C agree very closely, as long as $\frac{D}{d}$ is not less than 2.

One kind of diaphragm used by Gaskell was ·56 inch thick—the downstream side of the orifice being bevelled so as to make a thin-wall orifice—and the holes for the pressure tubes were drilled into it radially from the outside and ran into holes, one of which opened upstream and one downstream. For experimental work this kind of diaphragm was not suitable, because whenever the diaphragm was changed the pressure tubes had to be disconnected. A thin-plate diaphragm was therefore used, and the holes were drilled in the pipe flanges, which were sufficiently thick.

Observations on diaphragms in a 5-inch pipe have been made by Judd.[1] The maximum drop in pressure from the upstream to the downstream side of the diaphragm was that due to a head of about 6 feet of water. Of this the percentage recovered was about 77 when the diameter (d) of the orifice was ·9D, 30 when d was ·5D, and 4 when d was ·2D. The recovery had always ceased at a point distant 4D from the diaphragm. On the upstream side of the

[1] *Trans. Am. Soc. Mech. Eng.*, 1916.

diaphragm the pressure usually fell—but very slightly—in going upstream, but became constant long before a distance of one pipe-diameter was reached. Judd observed some pressures very near the diaphragm and some further away. The pressure drop through the orifice was often high compared with the actual pressure. The co-efficients are somewhat irregular, but on the whole confirm Gaskell's equation 82c above.

If the pressure is observed so far downstream as CD (Fig. 90), the pressure drop may not be sufficient to give accurate results. If observed—as in the cases of both sets of experiments above-mentioned—nearer the diaphragm, the pressure observed is that in the eddy. This pressure probably differs slightly from that in the jet, but this need not prevent complete and reliable sets of co-efficients being obtained by further experiment. Uniformity of procedure is desirable. It seems suitable to observe the downstream pressure at a point opposite the contracted section. Upstream of the diaphragm the pressure in the eddy is—judging from the case of a weir (chap. iv. art. 4)—greater than in the actual stream. The pressure should be observed clear of the eddy, say $\cdot 5D$ to $1D$ upstream of the diaphragm.

A diaphragm is vastly less costly and easier to instal than a Venturi meter. At present it is not so accurate. It causes more loss of head.

The lower ends of Judd's experimental pipes were fitted with caps having orifices in them of the same sizes as the diaphragm orifices. The co-efficients obtained in these and some previous experiments with 3-inch and 4-inch pipes are somewhat irregular, and when averaged slightly in excess of those obtained for diaphragms.

Judd made some experiments with eccentric and segmental orifices.* These are not dealt with above. It would seem to be desirable to attend first to the ordinary concentric orifices and obtain reliable co-efficients for these.

Notes to Chapter VIII

Self-recording Gauge (art. 5).—The float can be made to turn a drum which, provided with a screw thread of varying pitch and with simple mechanism, causes the pencil to move equal distances for equal increments in the discharge, and such distances can be magnified (*Journal Am. Soc. Mech. Eng.*, 1912, vol. 34).

Floats.—A rising float consists of a hollow ball, say of copper. It is held down on the bed of the stream and released at a given moment. Its position on reaching the surface is noted and the distance of this point downstream from the point of release gives the horizontal distance travelled by the float. The time taken is independent of the velocity and depends only on the depth, and can be ascertained beforehand, so that no time observations on the spot are needed. The arrangement is suitable for slow currents where current meters would not be reliable. In case the point of release cannot be exactly located, two balls of different specific gravities can be used and the difference between their points of emergence noted.

CHAPTER IX

UNSTEADY FLOW

SECTION I.—FLOW FROM ORIFICES

1. Head uniformly varying.—Let the head over an orifice during a time t vary from H_1 to H_2, and let the discharge in this time be Q. The mean head or equivalent head H' is that which would, if maintained constant during the time t, give the discharge Q. Let the head H vary uniformly, that is, by equal amounts in equal times, as, for instance, in the case of an orifice in the side of an open stream, whose surface is falling or rising at a uniform rate. In this case $h=Ct$ where C is constant. Let a be the area of the orifice and c the co-efficient of discharge, which is supposed constant. The discharge in the short time dt under the head h is

$$dQ=ca\sqrt{2gh}\,dt=ca\sqrt{2gC}\,t^{\frac{1}{2}}\,dt.$$

The discharge between the times T_1 and T_2 is

$$Q=\int_{T_2}^{T_1} ca\sqrt{2gC}\,t^{\frac{1}{2}}\,dt=\tfrac{2}{3}ca\sqrt{2gC}\,(T_1^{\frac{3}{2}}-T_2^{\frac{3}{2}})$$

$$=\tfrac{2}{3}\,ca\sqrt{2gC}\,\frac{H_1^{\frac{3}{2}}-H_2^{\frac{3}{2}}}{C^{\frac{3}{2}}}.$$

Under a fixed head H'

$$Q=ca\sqrt{2gH'}\,(T_1-T_2)=ca\sqrt{2gH'}\,\frac{H_1-H_2}{C}.$$

Equating the two values of Q

$$\sqrt{H}=\tfrac{2}{3}\,\frac{H_1^{\frac{3}{2}}-H_2^{\frac{3}{2}}}{H_1-H_2}\quad . \; . \; . \; (83).$$

If $H_2=0$, that is, if the head varies uniformly from H_1 to O or from O to H_1,

$$\sqrt{H}=\tfrac{2}{3}\sqrt{H_1}\quad . \; . \; . \; (84),$$

or the equivalent head is $\frac{4}{9}H_1$.

2. Filling or Emptying of Vessels.—Let water flow from an orifice in a prismatic or cylindrical vessel whose horizontal sectional area is A. The discharge in time dt is $d\,Q=A\ dh=ca\sqrt{2gh}\,dt$:

$$dt=\frac{A\ dh}{ca\sqrt{2g\,h}}=\frac{A\ h^{-\frac{1}{2}}\,dh}{ca\sqrt{2g}}.$$

The time occupied in the fall of the surface from H_1 to H_2 is

$$t=\int_{H_2}^{H_1}\frac{A}{ca\sqrt{2g}}\,h^{-\frac{1}{2}}\,dh=\frac{2\,A}{ca\sqrt{2g}}(H_1^{\frac{1}{2}}-H_2^{\frac{1}{2}}).$$

Under a fixed head H'

$$t=\frac{A\,(H_1-H_2)}{ca\sqrt{2gH'}}.$$

Therefore $\sqrt{H'}=\dfrac{H_1-H_2}{2\,(H_1^{\frac{1}{2}}-H_2^{\frac{1}{2}})}$ . . . (85).

This is useful for canal locks.

If $H_2=0$, that is, if the vessel is emptied down to the level of the orifice,

$$\sqrt{H'}=\frac{\sqrt{H_1}}{2}\quad . . . (86).$$

The following are the ratios of $\sqrt{H'}$ to $\sqrt{H_1}$ for certain cases :—

For a prism or cylinder, ...	$\frac{1}{2}$
For a sphere, ...	$\frac{5}{8}$
For a hemisphere concave downwards, ...	$\frac{5}{7}$
For a hemisphere concave upwards, ...	$\frac{5}{12}$
For a cone with apex downwards, ...	$\frac{5}{6}$
For a cone with apex upwards, ...	$\frac{5}{16}$
For a wedge with point downwards, ...	$\frac{3}{4}$
For a wedge with point upwards, ...	$\frac{1}{3}$
For a vessel whose vertical section is a parabola with vertex downwards :—	
When all vertical sections are the same, . . (Paraboloid of revolution).	$\frac{3}{4}$
When the horizontal sections are rectangles, . . (Two opposite sides of the vessel rectangles and two parabolas).	$\frac{2}{3}$

In the last case the surface falls at a uniform rate as in the case considered in art. 1.

In all cases the times occupied in emptying the vessels are greater than with a constant head H_1, in the inverse ratios of the above fractions. If a vessel is filled, through an orifice in its bottom, from a tank in which the water remains level with the top of the vessel, the ratio of $\sqrt{H'}$ to $\sqrt{H_1}$ is the same as for filling the vessel when inverted. Thus for a cylinder, prism, or sphere the time for filling is the same as for emptying.

If two prismatic vessels communicate by an orifice, and H_1 is the difference in the water-levels of the vessels, and A_1 and A_2 their horizontal areas, the time which elapses before the two heads become equal is

$$t = \frac{2 A_1 A_2 \sqrt{H_1}}{ca\sqrt{2g}\,(A_1+A_2)} \quad . \; . \; . \; (87),$$

and is the same whichever is the discharging vessel. This equation may be used for double locks.

SECTION II.—FLOW IN OPEN CHANNELS

3. **Simple Waves.**—Let ABC (Fig. 147) represent the surface of a uniform stream in steady flow, the reach commencing from a fall

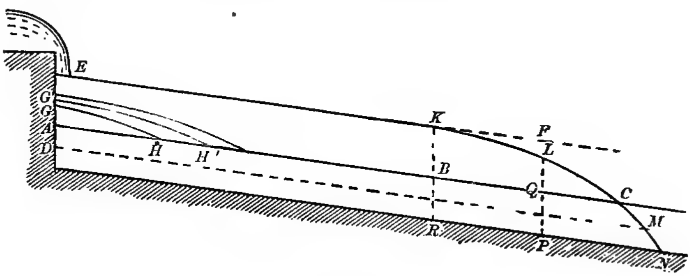


FIG. 147.

over which is introduced an additional steady supply q, such that the surface will eventually be EF. A wave is formed below A, the surface assuming successively the forms GH, $G'H'$, etc. The point H travels downstream at first with a very high velocity—since the slope GH cannot remain steep for any but an extremely short time—but its velocity decreases as the slope at H becomes less. The point G rises at a continually decreasing rate, because in equal times the volumes of water represented by $GG'H'H$, etc., are equal. Obviously the velocity of the point H is greater as q is greater, that is, it depends on the amount of the eventual rise. It must not be supposed that the actual velocity of the stream even at its surface, or velocity of 'translation,' is anything unusual. As in other cases of wave motion it is the form of the surface which changes rapidly.

When the surface has risen to E the wave advances only downstream, and there is formed a reach EK, in which the flow is steady and uniform. On consideration it will be seen that if the channel is long enough, the elongation of the wave ceases, its profile KC becomes fixed, and it progresses at the same rate as the mean velocity in the risen stream EK. The motion of such a wave is uniform, and the mean velocity of the stream is the same at all cross-sections. The proof given in chapter ii. (art. 9) applies to any short portion of the wave. The pressure on the upstream end is greater than on the downstream end, but the surface-slope is greater than the bed-slope, and the equation comes out exactly the same, S being the surface-slope. At different cross-sections in the wave S is greater as R is less, so that V is the same everywhere. Obviously the wave is convex upwards. If at any cross-section in the wave the slope were less than that required by the above consideration, the velocity there would be reduced, the upstream water would overtake it and increase the slope. If the slope at any cross-section were too great, the velocity there would be increased, and the water would draw away from that upstream of it. Thus the wave is in a condition of stable equilibrium, and always tends to recover its form, should this be accidentally disturbed. The curve KC produced to M and N gives the profile of the wave, supposing the original water-surface to have been DM, or the channel to have been dry.

Thus the flood-wave has two distinct characters according as its profile is forming or formed. The forming wave rises as well as progresses, its velocity is at first very high, and it depends on the amount of the rise that is on the height AE. The formed wave progresses at a uniform rate, and its velocity depends only on that of the risen stream, and not on the amount of the rise. The surface is in all cases convex upwards. Since any change in the form of the wave occurring at either end would be communicated to the whole of it, it is probable that, in ordinary cases, the moment of time when the point H commences to move with a uniform velocity coincides nearly with the moment when the point G ceases to rise, or the wave becomes formed.

As to the form of the curve KC, the case is analogous to that of the surface-curve in variable steady flow (chap. vii. art. 13). The slope at L is such as will, with uniform flow and depth LP, give the same velocity as the depth KR with slope EK. Thus the surface-slope corresponding to any depth is known, and tangents to the curve can be drawn, but the distance between two points where the depths are given is not known. In a case of steady flow, with a drawing down KB, the surface-slope at L must be greater than in the wave now under consideration, because in that case V is greater than at K instead of being the same, and also because V is continually increasing and work being stored.

In the case of a reduced steady supply at S (Fig. 148) the surface assumes the forms ST, $S'T'$, etc., the point T travelling with a

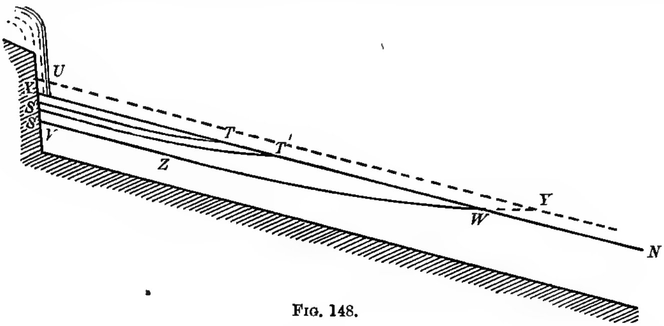


FIG. 148.

decreasing velocity and S falling with a decreasing velocity. The surface eventually assumes the form VZW, the portion VZ being in uniform flow. If the original surface is UY the curve is ZWY. The velocity in ZW is lower than in WN, so that WN continually draws away while ZW lengthens and flattens. The angle at W is no doubt rounded off, so that there is a wave-like form.

Ordinarily the curve of a wave is of great length, and the convexity or concavity slight. If the point L is such that the volumes KFL and LQC are equal, the time at which this point in the wave will reach any place, after the wave is formed, is found by dividing the distance of the place from E by the velocity of the risen stream.

If the additional supply introduced, or the supply abstracted, instead of being steady, is supposed to change gradually as would be the case if it were caused by a wave coming down the upper reach or by the opening or closing of gates or shutters, the wave below A or X does not at its commencement travel with such rapidity, and it more quickly assumes its fixed form, unless the water is introduced or abstracted too slowly to allow it to do so.

The form of a flood-wave may be observed by means of a number of gauges, but the wave, except when it is first formed—and even then if the change in the supply is not made with great abruptness—is of great length, and its form, or even the times of passage of its downstream end, can be accurately found only by very exact gauge readings. Slight changes in the supply, owing to rainfall or similar causes, are sufficient to vitiate the observations. Absorption of water by the channel, especially in the case of a wave travelling down a channel previously dry, may also greatly affect the movement and form of the wave. On the Western Jumna Canal in India, with a mean depth of water of about 7 feet, and a velocity of about 3·5 feet per second, a rise or fall in the surface of ·25 foot to ·55 foot, caused by the manipulation of regulating apparatus, and occupying in each case less than an hour, was found to occupy 5 or 6 hours at a point 12 miles downstream, and 6 to 7 hours at a point 40 miles downstream. Attempts made to observe the form of the wave failed owing to the causes just mentioned.

4. Complex Cases.—Let a rise be quickly succeeded by a fall (Fig. 148A). As ZW flattens, the point W overtakes P. The wave PC, no longer having behind it the steady stream WP, also flattens and the velocity of C decreases. The whole wave flattens and its velocity continually decreases.

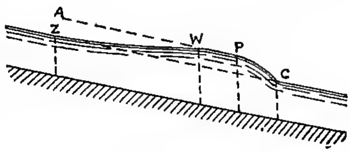


FIG. 148A.

If a fall is quickly succeeded by a rise the wave overtakes the trough. But it cannot fill it up. This would imply that the discharge passing a place lower down was the same as if no temporary diminution had occurred. The wave, as soon as it overtakes the other, begins to rise on it, suffers a decrease of slope, and is checked while the front wave receives an increase of slope and is accelerated. The trough lengthens indefinitely. At places a long way down the fluctuations in the water-level are slight in amount but long in duration.

Given the height of a flood at A (Fig. 147), the full effect of the flood will be felt at any place K only when the height at A is maintained for a sufficiently long period. If this period is prolonged indefinitely the rise at K will not be increased, except in so far as may be due to the cessation of absorption by the flooded soil, but if the period is shortened the rise at K may be greatly reduced. Empirical formulæ intended to give the height of a flood at any place, in terms of the heights in some reach upstream of it, must include the time as a factor, or, what is probably a better plan, must include gauge readings at several places upstream, and not at one place only. This plan has been adopted on various rivers, the places selected being generally those where tributaries enter. Sometimes it is sufficient merely to add together the different readings and take a given proportion.

If the channel is not uniform the form of the wave, even if it has once become fixed, changes. At a reduction of slope the wave assumes a more elongated, and, at an increase of slope, a more compact form. At an increase of surface-width, supposing the mean velocity to be unaltered, the wave is checked because additional space has to be filled up. At a decrease of width the velocity of the wave increases.

When an additional supply is introduced or abstracted at a place where there is not a fall, the water-surface upstream is headed up or drawn down, and the form which it eventually assumes may be found by the methods explained in chapter vii. (art. 13). The volume of water eventually added to the stream upstream of the point of change can thus be found, but the time in which it is added cannot easily be found, because it is not known how much of the supply passes downstream. The commonest case of the kind is that of the tide at the mouth of a river. When the tide begins to rise the water in the river is headed up and its velocity reduced. As the rise of the tide becomes more rapid the discharge of the river is insufficient to keep the channel filled up so as to keep pace with the rise of the tide, the water in the mouth of the river becomes first still and level, and then takes a slope away from the sea and flows landwards. At a place some way inland the water-surface forms a hollow and water flows in from both directions. This may obviously continue for some time after the tide has turned, and high-water then occurs later at the inland place than at the mouth of the river, a fact which is sometimes unnecessarily ascribed to 'momentum.' A sudden and high flood in the Indus once caused a backward flow up the Cabul River where it joins the Indus.

If in a long reach of a river the flood water-way is reduced (say by embankments which prevent flood-spill, or by training-walls which cause the channel inside them to silt up) a flood of any kind will, in most of that reach, rise higher and travel more quickly than before. The same effect will be produced, but to a less degree, at places further downstream. When the rise is followed by a fall the wave will not flatten out to the same extent as before. In the case of a permanent rise, except in so far as there will have been less absorption than before in the flooded area, matters will be as before.

5. **Remarks.**—Sometimes a wave motion is seen in a stream at some abrupt change where air, becoming imprisoned, escapes at intervals.[1] (Cf. unstable conditions at weirs, chap. iv. arts. 10 and 13.) It is believed that in a falling stream the surface is

[1] In flow through a bridge the water surface may rise in a wave in one span while it falls in the other and *vice versâ*, the movement continuing rhythmically.

slightly concave across, and in a rising stream convex, but the curvature is extremely small.

The action of an unsteady stream on its channel is, no doubt, subject to the same laws as in a steady stream. At the front end of a rising wave the relation of V to D is exceptionally high, and scour is likely to occur. At the advancing end of a falling wave the reverse is the case, and hence a falling flood frequently causes deposits. In discussions on the training of estuaries the idea has often been put forward as a general law that it is wrong to diminish the flow of tidal water. No doubt it is the tidal water which has made the estuary. If only the upland water flowed through it the size would be far too great for the volume. The salt water may enter an estuary comparatively clear and return to sea silt-laden. But if training-walls are made so as to reduce the volume of tidal water entering the estuary, the width to be kept open is also reduced. No such sweeping law as that above stated can be upheld. The Thames embankments in London contracted the channel and to some extent interfered with the tidal flow, but the channel was scoured and improved.

If a stream is temporarily obstructed by gates, and the water headed up, the silt deposited, if any, is removed again when the gates are opened. The same is true of obstruction caused by the rise of tides. If a given volume of water is available for the flushing of a sewer, it can probably be utilised best by introducing it intermittently, suddenly, and in considerable volumes at various points in the course of the sewer, commencing from near the tail and proceeding upwards. If there are any falls or gates it is clearly best to introduce it just below a fall or below a closed gate.

Ordinarily, in a rising or falling stream, the relative velocities at different points in a cross-section are probably normal or nearly so, but where the fresh water of a river meets the sea the relations are apt to be much disturbed, especially near the turns of the tide. The fresh water, being lighter, may rise on the salt water, which may have a movement landwards, while the fresh water above it is moving seawards. Such a landward current is obviously not the result of the surface-slope, and must be due to momentum and hence temporary. Even where the water is all fresh the relative velocities may be disturbed. At the turn of the tide the surface water may begin to move before the lower water.

CHAPTER X

DYNAMIC EFFECT OF FLOWING WATER

SECTION I.—GENERAL INFORMATION

1. Preliminary Remarks.—Hitherto we have been concerned almost entirely with questions relating to velocities, discharges, and water-levels. In this chapter will be considered questions relating to the Dynamic Effects of Flowing Water. In all cases the effect of friction will be neglected.

By dynamic pressure is meant the pressure produced by a stream of water when its velocity or its direction of motion is altered. This is, of course, entirely different from static pressure. Let V, A, and Q be the velocity, sectional area, and discharge of a stream, and W the weight of one cubic foot of the liquid. The volume discharged per second is AV, and its momentum is $WA\frac{V^2}{g}$. The force which, acting for one second, will produce or destroy this momentum is $F=WA\frac{V^2}{g}$. On this principle the pressures developed in various practical cases can be ascertained. Before proceeding to them it will be convenient to give two theorems regarding currents, though these do not strictly fall under the heading of this chapter, and might have been given in chapter ii. if they had been required sooner.

2. Radiating and Circular Currents.—Suppose water to be supplied by the pipe AB (Fig. 149), and then to flow out radially between two parallel horizontal surfaces CD and EF, whose distance apart is d. Of radii R_1, R_2, let R_2 be the greater, and let the velocities be V_1, V_2, and the pressures P_1, P_2. Since the discharges past all vertical cylindrical sections are equal, therefore $\frac{R_1}{R_2}=\frac{V_2}{V_1}$. Also since by Bernouilli's theorem the hydrostatic head is

$$H=\frac{P_1}{W}+\frac{V_1^2}{2g}=\frac{P_2}{W}+\frac{V_2^2}{2g}=\frac{P_2}{W}+\frac{R_1^2}{R_2^2}\frac{V_1^2}{2g}.$$

Therefore $$\frac{P_1}{W}=H-\frac{V_1^2}{2g}.$$

And $$\frac{P_2}{W}=H-\frac{V_1^2}{2g}\cdot\frac{R_1^2}{R_2^2},$$

or the heights in pressure columns increase from the centre outwards and tend to reach, though never reaching, the value H. If

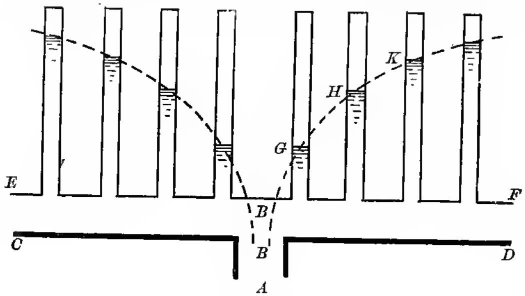


FIG. 149.

the water flows inwards and passes away by the pipe the law is the same. A curve through the points G, H, K, etc., is known as Barlow's curve.

In a vessel (Fig. 150) which, with its contents, is revolving about a vertical axis with angular velocity a, the forces acting on a particle A whose velocity is u are its weight w or AC, acting vertically, and a horizontal centrifugal force $w\,\frac{u^2}{gx}$ or $w\,\frac{a^2}{g}x$ or AB. The water-surface takes a form normal to the resultant AD of

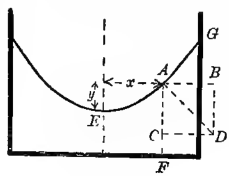


FIG. 150.

the above, that is, the angle DAC is $\tan^{-1}\frac{a^2}{g}x$. Hence $\frac{dy}{dx}=\frac{a^2}{g}x$.

Integrating, $y=\frac{a^2}{2g}x^2$, or the curve EA is a parabola with apex at E. Since $u=ax$, therefore $y=\frac{u^2}{2g}$, or the elevation of any point above E is the head due to its velocity of revolution. The theoretical velocity of efflux from an orifice at F or B is that due to a head AF or GB.

A similar condition occurs in a mass of water driven round by radiating paddles. In either case the condition is termed a 'forced vortex.' Questions connected with the pressure in a radiating current or in a forced vortex enter, though not to a very important degree, into the theories of certain hydraulic machines. In a centrifugal pump the pressures in the pump-wheel follow the law of the radiating current, while those in the whirling chamber outside the wheel depend on the law of the forced vortex.

SECTION II.—REACTION AND IMPACT

3. **Reaction.**—Let a jet issue without contraction from an orifice A (Fig. 151) in the side of a tank. The force F causing the flow is the pressure on B. This force is called the reaction of the jet. It tends to move the tank in the direction AB. It is equal to $WA\frac{V^2}{g}$, or to $2WAH$ where H is the head due to V. If the tank is supposed to move with velocity v in the direction AB, the absolute velocity of the issuing jet is $V-v$, but the quantity issuing is still AV. Hence the momentum of the discharge per second is $WA\frac{V(V-v)}{g}$.

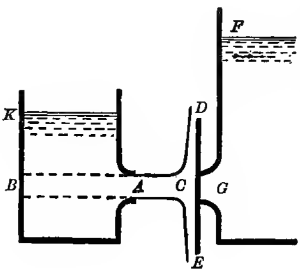


FIG. 151.

The principle of reaction has been utilised in driving a ship, water being pumped into the ship and driven out again sternwards. The energy of the water just after leaving the ship is $WAV\frac{(V-v)^2}{2g}$.

The work done on the ship is

$$Fv = WA\frac{V(V-v)v}{g} \quad . \quad . \quad . \quad (88).$$

The total work done on the water is the sum of the above or

$$WAV\frac{V^2-v^2}{2g} \quad . \quad . \quad . \quad (89).$$

The efficiency of the machine is the ratio of (88) to (89) or $\frac{2v}{V+v}$. The nearer v approaches V the nearer the efficiency is to 1·0, but the less the actual work done on the ship. If $V=v$ the efficiency is 1·0, but the work done is nil. In the *Waterwitch* V was $2v$, so that the efficiency was $\frac{2}{3}$.

The principal of reaction has also been applied in driving a

'Reaction Wheel' or 'Barker's Mill' (Fig. 152). The preceding formulæ and remarks apply to this case, v being the velocity of the rotating orifices. If AC is the head in the shaft the head over the orifice D is BD, AB being an imaginary water-surface found by the principles of article 2. If $AC=H$ the velocity of efflux at D is $\sqrt{2gH+v^2}$.

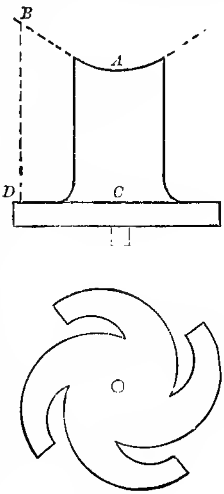


FIG. 152.

4. **Impact.**—When a jet of water (Fig. 153) meets a solid surface which is at rest, it spreads out over the surface. There is not, strictly speaking, any shock, but there is loss of head owing to abrupt change. If the surface is horizontal and a jet strikes it vertically, it spreads out equally in all directions. In other cases the amount and directions of spreading depend on the circumstances. In all cases, without exception, the velocity of the jet relatively to the surface is the same after impact as before. The flow after impact is along the surface which, being smooth, cannot alter the velocity of the water, but only force it to change its direction. The pressure between the fluid and the surface in any direction is equal to the change of momentum in that direction of so much fluid as reaches the surface in one second.

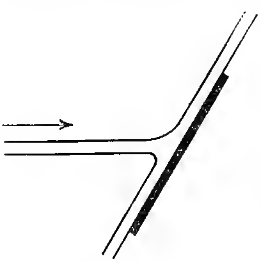


FIG. 153.

Let a jet AC (Fig. 151) meet a fixed plane surface at right angles. The momentum in the direction AC is wholly destroyed and the pressure on the plane is $WA\frac{V^2}{g}$, or the same as the pressure (reaction) on B, or twice the pressure due to the hydrostatic head which produces V. Thus the pressure on DE will balance the pressure due to the head FG where FG is twice KB. In the case shown in Fig. 97 (p. 141) the two heads are equal. In that case the head HG has to be produced, the discharge rising through GH. In the present case the head FG has merely to be maintained.

If the plane is moving with velocity v in the same direction as the jet the discharge meeting the plane per second is $A(V-v)$ and the pressure is $WA\frac{(V-v)^2}{g}$. The work done on the plane per second is $WA\frac{(V-v)^2}{g}v$. The total energy of the water before impact is $WAV\frac{V^2}{2g}$. The efficiency is $\frac{2(V-v)^2v}{V^3}$. This is a maximum when $V=3v$ and the efficiency is then $\frac{8}{27}$.

If for the vane there is substituted a series of vanes, as in the case of a jet directed against a series of radial vanes of a large wheel, the discharge reaching the vanes per second is AV and the whole pressure is $WAV\frac{(V-v)}{g}$. The work done per second is $WAV\frac{(V-v)v}{g}$ and the efficiency is $\frac{2V(V-v)v}{V^3}$ or $2v\frac{V-v}{V^2}$. It is a maximum when $v=\frac{V}{2}$, and is then $\frac{1}{2}$.

If the vane is cup-shaped (Fig. 154), so that the water leaving the vane is reversed in direction, the velocity of the water leaving the vane has relatively to the vane a velocity $V-v$ in a backward

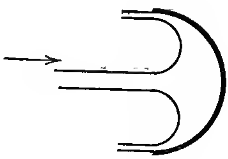


Fig. 154.

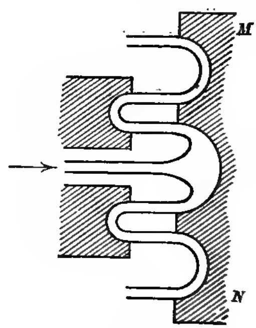


Fig. 155.

direction and an absolute velocity $v-V+v$ or $2v-V$. The change of momentum per second is $WA\frac{(V-v)}{g}\{V-(2v-V)\}$ or $2WA\frac{(V-v)^2}{g}$, and the pressure on the cup is double that on the plane considered above. The work done on the cup is $2WA\frac{(V-v)^2}{g}v$. The efficiency is $\frac{4(V-v)^2v}{V^3}$. It is a maximum when $V=2v$, and is then $\frac{1}{2}$. In the case represented by Fig. 155 the pressure on the solid MN is double that due to a single cup.

If there is a series of cups the discharge per second reaching them is AV the whole pressure is $WA\frac{V}{g}\{V-(2v-V)\}$ or $2WA\frac{V(V-v)}{g}$. The efficiency is $\frac{4V(V-v)v}{V^3}$. It is a maximum when $V=2v$, and is then 1·0.

The preceding cases illustrate the great principle to be adopted in the design of water-motors such as turbines and Poncelet wheels, namely, that the water shall leave the machines deprived, as far as possible, of its absolute velocity. If it has on departure any velocity it carries away work with it. In the last case it had no velocity and the efficiency is 1·0.

Another principle is that the water shall impinge on the vane so as to create as little disturbance as possible—that is, as nearly as possible tangentially to the vane—and thus minimise loss of energy by shock. When the jet strikes tangentially it has no tendency to spread out laterally, but slides along the vane. In practice an exact tangential direction is impracticable, but the vanes are provided with raised edges which prevent lateral spread and cause the water to be deflected entirely in one plane.

A third principle is that all passages for water shall, as far as possible, be free from abrupt changes in section or direction, so that loss of head from shock shall be avoided.

Let AA' (Fig. 156) be a surface or vane moving in the direction

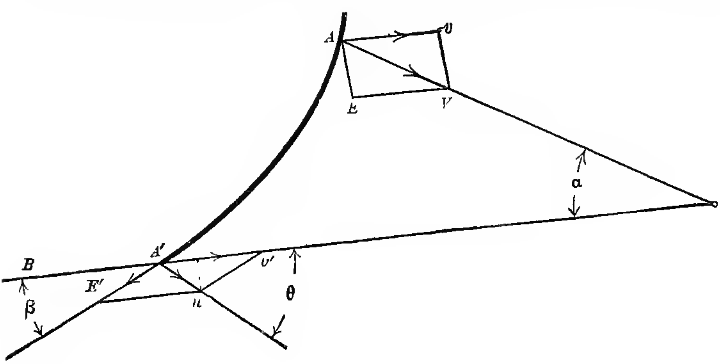


FIG. 156.

and with the velocity v, represented by Av, and let AV represent the direction and velocity V of a jet impinging on the vane. Let

α be the angle between the two lines. The line vV represents the velocity V' of the jet relatively to the vane at A. Let it be assumed that the jet is deviated entirely in planes parallel to the figure. The jet leaves the vane at A' with the velocity V', represented by the line $A'E'$. Draw $A'v'$ equal and parallel to Av. Then $A'u$ represents the absolute velocity of the water leaving the vane. Let the angle $v'A'u=\theta$ and $BA'E'=\beta$. If the quantity of water reaching the vane per second is w, the original and final momenta of the water resolved in a direction parallel to Av are $\frac{w}{g}V\cos\alpha$ and $\frac{w}{g}V'\cos\theta$. The change of momentum or pressure in the direction Av is $\frac{w}{g}(V\cos\alpha-V'\cos\theta)$ or $\frac{w}{g}(V\cos\alpha-v+V'\cos\beta)$. These are general expressions covering all cases, and the preceding ones can be derived from them.[1]

When a jet impinges on a plane, as in Fig. 157, the issuing velocity of the jet is theoretically $\sqrt{2gH_1}$, but on reaching the plane the velocity V is about $\sqrt{2gH}$. The outer streams at A press on the inner by reason of centrifugal force, and the intensity of pressure increases towards the centre of the jet. It cannot exceed the amount due to $\frac{V^2}{2g}$ or H, because otherwise the direction of flow would be reversed. Experiments made by Beresford[2] with jets ·475 inch to 1·95 inch in diameter falling on a brass plate show

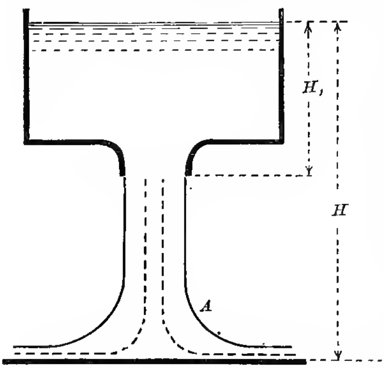


Fig. 157.

[1] Some machines which illustrate the principles of dynamic pressure have been referred to above. There are many machines such as water-meters, modules, rams, presses, pumps, water-wheels, and water-pressure engines which, though water passes through them, illustrate no principle of hydraulics, the questions involved in their design being engineering and dynamical. In fact, the principles involved in the above formulæ regarding vanes are dynamical, and are given here to bridge over a gap between hydraulics and another science. The same remark applies to parts of the succeeding article.

[2] Professional Papers on Indian Engineering, No. cccxxii.

that, at the axis of the jet, the pressure is very nearly that due to H, and the pressure becomes negligible at a distance from the axis equal to about twice the diameter of the jet. The pressure is thus distributed over an area of about four times that of the section of the jet. The pressures were measured by means of a water-column communicating with a small hole in the plate whose position could be altered.

5. Miscellaneous Cases.—When water flows round a bend in a channel the dynamic pressure produced on the channel is the same as if the channel was a curved vane. At bends in large pipes anchors are sometimes required to hold the pipe.

When a mass of water flowing in a pipe is abruptly brought to rest by the closure of a gate or valve the pressure produced is $f=\frac{v}{L}\frac{MTm}{2rm+MT}$ where L is the length of the pipe affected by the pulsation, m and M the moduli of elasticity for water and for the material of the pipe in pounds per square inch, T the thickness of the pipe in inches, r the radius of the pipe in feet, and v the velocity of the water in feet per second, f being in pounds per square inch over and above the static pressure.[1]

When a thin plate (Fig. 158) is moved normally through still water with velocity V, a mass of water in front of the plate is put in motion, and those portions of it which flow off at the sides of the plate cannot turn sharp round and fill up the space behind the plate. Instead of doing this they penetrate into the rest of the water and so communicate forward momentum to it, while other portions of still water have to be set in motion to fill up the space behind. Thus there is produced a resistance which is independent of friction or viscosity. Practically it is found that the resistance is $KWA\frac{V^2}{2g}$ where K is 1·2 to 1·8, the best results giving 1·3 to 1·6. The resistance is less than that caused by the impinging on a fixed plane of a jet of the same section as the area of the plate with a velocity V

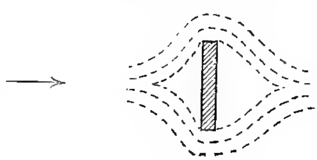


Fig. 158.

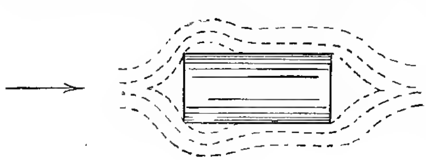


Fig. 159.

[1] *Min. Proc. Inst. C.E.*, vol. cxxx.

If for the plate there is substituted a cylinder (Fig. 159) whose length is not more than about three diameters, the resistance is less than in the case of the plate. It is further reduced if the downstream end of the cylinder is pointed.[1]

In the above cases, if the plane or cylinder is fixed and the water moving, the pressures are the same.

The following statement shows the approximate results of some experiments made by Hagen to show the position assumed by a rectangular plane surface when pivoted (Fig. 160) and placed in flowing water :—

$\frac{x}{y}=1{\cdot}0$	·9	·8	·7	·6	·5	·4	·3	·2
$\phi=90°$	74°	59°	46°	27°	13°	7°	6°	4°

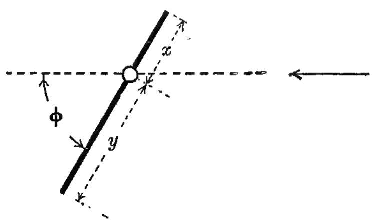


Fig. 160.

When a thin sharpened plate or a spindle-shaped or ship-shaped body is moved endways through still water the resistance is almost wholly frictional and is nearly as V^2, but if the body is only partly submerged waves are produced, and when V exceeds a certain limit (which bears a relation to the size of the body) the wave resistance increases and the total resistance increases faster than V^2. If the body, though sharp at both ends, tapers more rapidly at one end than at the other, it probably causes least resistance when the blunter end is forward.

In experiments made by Froude by towing boards through still water, it was found that the power of the velocity to which the friction is proportional varies for different surfaces, being sometimes less than 2 and sometimes more.[2] Also that for long boards f (chap. ii. art. 9) is much less than for short ones, the reason being that the forward part of a long board communicates motion to the water, and the succeeding portion thus experiences less resistance.

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[1] For results of some recent experiments on cylinders with square and pointed ends see *Min. Proc. Inst. C.E.*, vol. cxvii.

[2] The powers are as follows, the boards being 50 feet long : varnish 1·83, tinfoil 1·83, calico 1·87, fine sand 2·06, medium sand 2·00 Tinfoil is the smoothest surface and medium sand the roughest. These figures do not help much in arriving at practical formulæ for flow.

APPENDIX A—Units

Metres and Feet.—To convert a formula based on the metre into one based on the foot—

For metres, $V = C_m R^{\frac{1}{2}} S^{\frac{1}{2}}$ . . . . . . (M)

For feet, $3{\cdot}2809\, V = C_f (3{\cdot}2809 R)^{\frac{1}{2}} S^{\frac{1}{2}}$ . . . . (F)

Dividing F by M, $3{\cdot}2809 = \frac{C_f}{C_m}(3{\cdot}2809)^{\frac{1}{2}}$. Or $\frac{C_f}{C_m} = (3{\cdot}2809)^{\frac{1}{2}} = 1{\cdot}811$.

Similarly, if $Q = K_m l H^{\frac{3}{2}}$

$$(3{\cdot}2809)^3 Q = K_f 3{\cdot}2809 l (3{\cdot}2809 H)^{\frac{3}{2}}.$$

$$(3{\cdot}2809)^3 = \frac{K_f}{K_m} 3{\cdot}2809 (3{\cdot}2809)^{\frac{3}{2}}.$$

$$\frac{K_f}{K_m} = (3{\cdot}2809)^{\frac{1}{2}} = 1{\cdot}811.$$

If in either formula the index is m instead of $\frac{1}{2}$, the ratio $\frac{C_f}{C_m}$ or $\frac{K_f}{K_m}$ is $(3{\cdot}2809)^{1-m}$. This furnishes yet another instance of the advantage of the simple indices.

Gallons and Cubic Feet.—1 cubic foot per minute = 6·25 gallons per minute = 375 gallons per hour = 9000 gallons per day.

APPENDIX B—Calculation of m and n

(Chap. iv. arts. 5 and 8)

The following is a specimen of the method of calculating:—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Height of Weir.	Head.	M (observed).	$M^2\frac{H^2}{(G+H)^2}$	m (assumed).	$\frac{M}{m}$ or $1+\frac{2}{3}nM^2\frac{H^2}{(G+H)^2}$	$\frac{2}{3}nM^2\frac{H^2}{(G+H)^2}$	$\frac{2}{3}n$ or col. 7 ÷ col. 4.	n
Metres.	Metres.							
1·135	·15	·4284	·00258	·4260	1·0056	·0056	2·18	1·45
·75	Do.	·4316	·00518	...	1·0130	·0130	2·50	1·67
·50	Do.	·4359	·0100	...	1·0228	·0228	2·28	1·52

The value assumed for m is constant as long as the contraction is complete, and it then increases according to the rules of art. 3.

There is a certain margin within which m may vary. The following statement shows the values of n, calculated as above and corresponding to different values of M, for all the five weirs used by Bazin and for four different heads:—

Height of Weir.	Head.	M as observed.	Three assumed sets of values for m, and for each the corresponding value of n.					
(1)	(2)	(3)	(4)		(5)		(6)	
Feet.	Feet.		m	n	m	n	m	n
3·72	·49	4284	4250	1·45	4270	·87	4284	...
2·46	Do.	4316	Do.	1·67	Do.	1·36	Do.	1
1·64	Do.	4359	Do.	1·52	Do.	1·37	Do.	1·14
1·15	Do.	4424	4273	1·29	4283	1·19	4297	1·08
·79	Do.	4522	4303	·89	4313	·86	4327	·84
Mean.	Do.	..	...	1·36	...	1·12	...	·86
3·72	1·31	4286	4185	1·28	4200	1·09	4286	...
2·46	Do.	4430	4207	1·42	4221	1·30	4308	·85
1·64	Do.	4585	4245	1·20	4280	1·15	4346	·96
1·15	Do.	4794	4305	1·04	4320	1·02	4406	·87
·79	Do.	5034	4395	·86	4410	·85	4500	·75
Mean.	Do.	...	...	1·16	...	1·08	...	·67
3·31 [1]	1·44	4310	4167	1·32	4200	1·02	4214	1
2·46	Do.	4452	4178	1·61	4233	1·28	4275	1·07
Mean.	Do.	...	...	1·47	...	1·15	...	1·04
3·31 [2]	1·80	4334	4100	1·51	4190	·98	4211	·89

[1] Length of weir reduced to 3·28 feet.
[2] Length of weir reduced to 1·64 feet.

It will be noticed that slight changes in m cause great changes in n. Obviously m cannot rise to the values shown in column 6, as it would then equal M for the highest weirs. If reduced much below the value of column 4 it would make n very high. The values of m and n which seem most suitable are those of column 5, the mean value of n being 1·1.

APPENDIX C—FORMULÆ

Flow in Pipes (chap. v. art. 11).—The formula for flow in pipes is sometimes put in the form $\frac{h}{L}=\frac{V^2}{2g}\cdot\frac{f}{D}$ In this h is the head lost in the length L, and f is a 'friction factor' which is equal to $\frac{8g}{C^2}$.

It is not the same as the f in equation 13, p. 21. Neither is it a 'co-efficient of friction' which depends only on the roughness of the surface and the velocity of the water relatively to it. It is a variable factor which increases as C decreases. When f is ·020 C is 113, and when f is ·035 C is 86.

Flow in Open Channels (chap. vi. art. **11**).—Houk states that at first glance Barnes' formulæ seem to agree well with experiments, but that the observations chosen are hardly representative of the available data and that, of the particular series chosen, only selected measurements were included in the comparison. These contain 'such gaugings as Dubuat's' and some in which S was determined by aneroid barometer.

APPENDIX D—Variable Flow

(Chap. vii. art. 5)

The Ganges Canal had falls like that shown in Fig. 125 (p. 250). Scour occurred upstream of the falls, and weirs were built on the crests. In the *Encyclopædia Britannica* (art. Hydromechanics) it is implied that the construction of a weir on the crest of the fall would necessarily give a curved surface upstream. If built to the correct height it would give the straight line BC.

APPENDIX E—Unsteady Flow

(Chap. ix. art. 1.) Let the water from a tank be discharged over a weir. When the water level oscillates—as when there are waves —the discharge over the weir is slightly greater than that given by the mean head.

(Chap. ix. art. 5, foot-note to page 332.) The bridge had three spans of about 20 feet each. When the water in the centre bay rose—the rise was about 6 inches—that in the side bays fell, the fall being some 3 inches. Twenty feet upstream and downstream of the bridge no oscillation was perceptible from the bank. The piers were of brickwork with acute angles at both ends, wing walls curved. The whole period of oscillation was about twenty seconds. The water was perhaps 6 feet deep. Possibly a small fallen tree was submerged in the centre bay, and its branches, pressed down by the stream, sprung back at intervals, but there was no surface disturbance.

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HYDRAULICS IN GENERAL

OBSERVATIONS

OPEN CHANNELS

[1] For recent authors see under their respective names.

OPEN CHANNELS—*continued*

ORIFICES

PIPES

PIPES—*continued*

WEIRS

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